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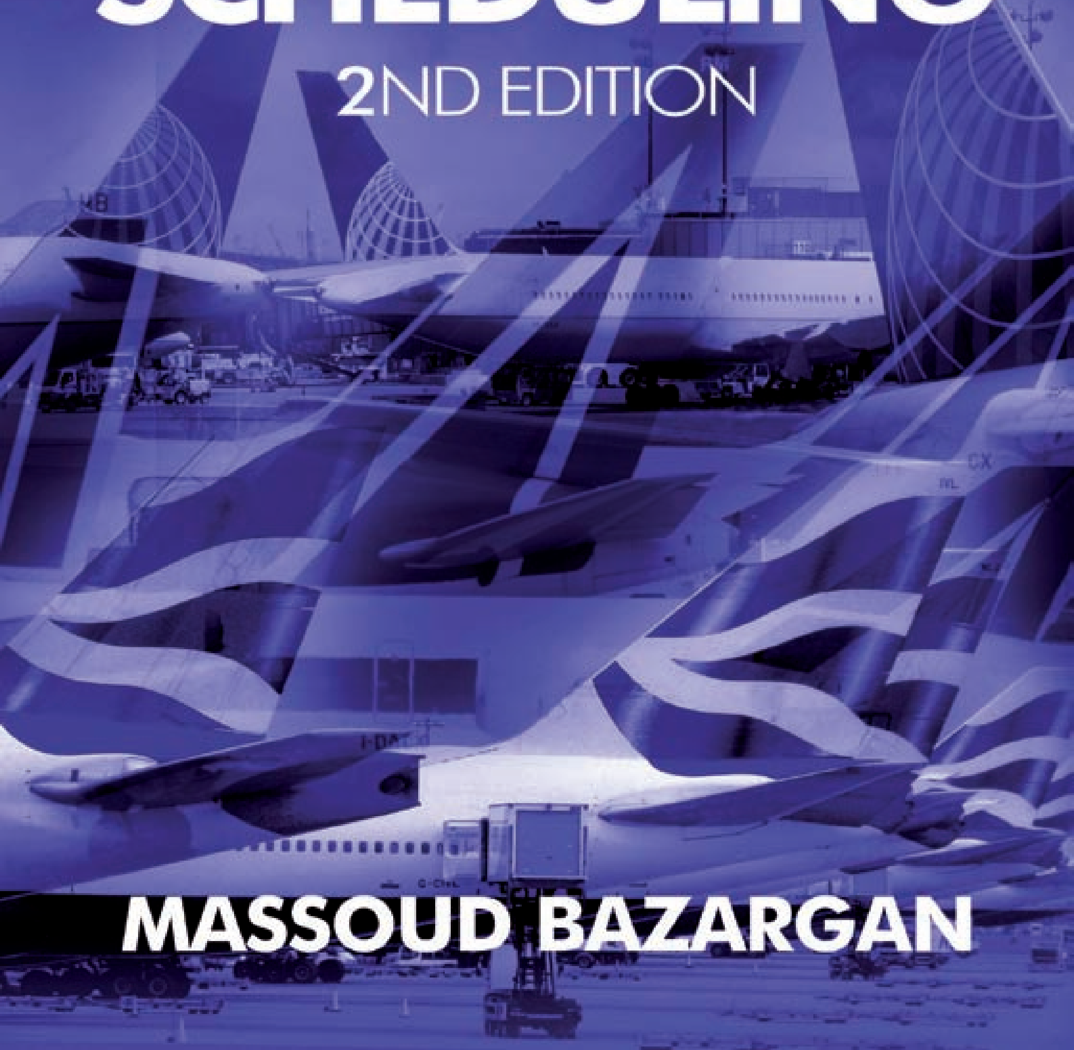
AIRLINE OPERATIONS AND SCHEDULING

2ND EDITION



MASSOUD BAZARGAN

AIRLINE OPERATIONS AND SCHEDULING

Dedicated to my wonderful family,
Soheila, Sina, Shiva, and Sarah
and to the memory of my Mother

Airline Operations and Scheduling
Second Edition

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ASHGATE

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Preface to Second Edition

The airline industry has evolved and gone through many challenges since the first edition of this book in 2004. It was felt that the time was right for a new edition of the book addressing these challenges. Four new chapters have been added, and the chapters in the first edition have been revised. The new chapters present real-world applications and projects that the author and his MBA students conducted for airlines and airports.

The book is divided into three major parts: planning, operation and dispatch optimization, and case studies. Two of the new chapters are in the area of operations and dispatch relating to fuel management systems and aircraft boarding strategy. A major challenge for the airlines is the significant rise in jet fuel price since the first edition was published. The chapter on fuel management systems explains how airlines purchase fuel and how they try to reduce cost by adopting fuel ferrying (tankering). The chapter on aircraft boarding strategy presents an interesting application of operations research to minimize delays in boarding passengers on to the aircraft.

The other two chapters are introduced in the case studies category. These cases relate to recent projects for airlines and airports. These two chapters are on aircraft tow tugs and airport runway-capacity planning. In both case studies, simulation modeling is utilized to identify economic and operational justification for purchasing aircraft tow tugs and to examine how capacity can be increased by changing airport runway layout configurations respectively.

The chapters in the first edition have been revised for typo errors and include more updated and recent references. In particular, Chapter 11 on gate assignment is revised to accommodate baggage handling in the mathematical model. Chapter 13 on computational complexity and heuristics has a new section about software vendors who develop solution suites for the airline industry.

The first edition of this book was published in 2004 as a result of developing an MBA course on Airline Planning and Operations in the College of Business at Embry-Riddle Aeronautical University. The course was initiated based on feedback received from alumni, mostly working at airlines, as well as students undertaking the author's operations research and operations management classes. The feedback indicated that a follow-up course, specifically focused on airline scheduling based on optimization methodologies, would be very appealing to them and to the aviation audience. The idea of developing such a course was additionally encouraged by the college's airline industry advisers. The development of the course was long and time-consuming. Owing to its unique nature, there were limited suitable texts, and related materials are very technical, thus beyond the scope of an MBA class. Some of the motivations for the first edition include:

- Introducing the importance and complexity of planning and operations at the airlines.
- Operations research techniques are extremely important tools for planning the operations in airlines. There are a large number of technical papers on airline optimization models. However, this literature is very advanced and therefore of interest only to a limited audience. This book attempts to fill this gap by simplifying the models and applying them to relatively simple examples, thus exposing them to a larger audience.
- There has been a growing concern among the operations research community that the materials offered in operations research courses at MBA or senior undergraduate business classes are too abstract, outdated, and at times irrelevant to today's fast and dynamic world. The book seeks to provide alternative and hopefully relevant materials for such courses.

Intended Audience

This book is intended to serve both as a textbook and as supporting material for graduate and undergraduate business, management, transportation, and engineering students. Currently, the airlines spend a long time training and acquainting new recruits with the planning and scheduling processes of various operations. This book can serve as an additional resource for such training. Other aviation audiences such as general aviation, flight schools, International Air Transport Association (IATA) and International Civil Aviation Organization (ICAO) training-course instructors, executive-jet and chartered-flight operators, air-cargo and package-delivery companies, and airline consultants may find the material in this book relevant and useful.

Required Background

The main background requirement on the part of the reader for a major portion of this book is basic familiarity with linear and integer programming. Linear and integer programming topics are widely covered in many disciplines at colleges and universities at different levels. Chapters 4 and 8 require some basic understanding of statistics in general and normal distribution in particular.

Adopting this Book as a Text

The author has offered the contents of this book in an MBA course as follows:

The students are grouped into teams, three students per team, each team representing operation managers of an airline company. As the course progresses, the teams are responsible for creating their own airlines, selecting routes, flight networks, fleet diversity, aircraft routings, maintenance locations, hub and spoke systems, air and ground crew scheduling, and gate assignments. The students need to conduct thorough research on passenger demand on city pairs, fleet cost, crew cost, determine ASM, CASM, RASM, yield, and so on, for their airlines. The teams should address how to determine their fares (revenue management) and how they accommodate unexpected interruptions in their flight schedule (irregular operations). If the teams are familiar with simulation software such as Arena (www.arenasimulation.com) then they enjoy simulating the operation of each airport within their network to assess the smooth operations such as adequate numbers of check-in counters, availability of gates, baggage handlers, and so on. The teams make a final presentation of their airlines and submit a comprehensive report detailing these operations.

Acknowledgements

I was very lucky to be constantly helped, supported, and encouraged by so many people throughout the writing of this book. I would like to thank my patient and hard-working assistants and friends Manolo Centeno, Rohan Dudley, Omar Haddadin, Lionel Charles, Henry Kosalim, and my daughter Shiva Bazargan. I would like to sincerely thank James Buckalew, Oscar Garcia, Tom Reich, Michael Gialouris, Glenn Martin, and Scott Wargo, for helping me with the industry side of the airline operations. I would like to thank Candas Ozdogu, Mauricio Angel, Deniz Saka, Pavel Hosa, Werner Leidenfrost, John Owens, Michelle Williams, Baohong Jiang, Prakash Subramanian, Mark Talaga, Yen-Ping Wu, Juan Ruiz, Victor Cole, Stefan Staschinski, Yan Li, and Shaun Londono for their help. I am also grateful to Dr. Yu at the University of Texas, Austin, and my colleagues at Embry-Riddle Aeronautical University, especially Drs. Petree, Raghavan, and Reynolds for their support.

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Massoud Bazargan

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Chapter 1
Introduction

Introduction

The United States Airline Deregulation Act of 1978 paved the way for major structural changes in the US airline industry. Airlines were allowed to select their network as well as their fares. This prompted a rush of new startup airlines to the market. After deregulation, the competition was not only between the pre-deregulation airlines, but also from the new entrants. Airlines were no longer protected, and if they wanted to be profitable, they had to manage their operations more efficiently.

Airlines use numerous resources to provide transportation services for their passengers. It is the planning and efficient management of these resources that determines the survival or demise of an airline. The airline industry is an excellent example of the 'survival of the fittest concept.' Table 1.1 shows the number of certificated airlines from 1976–2007 in the United States. The table also presents the number of airlines that were closed or merged with other airlines, and the number of newly established airlines. As the table implies, the airline industry operates in a very dynamic and uncertain environment. Furthermore, low flexibility to respond to changes, tightly coupled resources and limiting FAA regulations make the airline industry a complex environment (Yu 1998). To handle the complexity, robust and efficient planning tools and techniques are required. Operations research tools and techniques have played an important role in handling such complexities.

Operations Research and Airlines

Airlines have been using operations research techniques since the 1950s (Barnhart and Talluri 1997). Operations research models have had a tremendous impact on planning and managing operations within the airlines. The advances in computer technology and optimization models have enabled airlines to tackle more complex problems and solve them in a much shorter span of time. The vast contribution of these models has led to the establishment of operations research departments in many airlines, which help save millions of dollars. These departments have helped create an important professional society within the field of operations research, the Airline Group of the International Federation of Operational Research Societies (AGIFORS). AGIFORS is a professional society that seeks to advance, promote, and apply operations research within the airline industry (see www.agifors.org). A brief look at their website shows that Operations Research techniques have been

Table 1.1 Number of US certificated (DOT) airlines in the years 1976–2007

Year	Total Number of U.S. Airlines	Closed or Merged	Newly Established
2007	80	2	16
2006	66	11	10
2005	67	18	16
2004	69	14	18
2003	65	18	11
2002	72	3	12
2001	63	10	2
2000	71	13	9
1999	75	6	6
1998	75	12	8
1997	79	13	4
1996	88	6	9
1995	85	7	16
1994	76	8	14
1993	70	3	11
1992	62	4	12
1991	54	9	7
1990	56	7	4
1989	59	7	3
1988	63	4	5
1987	62	16	6
1986	72	15	17
1985	70	16	13
1984	73	16	21
1983	68	10	14
1982	64	28	14
1981	78	1	13
1980	66	1	13
1979	54	0	17
1978	37	10	5
1977	42	2	5
1976	39	4	1

Source: Bureau of Transportation Statistics.

successfully applied to many diverse problems such as revenue management, crew scheduling, aircraft routing, fleet planning, maintenance, and so on, within the airline industry. Barnhart (2008) discusses the accomplishment, opportunities and challenges of Operations Research in airline scheduling.

Outline of this Book

This book explores a variety of optimization models adopted by the airlines for scheduling and planning. The chapters discussing these models start with an example and then explain the process of developing a mathematical model. At the end of the chapter the general mathematical model is presented. The contents of this book are divided into three parts as follows:

Part 1 – Planning Optimization

- Chapter 2 – Network Flows and Integer Programming Models: This chapter is intended as a review of the basic concepts in network flows and integer programming models. These models are adopted later on in the following chapters.
- Chapter 3 – Flight Scheduling: Construction of flight schedules is the starting point for all other airline optimization problems. This chapter discusses the construction of flight schedules for a fictitious airline. This schedule is then used in the following chapters to address fleet assignment, aircraft routing, crew scheduling, and manpower planning.
- Chapter 4 – Fleet Assignment: Airlines typically operate a number of different aircraft, each having different characteristics, seating capacity, landing weights, and crew and fuel costs. This chapter introduces the basic fleet assignment model and its application to the fictitious airline.
- Chapter 5 – Aircraft Routing: This chapter presents the process of assigning individual aircraft to fly each flight segment assigned to the fleet. The chapter discusses mathematical models and their applications to the fictitious airline.
- Chapter 6 – Crew Scheduling: This chapter discusses the process of assigning crew to flight segments in two phases. First, crew pairing is introduced to determine which flight segments should be paired. The second phase, crew rostering, discusses how these pairings are assigned to the crew incorporating various rules and regulations.
- Chapter 7 – Manpower Planning: This chapter discusses manpower planning for ground crew through the fictitious airline case.

Part 2 – Operations and Dispatch Optimization

- Chapter 8 – Revenue Management: This chapter introduces revenue management, probabilistic models, and case studies.
- Chapter 9 – Fuel Management Systems: This chapter introduces jet fuel cost, hedging strategies, case study, and a mathematical model for fuel tankering.
- Chapter 10 – Airline Irregular Operations: When faced with a lack of resources and/or disruptions caused by various internal and external factors, airlines often are not able to fly their published flight schedule. This chapter provides an introduction to irregular operations, delays, cancellations, a mathematical model for irregular operations, and a case study.

- Chapter 11 – Gate Assignment: This chapter introduces the gate assignment mathematical model through a case study.
- Chapter 12 – Aircraft Boarding Strategy: This chapter explores various aircraft boarding strategies adopted by the airlines. It introduces a mathematical approach for an efficient aircraft boarding strategy applied to an Airbus A-320.

Part 3 – Computation Complexity and Simulation

- Chapter 13 – Computational Complexity, Heuristics, and Software: This chapter discusses inherent computational complexity with the airline problems and how heuristics are implanted to solve large scale problems. It also highlights some of the software vendors who provide solution suites for different airline problems.
- Chapters 14–18: These chapters introduce case studies on a start-up airline, and simulation modeling for airlines and airports. Simulation studies have become an alternative and/or integrated part of mathematical models when faced with complex problems.
- Appendix: provides the full name of the airports presented as their three/four letter codes in this book.

Software

Throughout this book references are made to software for solving linear/integer program models. Many of these models can be solved using student/trial versions of optimization software, which are typically available at colleges, universities, and airlines. There are many software vendors who provide these student/trial versions free to download on their websites (see, for example, www.lindo.com or www.maximal-usa.com). For larger problems, which exceed the student/trial version limits, we used full version of MPL software (www.maximal-usa.com) with CPLEX solver (www.ilog.com).

References

Barnhart, C. and Talluri, K.T. (1997). Airline operations research in design and operation of civil and environmental engineering system, in C. Revelle and A. McGarity. Wiley, 435–69.

Barnhart, C. (ed.). (2008). Proceedings from *CPAIOR '08: Integration of AI and OR Techniques in Constraint Programming for Combinatorial Optimization Problems* – 5th International Conference.

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PART I
Planning Optimization

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Chapter 2

Network Flows and Integer Programming Models

Introduction

A large part of the problems that airlines face can be translated into network and integer programming models. These models are mentioned and used throughout this book. This chapter attempts to provide a review of some of the optimization models discussed in this book. It should be noted that these topics only represent a small selection of models from the vast area of network and integer programming techniques. For a complete discussion of various network models, interested readers are referred to the list of books referenced in this chapter.

Networks

A network (also referred to as a graph) is defined as a collection of points and lines joining these points. There is normally some flow along these lines, going from one point to another. Figure 2.1 represents a network.

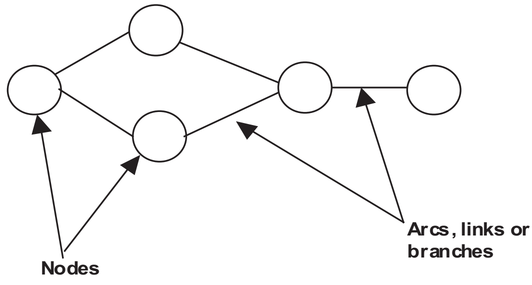

Figure 2.1 Basic elements of a network

Network Terminology

Before explaining the models, some terminologies commonly used in network study are described.

Nodes and Arcs: In a network, the points (circles) are called nodes and the lines are referred to as arcs, links or arrows (see Figure 2.1).

Flow: The amount of goods, vehicles, flights, passengers and so on that move from one node to another (see Figure 2.2).

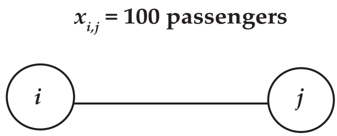

Figure 2.2 Flow between two nodes

Directed Arc: If the flow through an arc is allowed only in one direction, then the arc is said to be a directed arc. Directed arcs are graphically represented with arrows in the direction of the flow (see Figure 2.3).

Figure 2.3 Directed flow

Undirected Arc: When the flow on an arc (between two nodes) can move in either direction, it is called an undirected arc. Undirected arcs are graphically represented by a single line (without arrows) connecting the two nodes (see Figure 2.4).

Figure 2.4 Undirected flow

Arc Capacity: The maximum amount of flow that can be sent through an arc. Examples include restrictions on the number of flights between two cities.

Supply Nodes: Nodes with the amount of flow coming to them greater than the amount of flow leaving them – or nodes with positive net flow (see Figure 2.5).

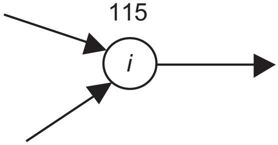

Figure 2.5 Supply node

Demand Nodes: Nodes with negative net flow or outflow greater than inflow (see Figure 2.6).

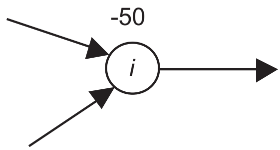

Figure 2.6 Demand node

Transshipment Nodes: Nodes with the same amount of flow arriving and leaving – or nodes with zero net flow (see Figure 2.7).

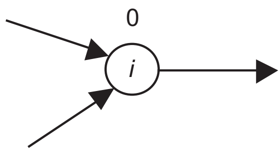

Figure 2.7 Transshipment node

Path: Sometimes two nodes are not connected by an arc, but could be connected by a sequence of arcs (see Figure 2.8). A path is a sequence of distinct arcs that connect two nodes in this fashion. Airliners utilize hubs to provide connections between city pairs in their network.

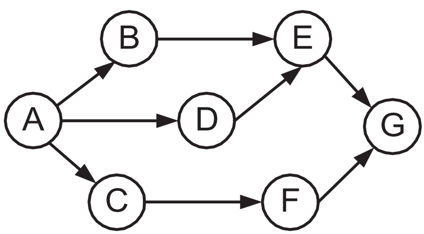

Figure 2.8 A network showing three paths from A to G

Source: Starting node in the path.

Destination: Last node in the path.

Cycle: A sequence of directed arcs that begins and ends at the same node (see Figure 2.9). Examples include aircraft that start from an airport which is a maintenance base and, after flying to several destinations, end up at the same airport from which they departed.

Connected Network: A network in which every two nodes are linked by at least one path (see Figure 2.10).

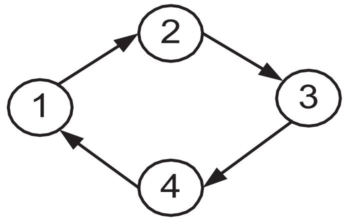

Figure 2.9 A cycle

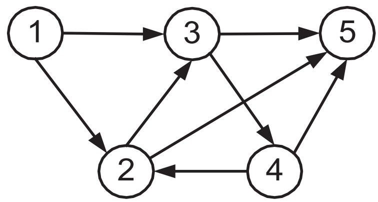

Figure 2.10 Connected network

Network Flow Models

In this section, select network models that are used in this book are discussed. It is assumed that the reader is familiar with basic linear and integer programming.

Shortest Path (Route) Problem

This problem attempts to identify a path, from source to destination, within the network, that results in minimum transport time/cost. This particular problem should be especially attractive to cargo handlers and origin/destination scenarios (see Figure 2.11). The problem consists of a connected network with known costs for each arc in the network. The objective is to identify the path with the minimum cost between two desired nodes.

Example

Consider the following network shown in Figure 2.11 (adapted from Winston and Albright 2001). The nodes represent the cities, and the arcs are the flights. The numbers on the arcs represent the flight time in minutes between the city pairs. We want to determine the best route that results in the shortest flying time from node 1 (source) to node 10 (destination).

We assume the following binary (0–1) decision variable:

$$x_{i,j} = \begin{cases} 1 \text{ if arc } (i,j) \text{ is part of the solution} \\ 0 \text{ otherwise} \end{cases}$$

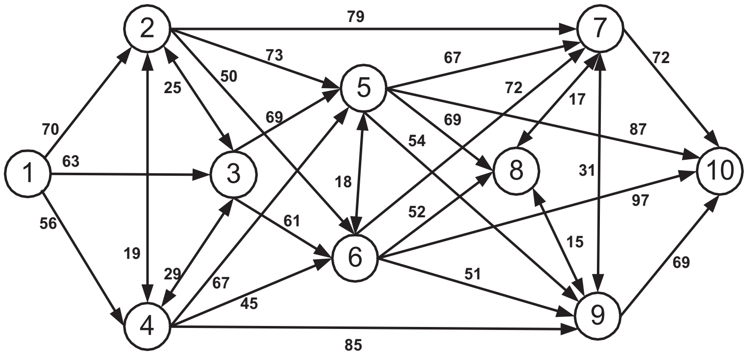

Figure 2.11 Network with flight times between city pairs

Then the objective function is to minimize the total flying cost (time) as follows:

$$\textit{Minimize } 70x_{1,2} + 63x_{1,3} + 56x_{1,4} + ...$$

We have three sets of constraints as follows:

Source node: The flow must originate from node 1. To make sure that the flow (in this case our starting flight) leaves the source we must have:

$$x_{1,2} + x_{1,3} + x_{1,4} = 1$$

Transshipment nodes: Every other node (except source and destination) is a transshipment node. That is the net flow in these nodes should be zero. As an example, node (2) in Figure 2.11 is a transshipment node. To address the constraint for this node we write:

$$x_{1,2} + x_{4,2} + x_{3,2} - x_{2,3} - x_{2,4} - x_{2,5} - x_{2,6} - x_{2,7} = 0$$

Similarly we write constraints for the other seven transshipment nodes.

Destination node: The flow must end up at the destination node (node 10). Therefore:

$$x_{5,10} + x_{6,10} + x_{7,10} + x_{9,10} = 1$$

Solving this problem using software, we find that the minimum cost is 198 minutes (56 + 45 + 97). The solution (route) for this example is presented in Figure 2.12.

The general mathematical model for the Shortest Path Problem (SPP) is represented by a binary (0–1) integer programming as follows:

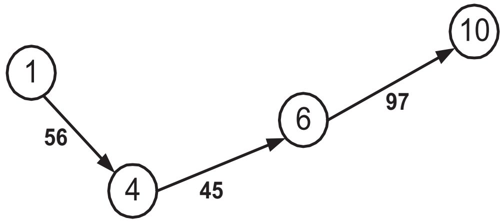

Figure 2.12 Graphical solution for the Shortest Path Problem

Sets

M = Set of nodes

Index

i,j,k = Index for nodes

Parameters

$c_{i,j}$ = Cost of flow along the arc joining node i to node j
m = Destination node

Decision Variable

$$x_{i,j} = \begin{cases} 1 \text{ if arc } (i,j) \text{ is part of the path} \\ 0 \text{ otherwise} \end{cases}$$

Objective Function

$$\text{Minimize} \sum_{i \in M} \sum_{j \in M} c_{i,j} \; x_{i,j} \tag{2.1}$$

Subject to

$$\sum_{j \in M} x_{1,j} = 1 \quad j \neq 1 \tag{2.2}$$

$$\sum_{j \in M} x_{i,j} - \sum_{k \in M} x_{k,i} = 0 \qquad \text{For all } (\forall)\ i,\ i \neq 1 \text{ and } i \neq m \tag{2.3}$$

$$\sum_{i \in M} x_{i,m} = 1 \tag{2.4}$$

The objective function (2.1) attempts to minimize the total cost. Constraint (2.2) ensures that the flow is shipped from the source (supply) node. The set of constraints (2.3) impose that all other nodes (except the source and the destination node) are transshipment nodes. Finally, constraints (2.4) ensure that the flow is received at the destination (demand) node.

Minimum Cost Flow Problem

The minimum cost flow network problem seeks to satisfy the requirements of nodes at minimum cost. This is a generalized form of transportation, transshipment, and shortest path problems. This problem assumes that we know the cost per unit of flow and capacities associated with each arc.

Example

Consider the following network presented in Figure 2.13 (adapted from Anderson et al. 2003). An airline is tasked with transporting goods from nodes 1 and 2 to nodes 5, 6 and 7 (see Figure 2.13). The airline does not have direct flights from the source nodes to the destination nodes. Instead, they are connected through its hubs in nodes 3 and 4. The numbers next to the nodes represent the demand/supply in tons. The numbers on the arcs represent the unit cost of transportation per ton. We want to determine the best way to transport the goods from sources to destinations so that the total cost is minimized. The aircraft flying to and from node 4 can carry a maximum of 50 tons of cargo.

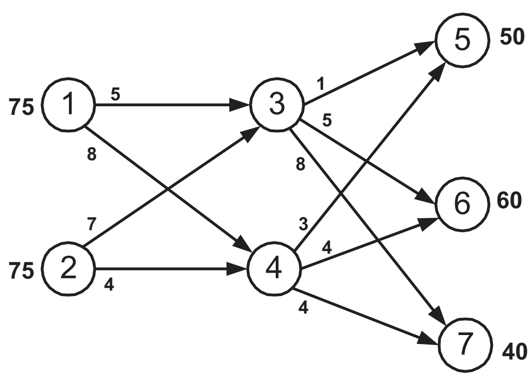

Figure 2.13 Network presentation for minimum cost flow

To formulate this problem, consider the following decision variable:

$x_{i,j}$ = Amount of flow from node i to node j

The objective function is then:

$$\textit{Minimize} \quad 5x_{1,3} + 8x_{1,4} + 7x_{2,3} + \ldots$$

We need to write one constraint for each node. For example, for node 1 we have:

$$x_{1,3} + x_{1,4} \leq 75$$

Similarly, we write constraints for the other six nodes. Note that the net flow for nodes 3 and 4 should be zero as these are transshipment nodes.

All the flights to and from node 4 can carry a maximum of 50 tons. Therefore, all the flow to and from this node must be limited to 50 as follows:

$$x_{1,4} \leq 50$$

$$x_{2,4} \leq 50$$

$$x_{4,5} \leq 50$$

$$x_{4,6} \leq 50$$

$$x_{4,7} \leq 50$$

Solving this problem using software generates a total minimum cost of $1,250. The solution for this problem is presented in Figure 2.14.

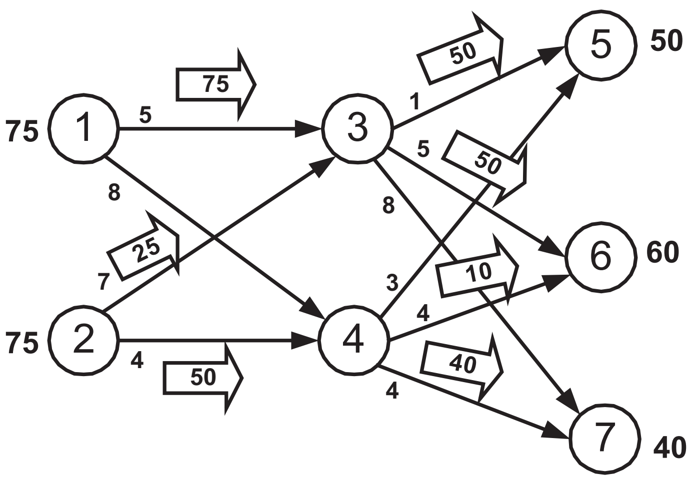

Figure 2.14 Solution to minimum cost flow

The general model is mathematically expressed as follows (Bazaraa et al. 1990).

Sets

M = Set of nodes

Index

i,j,k = Index for nodes

Parameters

$c_{i,j}$ = Unit cost of flow from node i to the node j
b_i = Amount of supply/demand for node i.
$L_{i,j}$ = Lower bound on flow through arc (i,j)
$U_{i,j}$ = Upper bound on flow through arc (i,j)

Decision Variable

$x_{i,j}$ = Amount of flow from node i to node j

Objective Function

$$Minimize \sum_{i \in M} \sum_{j \in M} c_{i,j} x_{i,j} \tag{2.5}$$

Subject to

$$\sum_{j \in M} x_{i,j} - \sum_{k \in M} x_{k,i} = b_i \qquad \forall\ i = 1, 2, .., M \tag{2.6}$$

$$L_{i,j} \le x_{i,j} \le U_{i,j} \tag{2.7}$$

The objective function (2.5) attempts to minimize the total cost of the network. Constraints (2.6) satisfy the requirements of each node by determining the amount of inflow and outflow from that node. The set of constraints (2.7) impose the lower and upper-bound restrictions along the arcs.

Maximum Flow Problem

The Maximum Flow problem is a special case of the Minimum Cost flow problem. It attempts to find the maximum amount of flow that can be sent from one node (source node) to another (destination node) when the network is capacitated, that is, the arcs in the network have a capacity restriction.

Example

This example is adapted from Winston and Venkataramanan (2003). An airline must determine the number of daily connecting flights that can be arranged between Daytona Beach (DAB), Florida, and Lafayette (LAF), Indiana. Connecting flights must stop in Atlanta (ATL), Georgia, and then make one more stop in either Chicago (ORD), Illinois, or Detroit (DTW), Michigan. Owing to its current policies with these airports, the airline has a maximum number of daily flights which it can operate between the city pairs shown in Table 2.1.

Table 2.1 Maximum number of flights per city-pair for Shuttle Hopper Airways

City-Pairs	Maximum number of daily flights
DAB - ATL	3
ATL - ORD	2
ATL - DTW	3
ORD - LAF	1
DTW - LAF	2

The airline wants to determine how to maximize the number of connecting flights daily from Daytona Beach, FL, to Lafayette, IN, respecting the current restrictions.

The following network represents this problem with arcs showing maximum daily flights along the city pairs.

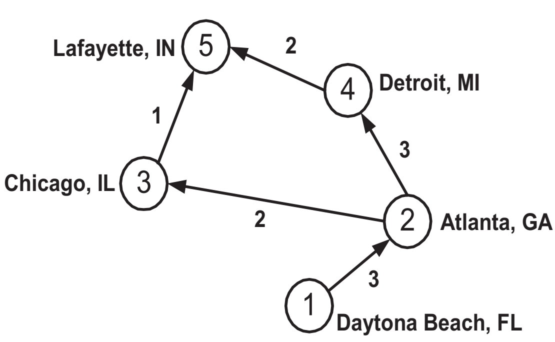

Figure 2.15 Network presentation from source to destination

To formulate the problem, let us assume the following decision variables:

$x_{i,j}$ = Number of flights (integer) from node *i* to node *j*
f = Number of daily flights from DAB to LAF

In this problem, the objective is to maximize the daily flights between DAB and LAF. Therefore:

Maximise f

Similar to the Shortest Path Problem, we have a set of constraints for source, transshipment and destination nodes:

Source node: DAB is our source node. *f* is the total flow leaving DAB, therefore:

$$x_{1,2} = f$$

Transshipment nodes: We write one constraint for each transshipment node. For example, for node 2 (ATL) we have:

$$x_{1,2} - x_{2,3} - x_{2,4} = 0$$

Similarly we write transshipment constraints for other nodes 3 and 4.

Destination node: The same number of daily flights *f* departing from DAB should now arrive at destination node LAF.

$$x_{4,5} + x_{3,5} = f$$

Arc capacity: The last set of constraints address the capacity of arcs as follows:

$$x_{1,2} \le 3$$

$$x_{2,3} \le 2$$

$$x_{2,4} \le 3$$

$$x_{3,5} \le 1$$

$$x_{4,5} \le 2$$

Solving this problem generates a maximum flow of three daily flights between DAB and LAF as follows:

- 1 flight assigned to the DAB-ATL-ORD-LAF route, and;
- 2 flights assigned to the DAB-ATL-DTW-LAF route.

The general model is mathematically expressed as follows (Ahuja et al. 1993):

Sets

M = Set of nodes

Index

i,j,k = Index for nodes

Parameters

$L_{i,j}$ = Lower bound on flow through arc (i,j)
$U_{i,j}$ = Upper bound on flow through arc (i,j)
m = Destination node

Decision Variables:

$x_{i,j}$ = Amount of flow from node i to node j
f = Amount of flow from source node to destination node

Objective Function

$$\text{Maximize } f \tag{2.8}$$

Subject to

$$\sum_{j \in M} x_{1,j} = f \quad \Leftrightarrow \text{ Origin Node} \tag{2.9}$$

$$\sum_{i \in M} x_{i,j} - \sum_{k \in M} x_{j,k} = 0 \quad \Leftrightarrow \text{ Transshipment nodes} \tag{2.10}$$

$$\sum_{i \in M} x_{i,m} = f \quad \Leftrightarrow \text{ Destination node} \tag{2.11}$$

$$L_{i,j} \le x_{i,j} \le U_{i,j} \tag{2.12}$$

The objective function (2.8) attempts to maximize flow from the source node (node 1) to the destination node (node m). The set of constraints (2.9) and (2.11) impose the outflow and inflow restrictions on the source and destination nodes. All other nodes are transshipment nodes. The set of constraints (2.10) imposes this restriction. Finally, constraints (2.12) restrict the flow along the arcs based on the imposed capacity.

Multi-Commodity Problem

All the network models explained so far assume that a single commodity or type of entity is sent through a network. Sometimes a network can transport different types of commodities. The multi-commodity problem seeks to minimize the total cost when different types of goods are sent through the same network. The commodities may either be differentiated by their physical characteristics, or simply by certain attributes. The multi-commodity problem is extensively used in transportation industry. In the airline industry, the multi-commodity model is adopted to formulate crew pairing and fleet assignment models.

Example

We modify the example that was presented for the Minimum Cost Flow problem discussed earlier to address the multi-commodity model formulation. Figure 2.16 presents the modified example:

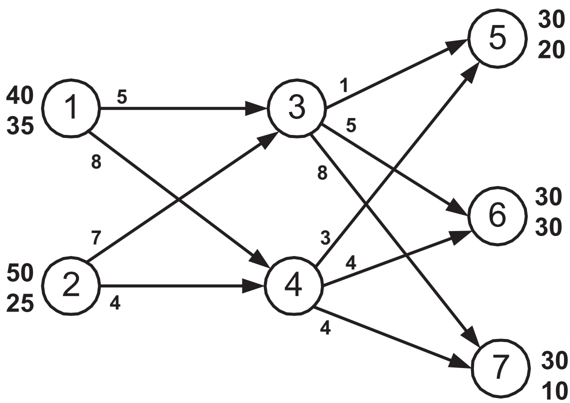

Figure 2.16 Network presentation for multi-commodity problem

As we see in this figure the scenario is very similar to the earlier case. The only difference is that instead of having only one type of cargo, in this case we have two types (two commodities). The numbers next to each node represent the supply/demand for each cargo at that node. As an example, node 1 supplies 40 and 35 tons of cargo 1 and 2 respectively. The transportation costs per ton are also similar. We want to determine how much from each cargo should be routed on each arc so that the total transportation cost is minimized.

To formulate this problem we assume the following decision variable:

$x_{i,j,k}$ = Amount of flow from node *i* to node *j* for commodity *k*

In this decision variable the indices *i* and *j* represent the nodes (*i,j* = 1,.,7) and *k* represents the type of commodity (*k* = 1,2).

The objective function is therefore:

$$Minimize \quad 5x_{1,3,1} + 5x_{1,3,2} + 8x_{1,4,1} + 8x_{1,4,2} + \ldots$$

We need to write one constraint for each node. For example, for node 1 we have:

$$x_{1,3,1} + x_{1,4,1} \leq 40$$

$$x_{1,3,2} + x_{1,4,2} \leq 35$$

We write similar constraint for the other six nodes.

Recall that all the flights to and from node 4 can carry a maximum of 50 tons. Therefore:

$$x_{1,4,1} + x_{1,4,2} \leq 50$$

$$x_{2,4,1} + x_{2,4,2} \leq 50$$

$$x_{4,5,1} + x_{4,5,2} \leq 50$$

$$x_{4,6,1} + x_{4,6,2} \leq 50$$

$$x_{4,7,1} + x_{4,7,2} \leq 50$$

Solving this problem using software generates a total minimum cost of $1,150. The solution for this problem is presented in Figure 2.17.

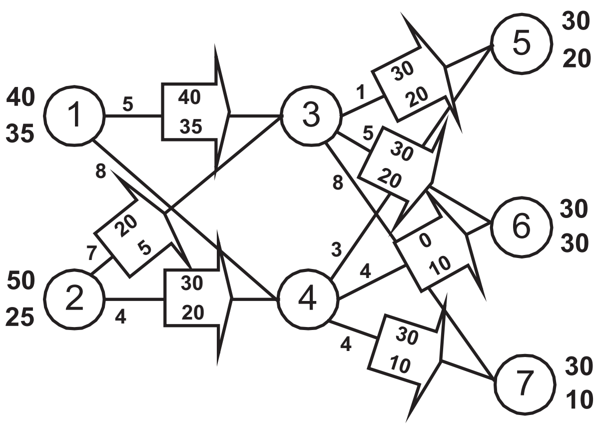

Figure 2.17 Solution to multi-commodity problem

The general model is mathematically expressed as follows (Ahuja et al. 1993):

Sets

M = Set of nodes
K = Set of commodities

Indices

i,j = Index for nodes
k = Index for commodities

Parameters

$c_{i,j,k}$ = Unit cost of flow from node i to node j for commodity k,
$b_{i,k}$ = Amount of supply/demand at node i for commodity k
$U_{i,j}$ = Flow capacity on arc (i,j)

Decision Variable

$x_{i,j,k}$ = Amount of flow from node i to node j for commodity k

Objective Function

$$Min \sum_{k \in K} \sum_{i \in M} \sum_{j \in M} c_{i,j,k} x_{i,j,k} \tag{2.13}$$

Subject to

$$\sum_{t \in M} x_{i,t,k} - \sum_{t \in M} x_{t,i,k} = b_{i,k} \qquad \text{For all } i \in M \text{ and } k \in K \tag{2.14}$$

$$\sum_{k \in K} x_{i,j,k} \le u_{i,j} \qquad \text{For all } i \in M \text{ and } \mathrm{j} \in M \tag{2.15}$$

In this model, the objective function (2.13) seeks to minimize the total network cost over all nodes and all commodities. The set of constraints (2.14) and (2.15) satisfies the supply/demand of the node and imposes capacity constraints on the arc.

Integer Programming Models

Integer programming models relate to certain types of linear programming in which all of the decision variables are required to be non-negative integers. The following represents a brief introduction to a small number of integer programming models adopted in the following chapters.

Set-Covering/Partitioning Problems

Set-covering problems relate to cases where each member of one set should be assigned/matched to member(s) of another set. Examples include the assignment of crew members to flights, aircraft to routes, and so on. The objective in a set-covering problem is to minimize the total cost of this assignment.

Example

The following is an example of set-covering adapted and modified from Winston and Venkataramanan (2003).

An airline wants to design its 'hub' system (hub-and-spoke systems are discussed in Chapter 3). Each hub will be used for connecting flights to and from cities within 1,000 miles of the hub. The airline wants to serve the following cities: Atlanta, Boston, Chicago, Denver, Houston, Los Angeles, New Orleans, New York, Pittsburgh, Salt Lake City, San Francisco, and Seattle. The airline wants to determine the smallest number of hubs it will need in order to cover all of these cities. By cover, we mean each city should be within 1,000 miles of at least one hub. Table 2.2 lists the distances between the cities.

Table 2.2 Distance-matrix between cities

		1	2	3	4	5	6	7	8	9	10	11	12
		A T	**BO**	**CH**	**DE**	**HO**	**LA**	**NO**	**NY**	**PI**	**SL**	**SF**	**SE**
1	**AT**	0	1037	674	1398	789	2182	479	841	687	1878	2496	2618
2	**BO**	1037	0	1005	1949	1804	2979	1507	222	574	2343	3095	2976
3	**CH**	674	1005	0	1008	1067	2054	912	802	452	1390	2142	2013
4	**DE**	1398	1949	1008	0	1019	1059	1273	1771	1411	504	1235	1307
5	**HO**	789	1804	1067	1019	0	1538	356	1608	1313	1438	1912	2274
6	**LA**	2182	2979	2054	1059	1538	0	1883	2786	2426	715	379	1131
7	**NO**	479	1507	912	1273	356	1883	0	1311	1070	1738	2249	2574
8	**NY**	841	222	802	1771	1608	2786	1311	0	368	2182	2934	2815
9	**PI**	687	574	452	1411	1313	2426	1070	368	0	1826	2578	2465
10	**SL**	1878	2343	1390	504	1438	715	1738	2182	1826	0	752	836
11	**SF**	2496	3095	2142	1235	1912	379	2249	2934	2578	752	0	808
12	**SE**	2618	2976	2013	1307	2274	1131	2574	2815	2465	836	808	0

We can now revise Table 2.2 above to identify which cities are covered by each hub. Simply replace all the distances in the above table by 1 if the distance is less than 1,000 miles (covered) and 0 otherwise. Table 2.3 presents the revised matrix.

To formulate this problem, we define the following binary decision variable:

$$x_j = \begin{cases} 1 \text{ if city } j \text{ (1,2,..,12) is selected as a hub} \\ 0 \text{ otherwise} \end{cases}$$

We want to minimize the number of hubs, therefore the objective function is:

$$\textit{Minimize } x_1 + x_2 + \ldots + x_{12}$$

Each city must be covered by at least one hub. Atlanta (Index 1), for example, is covered by cities 1, 3, 5, 7, 8, and 9 (see Table 2.3). Therefore, the constraint for Atlanta is:

$$x_1 + x_3 + x_5 + x_7 + x_8 + x_9 \geq 1 \text{ (Atlanta)}$$

Table 2.3 Binary-matrix showing cities covered by each hub

		1	2	3	4	5	6	7	8	9	10	11	12
		AT	**BO**	**CH**	**DE**	**HO**	**LA**	**NO**	**NY**	**PI**	**SL**	**SF**	**SE**
1	**AT**	1	0	1	0	1	0	1	1	1	0	0	0
2	**BO**	0	1	0	0	0	0	0	1	1	0	0	0
3	**CH**	1	0	1	0	0	0	1	1	1	0	0	0
4	**DE**	0	0	0	1	0	0	0	0	0	1	0	0
5	**HO**	1	0	0	0	1	0	1	0	0	0	0	0
6	**LA**	0	0	0	0	0	1	0	0	0	1	1	0
7	**NO**	1	0	1	0	1	0	1	0	0	0	0	0
8	**NY**	1	1	1	0	0	0	0	1	1	0	0	0
9	**PI**	1	1	1	0	0	0	0	1	1	0	0	0
10	**SL**	0	0	0	1	0	1	0	0	0	1	1	1
11	**SF**	0	0	0	0	0	1	0	0	0	1	1	1
12	**SE**	0	0	0	0	0	0	0	0	0	1	1	1

Note that we use the greater than or equal-to sign because a city can be covered by more than one hub. Similarly for Boston (Index 2), we have:

$$x_2 + x_8 + x_9 \geq 1 \text{ (Boston)}$$

Hence, we can write similar constraints for all the other 10 cities.

Solving this binary integer program using software generates three hubs as follows:

> *Atlanta* covers Chicago, Houston, New Orleans, New York, and Pittsburgh;
> *Pittsburgh* covers Atlanta, Chicago, Boston, and New York;
> *Salt Lake City* covers Denver, Los Angeles, San Francisco, and Seattle.

We see that some cities are covered by more than one hub. As an example, Chicago is covered by both Atlanta and Pittsburgh hubs.

In the case where we want to cover each city by exactly one hub, all the inequalities in the above model become equal signs. This special case where each member of one set is covered exactly once is called *set-partitioning*.

If we run the above program with this restriction, that is, changing all greater than or equal to signs with strictly equal to signs, we find that the minimum number of hubs to cover all cities exactly once is also three. The hubs are:

> *Boston* covers New York and Pittsburgh
> *New Orleans* covers Atlanta, Chicago, and Houston
> *Salt Lake City* covers Denver, Los Angeles, San Francisco, and Seattle

Therefore, as the name implies, set-partitioning attempts to make disjoint sets such that no member appears in two sets.

The general model for set-covering is as follows (Ignizio and Cavalier 1994):

Sets

M = Members of set 1
N = Members of set 2

Indices

i = Index for set 1
j = Index for set 2

Parameters

c_j = Cost associated with selecting member j

$$a_{i,j} = \begin{cases} 1 \text{ if member } j \text{ covers member } i \\ 0 \text{ otherwise} \end{cases}$$

Decision Variable

$$x_j = \begin{cases} 1 \text{ if member } j \text{ is selected} \\ 0 \text{ otherwise} \end{cases}$$

The integer binary programming model is as follows:

Objective Function

$$Min \sum_{j \in N} c_j x_j \qquad (2.16)$$

Subject to

$$\sum_{j \in N} a_{i,j} x_j \geq 1 \qquad \text{For all } i \in M \qquad (2.17)$$

In this model, the objective function (2.16) seeks to minimize the total covering cost. The set of constraints (2.17) imposes that each member of set 1 is covered by at least one member of set 2.

The set-partitioning formulation of the above problem is similar, except that (2.17) is now rewritten with a strictly equal to sign as follows:

$$\sum_{j \in N} a_{i,j} x_j = 1 \qquad \text{For all } i \in M$$

Throughout this book both set-covering and set-partitioning models are used extensively. In these models references are made to matrices. By a set-covering or set-partitioning matrix, we mean a matrix of $a_{i,j}$ parameters where the members of one set (index *i*) are represented by rows and members of the other set (index j) are represented by columns as shown below. In this matrix a value of 1 means that the specific member in set 1 is covered by the specific member in set 2. A value of 0 means that this coverage does not exist.

Members of set 2 indexed by *j*

$$\left\{ \begin{array}{l} \text{Members of} \\ \text{set 1 indexed by} \\ i \end{array} \right. \qquad \begin{pmatrix} 1 & 0 & \ldots\ldots\ldots\ldots \\ 0 & 0 & \ldots\ldots\ldots \\ \ldots\ldots\ldots & & \\ 1 & 1 & \ldots\ldots\ldots\ldots \end{pmatrix}$$

Traveling Salesman Problem

The Traveling Salesman problem is a classical problem in operations research, and has received considerable attention in the literature. It has vast applications in sequencing series of jobs or routes. The Traveling Salesman problem is as follows:

Starting from his hometown, a traveling salesman wants to visit a series of cities just once, and finally return to his hometown. The problem is to determine the best sequence for visiting these cities so that the total cost (total distance or total time traveled) is minimized.

Despite the simplicity of the problem's scope, the solution to this problem is very challenging and falls among one of the most computationally intensive combinatorial problems (discussed further in Chapter 13). To clarify this problem, consider the following example.

Example

A cargo airline based in Atlanta (ATL) wants to determine the sequence of flights to cities in its network such that the total distance flown in its cycle is minimized. A restriction to this operation is that the flight sequences must start and end in Atlanta. The cities in the airline's network, and their distances are presented in Table 2.4.

This case can be formulated as the Traveling Salesman problem. We define the following binary decision variable:

$$x_{i,j} = \begin{cases} 1 \text{ if city j should immediately follow city i} \\ 0 \text{ otherwise} \end{cases}$$

Table 2.4 Sequence of flights to cities in cargo airline network

		1	2	3	4	5	6	7
		ATL	**ORD**	**CVG**	**HOU**	**LAX**	**MON**	**JFK**
1	**ATL**	-	702	454	842	2396	1196	864
2	**ORD**		-	324	1093	2136	764	845
3	**CVG**			-	1137	2180	798	664
4	**HOU**				-	1616	1857	1706
5	**LAX**					-	2900	2844
6	**MON**						-	396
7	**JFK**							-

The objective function is therefore to minimize the total distances flown:

$$\textit{Minimize}\ 702x_{1,2} + 454x_{1,3} + \ldots$$

The first set of constraints is to make sure that each city is visited only once. For example, for Atlanta we have:

$$x_{1,1} + x_{2,1} + x_{3,1} + x_{4,1} + x_{5,1} + x_{6,1} + x_{7,1} = 1$$

We write similar constraints for the other six cities.

The second set of constraints must route the aircraft after visiting a city. Without these constraints, the aircraft will be stuck in one city. The constraint to route the aircraft after visiting Atlanta, for example, is as follows:

$$x_{1,1} + x_{1,2} + x_{1,3} + x_{1,4} + x_{1,5} + x_{1,6} + x_{1,7} = 1$$

We write similar constraints for the other six cities as well.

Solving this problem generates the following solutions:

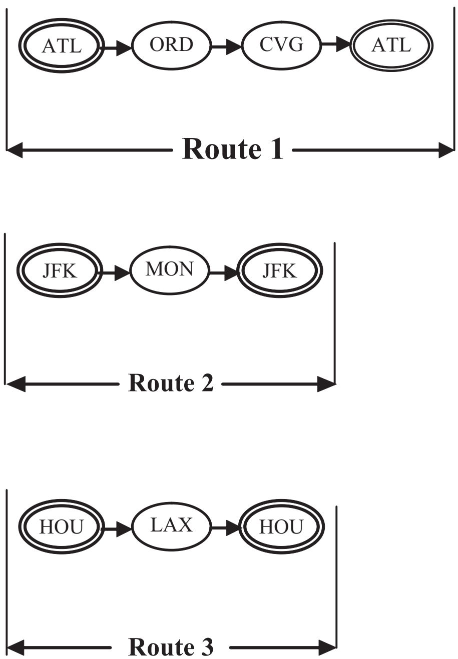

Figure 2.18 Solution showing three disjoint sequences or sub-tours

This solution shows three disjointed sequences. It does not offer the expected complete tour sequence among all of the seven cities. This is a common difficulty with the Traveling Salesman problem. Instead of one tour of all the cities, the solution generates sub-tours. To address this difficulty, the common approach is to prevent the formation of sub-tours. First, the problem is solved, and then we add additional constraints to break these sub-tours, if they are formed. As an example, we have the JFK-MON-JFK sub-tour in our solution. To break this sub-tour we add the following constraint in our model:

$$x_{6,7} + x_{7,6} \leq 1$$

We solve the problem once again, adding this new constraint. The following solution is generated:

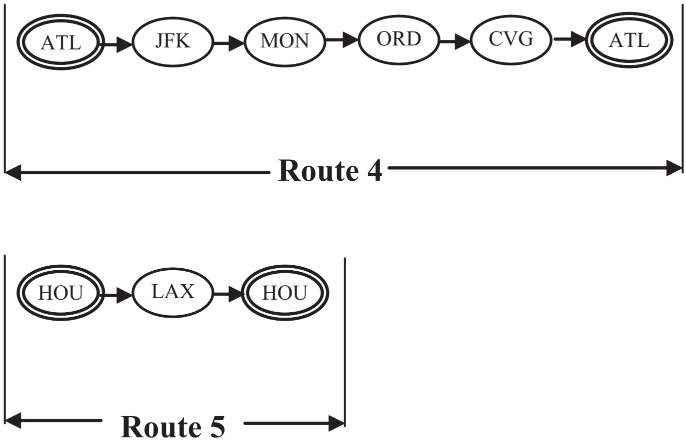

Figure 2.19 Solution showing two sub-tours after adding first breaking constraint

We now have two sub-tours. Hence, we add the following constraint to break the second sub-tour:

$$x_{5,4} + x_{4,5} \leq 1$$

Adding this constraint results in the following complete tour solution, presented in Table 2.5.

Table 2.5 Final tour sequence of flights with distances

Origin	Destination	Miles
ATL	CVG	454
CVG	JFK	664
JFK	MON	396
MON	ORD	764
ORD	LAX	2136
LAX	HOU	1616
HOU	ATL	842
TOTAL		6872

The general model for the Traveling Salesman Problem, adapted from Ignizio and Cavalier (1994), is as follows:

Sets

N = Number of cities

Index

i,j = Index for cities

Parameters

$c_{i,j}$ = Cost of traveling from city i to city j

Decision Variable

$$x_{i,j} = \begin{cases} 1 \text{ if city } j \text{ follows city } i \\ 0 \text{ otherwise} \end{cases}$$

The integer programming model is as follows:

Objective Function

$$Min \sum_{i \in N} \sum_{j \in N} c_{i,j} x_{i,j} \tag{2.18}$$

Subject to

$$\sum_{j \in N} x_{i,j} = 1 \quad \text{For all } i = 1, \ldots, \text{N} \tag{2.19}$$

$$\sum_{i \in N} x_{i,j} = 1 \quad \text{For all } j = 1, \ldots, \text{N} \tag{2.20}$$

$$t_i - t_j + N x_{i,j} < N - 1 \quad \text{For } i, j \ = \ 2, 3, .., N \tag{2.21}$$

In this model, the objective function (2.18) seeks to minimize the total traveling cost. The set of constraints (2.19) ensure that each city i is followed by exactly one city j. Similarly, the set of constraints (2.20) ensures that each city j is visited exactly once. The set of constraints (2.21) imposes the restriction on the sub-tours. Variables t_i and t_j are arbitrary fixed numbers used for breaking the sub-tours.

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Chapter 3
Flight Scheduling

Introduction

Flight scheduling is the starting point for all other airline planning and operations (Barnhart 2008, Yu and Thengvall 2002). The flight schedule is a timetable consisting of what cities to fly to and at what times. An airline's decision to offer certain flights will mainly depend on market demand forecasts, available aircraft operating characteristics, available manpower, regulations, and the behavior of competing airlines. The number of airports and flight frequencies served by an airline usually expresses and measures the physical size of the airline network (Janic 2000). For large air carriers, the flight-scheduling group and route development may contain more than 30 employees (Kuzminski 1999).

Table 3.1 shows a small portion of the daily flight schedule for Delta Air Lines. The level of detail in constructing the flight schedule varies among the airlines, but it will be a complete schedule for a full cycle (Grandeau et al. 1998). A cycle is normally one day for domestic and one week for international services.

The schedule construction phase begins with the route system. The cities in the airline network determine the route system. The economics of an air carrier are driven by its route system. All the short- and long-term costs attributed to fleet, avionics, labor contracts, and operations are tied to the route systems of an airline. The marketing department plays an important role in the construction of this schedule. Before the 1978 Airline Deregulation Act, airlines had to fly routes as assigned by the Civil Aeronautics Board (CAB) regardless of the demand for the service! During this period, most airlines emphasized long point-to-point routes. Since deregulation, airlines have gained the freedom to choose which markets to serve and how often to serve them. This change has led to a fundamental shift in most airlines' routing strategies from point-to-point flights to hub-and-spoke oriented networks (Etschamaier and Mathaisel 1985).

The schedule construction phase is a rough first schedule, which requires extensive modification to be both operationally feasible and economically viable (Etschamaier and Mathaisel 1985).

Hub-and-Spoke

Most airlines adopt some variation of a hub-and-spoke system. Major carriers operate up to five hubs, while smaller ones typically have one hub located at the center of the region they serve. Each hub has a set of cities that it serves, normally

Table 3.1 A sample flight schedule

Carrier	Depart		Arrive	
Flight #	**Time**	**Airport**	**Time**	**Airport**
Delta 442	6:20 AM	ATL	7:39 AM	MCO
Delta 171	6:25 AM	ATL	7:46 AM	DFW
Delta 193	8:55 AM	CVG	10:28 AM	ATL
Delta 353	4:35 PM	CVG	6:10 PM	ATL
Delta 267	5:45 AM	DFW	8:52 AM	ATL
Delta 1264	7:45 PM	DFW	10:53 PM	ATL
Delta 1981	3:00 PM	JFK	5:28 PM	ATL
Delta 137	5:30 PM	JFK	8:40 PM	LAX
Delta 292	7:00 AM	LAX	2:20 PM	ATL
Delta 1886	3:15 PM	LAX	10:28 PM	ATL
Delta 929	7:35 AM	MCO	9:13 AM	ATL
Delta 622	10:05 AM	MCO	11:35 AM	ATL
Delta 2246	8:20 AM	MIA	10:13 AM	ATL
Delta 858	5:20 PM	MIA	7:22 PM	ATL

Source: www.delta.com

referred to as spokes. Figure 3.1 shows an airline network with Chicago O'Hare and Washington Dulles as hubs.

Air carriers normally assign large capacity non-stop flights between their hubs. Smaller airplanes are assigned to hub-and-spoke flights. Major advantages for the airlines adopting hub-and-spoke operations include higher revenues, higher efficiency, and lower number of aircraft needed as compared with point-to-point operations. Disadvantages of these operations include discomfort to the passengers, as they may require multiple connecting flights at different hubs, congestions and delays at hub airports, and higher personnel and operational costs for the airlines (Radnoti 2002).

Route Development and Flight-Scheduling Process

There are two types of route development activities: *strategic* and *tactical*. Strategic development focuses on future schedules which may range from a few months to ten years depending on the air carriers' policies. Strategic developments respond to major changes in both business and operational environments. Tactical strategies, on the other hand, focus on short-term changes to the schedule and

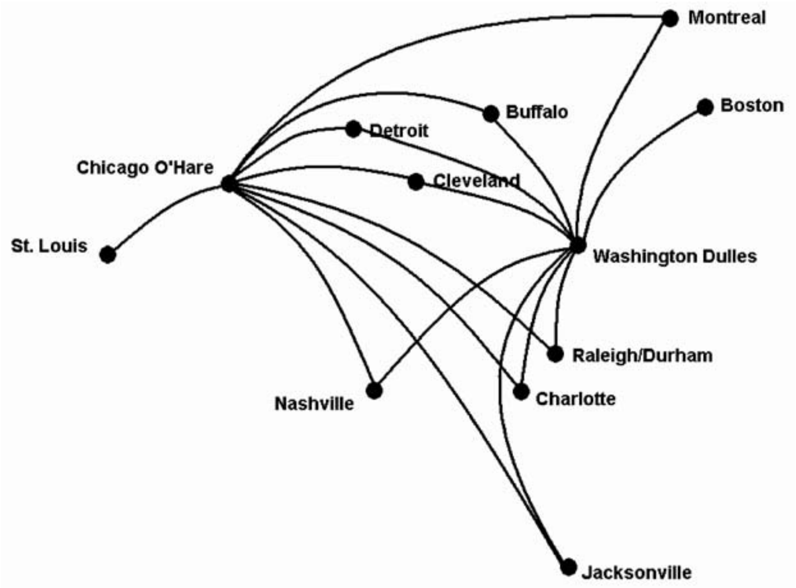

Figure 3.1 A sample airline network with two hubs and nine spokes

routes, sometimes on a daily basis. This is done by constantly monitoring markets, competitors, and operations. The tactical strategy includes adding, dropping flights, and making changes to city-pair markets and their frequencies.

The following section briefly describes the phases of developing a flight schedule and the decisions made at each phase.

60+ months	36–12 months	12–3 months	4–1 months
Long range Planning	Market Evaluations	Schedule Optimization	Schedule issues

Long-Range Schedule Planning

- Fleet diversity
- Manpower planning
- Protecting hubs
- Adding or changing hubs
- Adequate facilities at airports.

Market Evaluations

- Frequency and time of service to each market
- Adding new and dropping existing markets

- Pricing policies
- Predicting competitors' behaviors
- Code-sharing agreements and alliances.

Schedule Optimization

- Developing initial schedule based on available fleet
- Assigning aircraft to flights
- Evaluating facilities and manpower capabilities.

Schedule Issues

- Crew issues
- Arrival departure times
- Maintenance issues.

As described earlier, flight schedule construction is the basis for all other operations. It is therefore important to include detailed airline operations in the process of flight scheduling. This, however, creates a complex system with a large number of variables in the model (Grosche et al. 2001). Owing to its complexity it is almost impossible to formulate the complete scheduling construction problem as a mathematical model. As a result, the schedule construction process is performed through a structured planning process involving various parts of the airline. This planning process is decomposed into sub-problems with less complexity, which are solved sequentially. Chapters 4 to 6 present these sub-problems.

One of the major drawbacks of this approach is that an individual sub-problem's solution might not be good for the overall airline operations (Papadakos 2009). To overcome this difficulty, the process of flight scheduling is performed on a feedback system. That is, if the solutions to some sub-problems are not desirable, the flight schedules are altered to see the impact of such changes. Figure 3.2 shows the process of flight schedule development and the hierarchy of various phases of airline planning. The chapters to follow show how this process is done.

Load Factor and Frequency

Average load-factor plays an important role in determining the frequency of flights between city pairs. Load factor is the average percentage of aircraft seats which are filled with passengers. The parameters affecting load factors include flight times, frequency, type of service and, of course, fare levels. It should be noted that a higher load-factor does not necessarily translate into higher revenues for the airlines. As an example, Table 3.2 shows the fares, expected demands and load factors for a 150-seat Airbus A-320. According to this table, an 85% load factor generates higher revenues of more than 100% for the airline! Demand and revenue management will be further discussed in Chapter 8.

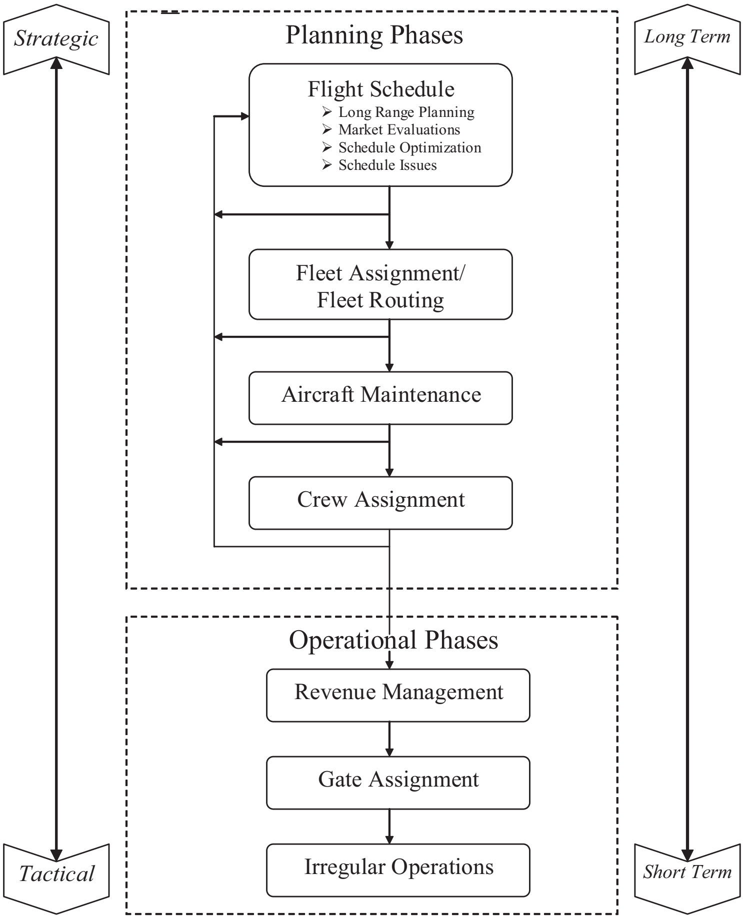

Figure 3.2 The hierarchy of airline planning

Table 3.2 Load factor and expected revenue

Average fare	Expected number of passengers	Load factor	Expected revenue
$240	100	0.67	$24,000
$220	115	0.77	$25,300
$200	128	0.85	$25,600
$180	140	0.93	$25,200
$160	150	1.00	$24,000

The load factor is utilized to determine the frequency between city pairs. Let the forecasted daily number of passengers between two cities be *PAX* and the airline's policy on average load-factor be *LF*. Further, let us assume the average aircraft capacity is *CAP*. Then the frequency (*FREQ*) of flights between these two cities is determined by:

$$\text{FREQ} = \frac{\text{PAX}}{\text{CAP} \times \text{LF}} \tag{3.1}$$

As implied by the above equation, the load factor and frequency have an inverse relationship. It is up to the marketing and scheduling departments to actually assign these frequencies between city pairs to different times of the day/week.

Case Study

In this section a fictitious airline is presented. We will use this airline in the following chapters to introduce the various phases of planning within the airlines.

Ultimate Air is a new airline that provides service to the most important domestic business destinations within the United States from its hub at JFK in New York. The cities serviced from JFK are Boston (BOS), Los Angeles (LAX), San Francisco (SFO), Miami (MIA), Atlanta (ATL), Washington D.C. (IAD), and Chicago (ORD). Figure 3.3 shows the airline's network.

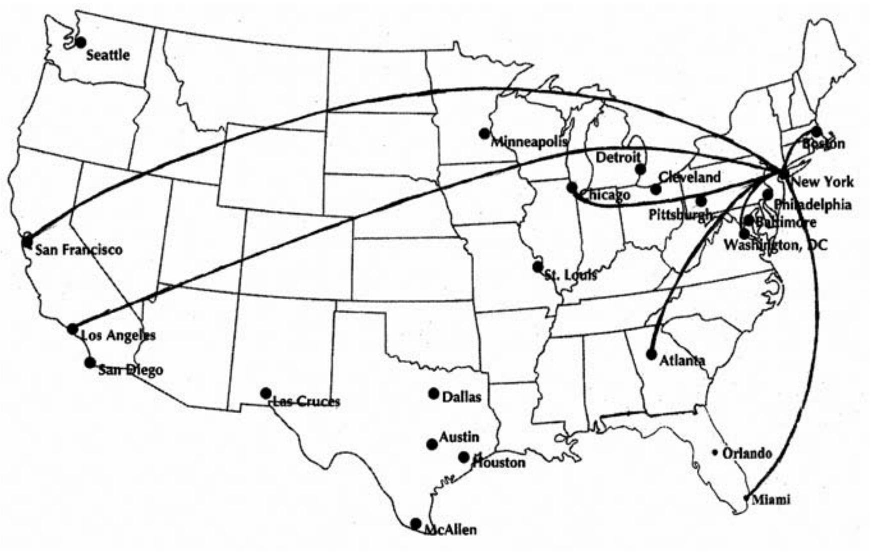

Figure 3.3 Ultimate Air route network

Based on forecasts, the airline's load-factor policy, and marketing analysis, the airline has proposed providing three daily round-trip flights from JFK to each city in the network. It has also developed a first draft of its schedule for the next quarter. The complete flight schedule route, incorporating the 42 flights per day, is presented in Table 3.3. All the arrival and departure times are local times.

Table 3.3 Flight schedule for Ultimate Air

Flight no.	Origin	Departure time	Destination	Arrival time	Flight hours
101	LAX	05:00	JFK	13:30	5.5
104	SFO	05:05	JFK	13:35	5.5
116	BOS	06:15	JFK	07:45	1.5
140	JFK	06:20	IAD	07:20	1
125	JFK	07:25	SFO	09:55	5.5
107	ORD	07:30	JFK	10:30	2
122	JFK	07:35	LAX	10:05	5.5
137	JFK	07:40	BOS	09:10	1.5
110	ATL	08:10	JFK	10:40	2.5
119	IAD	08:15	JFK	09:15	1
113	MIA	09:10	JFK	12:10	3
131	JFK	09:30	ATL	12:00	2.5
102	LAX	09:45	JFK	18:15	5.5
105	SFO	09:50	JFK	18:20	5.5
117	BOS	10:00	JFK	11:30	1.5
128	JFK	10:05	ORD	11:05	2
134	JFK	10:35	MIA	13:35	3
141	JFK	12:00	IAD	13:00	1
108	ORD	12:20	JFK	15:20	2
138	JFK	12:30	BOS	14:00	1.5
111	ATL	13:10	JFK	15:40	2.5
120	IAD	14:25	JFK	15:25	1
114	MIA	14:30	JFK	17:30	3
132	JFK	14:35	ATL	17:35	2.5
118	BOS	15:00	JFK	16:30	1.5
129	JFK	15:05	ORD	16:05	2
135	JFK	15:10	MIA	18:10	3

Table 3.3 *Concluded*

Flight no.	Origin	Departure time	Destination	Arrival time	Flight hours
142	JFK	15:15	IAD	16:15	1
103	LAX	15:20	JFK	23:50	5.5
106	SFO	15:25	JFK	23:55	5.5
126	JFK	15:30	SFO	18:00	5.5
123	JFK	16:00	LAX	18:30	5.5
109	ORD	17:10	JFK	20:10	2
112	ATL	18:00	JFK	20:30	2.5
133	JFK	18:05	ATL	20:35	2.5
136	JFK	18:10	MIA	21:10	3
115	MIA	18:15	JFK	21:15	3
121	IAD	18:30	JFK	19:30	1
124	JFK	19:00	LAX	21:30	5.5
127	JFK	20:00	SFO	22:30	5.5
130	JFK	21:00	ORD	22:00	2
139	JFK	21:30	BOS	23:00	1.5

Table 3.4 presents the demand distribution for each flight as well as distances between cities. It is assumed that demand for each flight is normally distributed with the given means and standard deviations.

Table 3.4 Destination in miles, demand means and standard deviations for Ultimate Air network

Flight no.	Origin	Destination	Distance (miles)	Demand	Standard deviation
101	LAX	JFK	2475	175	35
102	LAX	JFK	2475	182	36
103	LAX	JFK	2475	145	29
104	SFO	JFK	2586	178	35
105	SFO	JFK	2586	195	39
106	SFO	JFK	2586	162	32
107	ORD	JFK	740	165	33

Table 3.4 ***Continued***

Flight no.	Origin	Destination	Distance (miles)	Demand	Standard deviation
108	ORD	JFK	740	182	36
109	ORD	JFK	740	170	34
110	ATL	JFK	760	191	38
111	ATL	JFK	760	171	34
112	ATL	JFK	760	165	33
113	MIA	JFK	1090	198	39
114	MIA	JFK	1090	182	36
115	MIA	JFK	1090	168	33
116	BOS	JFK	187	115	23
117	BOS	JFK	187	146	29
118	BOS	JFK	187	120	24
119	IAD	JFK	228	135	27
120	IAD	JFK	228	109	21
121	IAD	JFK	228	98	19
122	JFK	LAX	2475	150	30
123	JFK	LAX	2475	145	29
124	JFK	LAX	2475	125	25
125	JFK	SFO	2586	148	29
126	JFK	SFO	2586	138	27
127	JFK	SFO	2586	121	24
128	JFK	ORD	740	132	26
129	JFK	ORD	740	129	25
130	JFK	ORD	740	117	23
131	JFK	ATL	760	168	33
132	JFK	ATL	760	160	32
133	JFK	ATL	760	191	38
134	JFK	MIA	1090	165	33
135	JFK	MIA	1090	184	36
136	JFK	MIA	1090	192	38

Table 3.4 Concluded

Flight no.	Origin	Destination	Distance (miles)	Demand	Standard deviation
137	JFK	BOS	187	147	29
138	JFK	BOS	187	135	27
139	JFK	BOS	187	146	29
140	JFK	IAD	228	105	21
141	JFK	IAD	228	115	23
142	JFK	IAD	228	118	23

We will use the above flight schedule as a basis to derive the planning for the fleet assignment as well as aircraft routing in the following chapters.

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Chapter 4
Fleet Assignment

Introduction

Following the construction of a flight schedule and its corresponding network, the next step is to assign the right fleet type to each flight in the schedule. The task of fleet assignment is to match each aircraft type in the fleet with a particular route in the schedule. It should be noted that this phase of planning concerns only fleet type and not a particular aircraft. The goal of fleet assignment is to assign as many flight segments as possible in a schedule to one or more fleet types, while optimizing some objective function and meeting various operational constraints (Abara 1989). Fleet assignment should not be confused with fleet planning (Clark 2001). Fleet planning is a strategic decision normally undertaken when an airline is conceived, and concerns the number and type of aircraft needed for operation. It entails the process of acquiring the appropriate aircraft-types in order to serve the anticipated markets based on the airline's strategic plan. Fleet planning addresses fleet size and fleet mix. In fleet assignment, however, we assume that the airline is operational with the existing aircraft in its fleet, and the problem is to assign a fleet type to each flight leg.

Airlines typically operate a number of different fleet types. Each fleet type has different characteristics and costs, such as seating capacity, landing weights, crew, maintenance, and fuel (Yu and Thengvall 1999). Table 4.1 presents the fleet diversity for select airlines. Maintenance cost is a major factor that persuades airlines to be less diverse when planning for their fleet. Fleet diversity requires the airlines to have skilled crew and personnel for each fleet type, plan for different maintenance checks, and have less flexibility in replacing an aircraft when a failure occurs. Sherali et al. (2006) provide an overview of fleet assignment models integrated with maintenance planning and crew scheduling.

Indicator Definitions

Before addressing the mathematical model for the fleet assignment problem, some terms commonly used in the airline industry are explained:

ASM (ASK): Available Seat Miles (Kilometers) represents the annual airline capacity, or supply of seats, and refers to the number of seats available for passengers during the year multiplied by the number of miles (kilometers) that those seats are flown.

RPM (RPK): Revenue Passenger Miles (Kilometers) represents the total number of paying passengers flown on all flight segments multiplied by the number

Table 4.1 Fleet diversity for select airlines

Airline	B737	A318/319/320/321	A300	A330/A340	A380	B757	B767	B777	B787	B747	A350	DC9	F-100	CRJ	EMB	C Series	Total
Air France	-	168	-	35	12	-	-	73	-	23	-	-	-	-	-	-	311
American Airlines	164	-	30	-	-	124	73	54	-	-	-	294	4	-	-	-	743
British Airways	26	90	-	-	12	11	21	52	24	57	-	-	-	-	-	-	293
Delta Air Lines	114	-	-	-	-	132	102	18	-	-	-	133	-	9	-	-	508
Lufthansa	64	144	13	67	15		-	-	-	51	-	-	-	-	28	30	412
Northwest	-	133	-	32	-	61	-	-	18	30	-	96	-	-	-	-	370
Southwest	641	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	641
United	78	194	-	-	-	97	35	52	-	30	-	-	-	-	-	-	486
US Airways	72	292	-	34	-	39	10	-	-	-	22	-	-	-	42	-	511
Continental Airlines	320	-	-	-	-	57	26	28	25	-	-	-	-	-	-	-	456

Source: 2009 Fleet-iNet – OAG Aviation Solutions.

of miles (kilometers) that those passengers are flown. RPM (RPK) is considered to be demand. It should be noted that RPM (RPK) is typically less than ASM (ASK). This is because airlines will not have all the seats filled on all flight segments during the entire year.

Yield: Yield is how much an airline makes per revenue passenger mile (kilometer). In other words, yield is how much an airline makes per mile (kilometer) on each seat sold. Yield is obtained by dividing total operating revenue divided by RPM (RPK).

RASM (RASK): Revenue per Available Seat Mile (Kilometer), or 'unit revenue' represents how much an airline made across all the available seats that were supplied. RASM (RASK) is calculated by dividing the total operating revenue by available seat mile (kilometer) or ASM (ASK). Since ASM (ASK) is generally larger than RPM (RPK), yield has a higher value than RASM (RASK).

CASM (CASK): Cost per Available Seat Mile (Kilometer) or 'unit cost' is the average cost of flying one seat for a mile (kilometer). CASM (CASK) is calculated by dividing the total operating cost by ASM (ASK).

Table 4.2 presents the above measures for select US airlines differentiated by market segments.

Table 4.2 2008 domestic operations key performance indicators for major US carriers

Carrier	ASM (million)	RPM (million)	RASM (cents)	CASM (cents)	Yield (cents)
Airtran	23,814.456	18,784.437	10.13	8.00	12.85
Alaska	21,815.762	16,742.678	10.80	7.76	14.07
American	101,855.071	83,313.426	10.89	8.22	13.32
Continental	52,987.792	44,215.689	10.95	7.79	13.12
Delta	128,976.090	105,697.565	10.73	7.17	13.10
JetBlue	32,435.674	26,069.180	9.45	6.69	11.76
Southwest	103,486.264	73,639.652	9.93	6.48	13.96
United	135,859.306	110,061.748	10.90	8.62	13.45
US Airways	74,148.295	60,567.144	10.77	8.51	13.18

Source: Form41 iNET.

The above figures are total measures across the various market segments and all fleets. Table 4.3 shows average ASM, RPM, and CASM by fleet type.

Table 4.3 US major carriers' unit revenues and expenses by fleet-type

Aircraft	ASM (million)	RPM (million)	CASM*
A300-600	10,239.113	8,056.686	7.5
A319	28,714.442	23,080.522	7.2
A320	43,670.093	36,306.733	6.27
A321	7,785.539	6,582.864	5.26
A330	24,786.437	20,929.953	4.65
B717-200	12,841.303	9,581.749	7.58
B737-200	15.110	9.889	N/A
B737-300	53,346.328	39,261.932	7.45
B737-400	12,301.534	9,226.063	7.7
B737-500	14,931.963	11,715.791	8.2
B737-700	83,872.844	62,853.756	4.9
B737-800/900	71,276.880	57,523.046	5.46
B747-200	917.667	664.746	9.16
B747-400	40,405.191	33,848.281	5.61
B757-200	139,883.555	116,023.006	5.95
B757-300	14,372.536	12,175.073	5.11
B767-200	12,786.649	10,378.550	6.12
B767-300	78,611.122	64,333.916	5.79
B767-400	19,667.409	16,325.545	4.97
B777	77,297.169	63,606.785	6.3
DC-10-30	524.388	308.648	6.8
DC-9	10,417.344	8,007.598	10.2
EMB-190	333.023	250.417	9.70
L-1011-500	1,242.207	704.213	7.70
MD-80	71,094.571	57,170.452	7.31

* CASM = Total of type × aircraft operating cost / Total of type × aircraft ASM.

Source: The Airline Monitor, August 2008 & Back Aviation Solutions, Form41 iNET.

Mathematical Model

A major concern in formulating the fleet assignment problem is keeping track of the fleet at different stations (airports) at any given point in time. Fortunately, researchers have developed an ingenious method of adopting a time-space network to formulate this problem. Figure 4.1 shows such a network for five cities.

This approach facilitates the process of modeling the fleet assignment problem. The above time-space network presents the airports as columns, and times of the day as rows. In this network, the arcs (arrows) are the flights, and nodes represent the arrival/departure of a flight segment at a specific airport, at a specific time of the day. A wrap-around arc is a ground arc which connects the last node to the first node in a given city. These arcs normally represent the aircraft that stay overnight in an airport, and connect the last arrival to the next day's departure flight (see Figure 4.1).

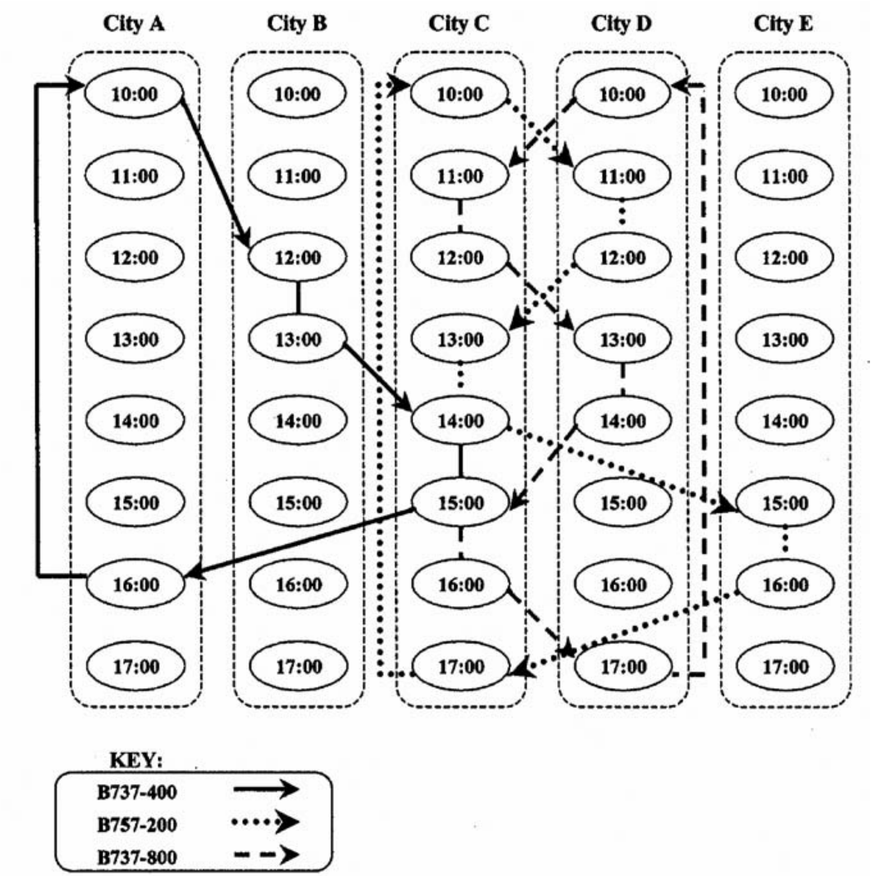

Figure 4.1 An example of a time-space network

The fleet assignment problem is basically formulated as a multi-commodity network problem (see Chapter 2). Each node represents supply/demand, which can be satisfied through a diverse fleet. The model seeks to minimize the total cost or maximize the net profit by assigning the most appropriate fleet type to each flight leg. The constraints ensure that each flight is assigned to a particular fleet type, and that the number of aircraft for each fleet does not exceed the number of available aircraft. Other side-constraints may include curfew, range, noise, forced turns, maintenance, and user-specific restrictions.

In the mathematical model presented here, the objective function represents the total cost of the network, which we seek to minimize. These costs include two parts: operating costs and spill costs.

Operating Costs

The operating costs for a flight mainly depend on the type of the fleet assigned to that flight and are determined as follows:

Operating costs of a flight = CASM of the fleet × distance × number of seats on the aircraft

Let us return to the fleet diversity for Ultimate Air, in the case study we introduced in Chapter 3. We have two types of fleet, namely Boeing 737–800 and Boeing 757–200. The seating capacities for these two fleet types are 162 and 200 seats respectively. Furthermore, we have the following information for this airline:

- Cost per available seat mile (CASM) for B737–800 and B757–200 are \$0.042 (4.2 cents) and \$0.044 (4.4 cents) respectively;
- Revenue per available seat mile (RASM) is \$0.15 (15 cents).

Using the above information we can determine the operating cost for each flight in the Ultimate Air schedule for the two fleet types. As an example, for flight 122 (JFK-LAX), the distance between JFK and LAX is 2,475 miles (see Table 3.4, Chapter 3). Hence, the operating costs of this flight for the two fleet types are:

- Operating cost for a B737–800 = \$0.42 × 2,475 × 162 = \$16,839
- Operating cost for a B757–200 = \$0.44 × 2,475 × 200 = \$21,780

Passenger-Spill Costs

An important issue in assigning fleet types to flights is the passenger demand for each flight segment. Assigning large capacity aircraft to flights with low demand leads to low utilization and consequently low load-factor for the airline. On the other hand, assigning small aircraft to flight legs with high demand leads to passenger spills. Spill is the degree of average demand, which exceeds the capacity offered. The spill cost is therefore the revenue of lost passengers due to insufficient aircraft capacity.

Let us once again consider flight 122 (JFK-LAX) in our Ultimate Air case study. Our historical data for flight 122 shows that the demand for this flight is normally distributed with a mean of 150 and a standard deviation of 30 passengers (see Table 3.4, Chapter 3). Figure 4.2 shows the demand distribution for this flight. The shaded areas show the probability of passenger spills for the two fleet types. The spill is basically the truncation of the demand distribution beyond the aircraft capacity.

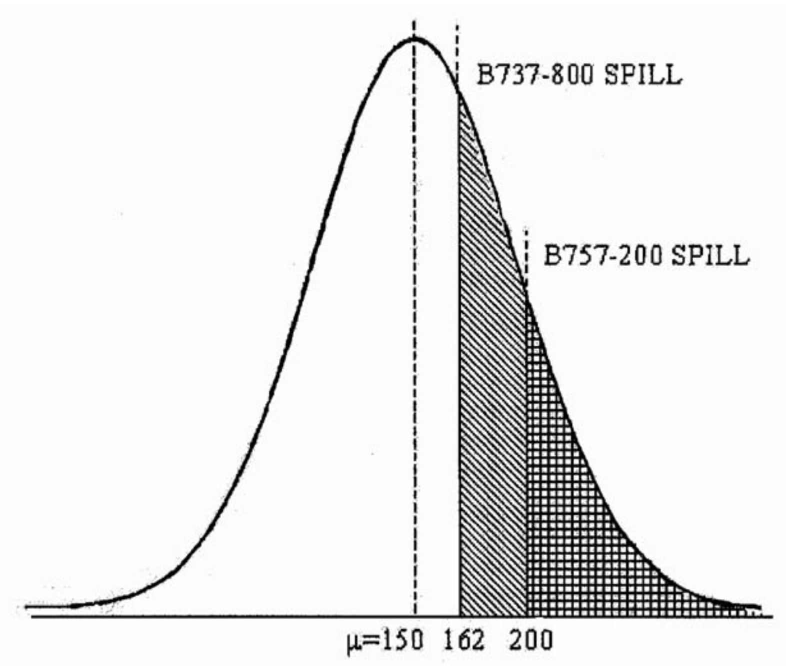

Figure 4.2 Demand distribution and passenger spills

The expected spill costs are determined as follows:

Expected spill cost for a fleet = expected number of passenger spill $\times$ RASM $\times$ distance

The expected number of passenger spill is calculated as follows:

$$\text{Expected number of passenger spill} = \int_{c}^{\infty} (x-c) f(x) dx$$

In the above equation, c is the fleet capacity and $f(x)$ is the probability distribution function of the demand. The above integral can be obtained using mathematical software (e.g., MAPLE) or some calculators. It is possible and perhaps easier to use a Microsoft Excel spreadsheet to approximate the above expected number of passenger spill using simulation. The following steps show the Excel functions used to determine the expected number of spilled passengers for a B737–800 fleet type with 162 seats:

- Cell A1: NORMINV (RAND(),150,30)
- Cell B1: IF (A1>162,A1–162,0)

Cell A1 randomly generates a demand from normal distribution with a mean of 150 and a standard deviation of 30. Cell B1 checks to see if the demand in cell A1 exceeds 162 seats. If it does, then cell B1 is assigned to their difference (i.e., passenger spill), otherwise passenger spill is zero. The above two cells are copied and pasted (downward) many times (we used 10,000 replications). The average of column B, denoted by AVERAGE(B:B), calculates the expected number of spilled passengers.

Using the above approximation method, the expected numbers of passenger spill (rounded to two decimal places) for the two fleet types are as follows:

- Expected passenger spill for B737–800 with 162 seat capacity = 6.91
- Expected passenger spill for B757–200 with 200 seat capacity = 0.60

The expected spill costs for the two fleet types are therefore calculated as:

- Expected spill costs for B737–800 = 6.91×.15×2475=$2,565.33
- Expected spill costs for B757–200 = .60×.15×2475=$222.75

It may seem that this model attempts to assign larger capacity fleet type to all flights since expected shortages are penalized, but excess capacity or surplus seats are not. It should be noted that the larger capacity fleet type was already penalized when we calculated the operating costs above.

Recapture Rate

A closely related topic to passenger spill is the recapture rate. The recapture rate represents the percentage of passengers that were spilled, but could be accommodated or recaptured on other flights by the same airline. That is, if a passenger cannot get a seat on a specific flight, the airline offers earlier or later flights (in some cases with bonuses) to the passenger for consideration. If the passenger accepts the offer for another flight, then this passenger is considered to be recaptured. The recapture rate among the major airlines is typically very high. This is due to high flight frequencies offered by these airlines as well as other marketing incentives such as frequent-flyer programs.

Returning to our Ultimate Air case study, owing to low flight frequencies the recapture rate is low. Let us assume that this rate is 15% for this airline. This rate means that 85% of passengers who request a reservation for a flight on Ultimate Air and are denied such a request, are lost to other airlines. Therefore the expected spill costs for the two fleet types for flight 122 are:

- Expected spill costs for B737–800 = $2,565.33 % .85 = $2,180.31
- Expected spill costs for B757–200 = $222.75 % .85 = $189.34

Now we can determine the total cost of assigning a fleet type to a flight leg by adding the operating and spill costs. The total cost for each fleet when assigned to flight 122 is:

- Total cost of assigning B737–800 to flight 122 = $16,839.90+$2,180.31 = $19,020.21
- Total cost of assigning B757–200 to flight 122 = $21,780.00+$189.34 = $21,969.34

Similarly, we determine the total costs for all other flights.

Objective Function

To setup the objective function for Ultimate Air, we need to first select our decision variables in a way that addresses the assignment of the fleet type to the flight leg. The following decision variables are commonly adopted for fleet assignment models.

$$x_{i,j} = \begin{cases} 1 \text{ if flight } i \text{ is assigned to fleet-type } j \\ 0 \text{ otherwise} \end{cases}$$

$G_{k,j} =$ integer decision variable representing number of aircraft of fleet-type j on ground at node k

In the binary decision variable $x_{i,j}$ index i represents the flight leg (42 flight legs for Ultimate Air), while index j represents the fleet type (for our case study we have two fleet types). For simplicity in our notation, we designate j to take the value 1 for B737–800, and the value 2 for B757–200 fleets. Based on this definition, $x_{101,1}$ represents the binary decision variable for flight 101 assigned to a B737–800 fleet. Similarly, $x_{101,2}$ represents the same flight, but assigned to fleet type 2 (i.e., B757–200) and so on. Decision variable $G_{k,j}$ will be used to address the set of constraints for aircraft balance. This set of decision variables will be discussed later in the constraints section.

The objective function is basically to minimize the total cost by assigning the most appropriate fleet type to flights as follows:

$$\textit{Minimize } 21485.26x_{101,1} + 22556x_{101,2} + 24222.37x_{102,1} + 23556x_{102,2} + \ldots + 1558.42x_{142,1} + 2006x_{142,2}$$

Constraints

There are three main sets of constraints in the fleet assignment model. They are discussed as follows:

Flight Cover

The first set of constraints is what is typically known as flight cover. Flight cover implies that each flight must be flown. To cover a flight, the sum of all the decision variables representing that flight must add up to 1. As an example, to cover flight 101 in our Ultimate Air case study, we write:

$$x_{101,1} + x_{101,2} = 1$$

This constraint ensures that flight 101 is covered. Furthermore, the flight will be covered by only one type of fleet since the sum of binary decision variables adds up to 1. Only one of the two binary decision variables in this constraint will take a value of 1, forcing the other variable to be zero. We write similar constraints for all other 41 flights in our case study.

Aircraft Balance

The next set of constraints concerns the aircraft balance or equipment continuity within the fleets. This set of constraints ensures that an aircraft of the right fleet type will be available at the right place at the right time. Earlier, we introduced the concept of a time-space network. We adopt this concept to address this set of constraints. Referring to Figure 4.1, each node represents an arrival or departure. Recall that each node represents a specific time at a specific airport. So, the number of aircraft at any node changes with respect to an instant before that node. To clarify this, consider Figure 4.3 opposite. In this figure we have an arrival node. Just before this node, there were two aircraft (of the same fleet type) at the airport. After this arrival, we now have another aircraft (of the same fleet type again) added to those already at this airport.

Referring to Figure 4.3, the set of constraints for aircraft balance or equipment continuity states that:

> Number of aircraft of a particular fleet type on the ground at a node = Number of aircraft in that fleet on the ground an instant before that node + arrival of aircraft of the same fleet type at that node – (minus) departures of aircraft of the same fleet type from that node.

For example, the balance constraint for the node in Figure 4.3 is:

> Number of aircraft at this node = 2 (number of aircraft before this node) + 1 (one arrival) – 0 (no departure from this node) = 3

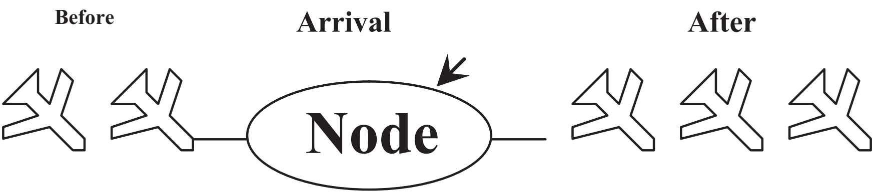

Figure 4.3 Example of aircraft balance

Adopting this approach, we can now write the constraints for balance for each airport in our Ultimate Air case study. Let us consider LAX. The flights in and out of LAX (extracted from our flight schedule in Chapter 3) are as shown in Table 4.4.

Table 4.4 Arrival/departure flights for LAX

Flight no.	Origin	Departure time	Destination	Arrival time	Duration of flight (hrs)
101	LAX	05:00	JFK	13:30	5.5
102	LAX	09:45	JFK	18:15	5.5
122	JFK	07:35	LAX	10:05	5.5
103	LAX	15:20	JFK	23:50	5.5
123	JFK	16:00	LAX	18:30	5.5
124	JFK	19:00	LAX	21:30	5.5

Figure 4.4 presents this table as a time-space network, similar to Figure 4.3 discussed earlier.

We have two types of fleet. We use the decision variable $G_{k,j}$ to write the constraints for aircraft balance for each fleet type. Let us first consider the B737–800 fleet type. Based on Figure 4.4, the first node at LAX is at L1. The number of B737–800 aircraft at this node, based on the rule for balance, is basically the number of aircraft carried over from the previous day (wrap-around arc from node L6) minus one departure (flight 101), so:

$$G_{L1,1} = G_{L6,1} - x_{101,1}$$

At node L2 (see Figure 4.4), we have another departure (flight 102) so:

$$G_{L2,1} = G_{L1,1} - x_{102,1}$$

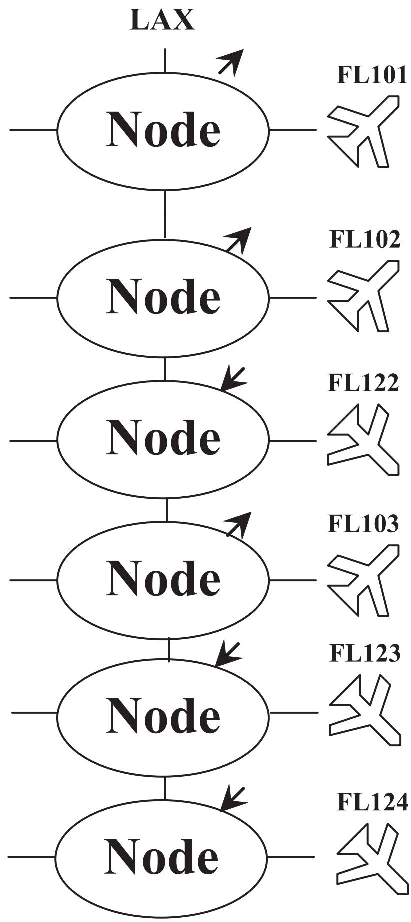

Figure 4.4 Time-space network for LAX

At node L3, we have an arrival (flight 122), therefore:

$$G_{L3,1} = G_{L2,1} + x_{122,1}$$

Similarly, we write the other three constraints for this fleet type as follows:

$$G_{L4,1} = G_{L3,1} - x_{103,1}$$

$$G_{L5,1} = G_{L4,1} + x_{123,1}$$

$$G_{L6,1} = G_{L5,1} + x_{124,1}$$

The constraints for the B757–200 fleet are similar to the B737–800 as follows:

$$G_{L1,2} = G_{L6,2} - x_{101,2}$$

$$G_{L2,2} = G_{L1,2} - x_{102,2}$$

$$G_{L3,2} = G_{L2,2} + x_{122,2}$$

$$G_{L4,2} = G_{L3,2} - x_{103,2}$$

$$G_{L5,2} = G_{L4,2} + x_{123,2}$$

$$G_{L6,2} = G_{L5,2} + x_{124,2}$$

Similarly, we write the balance constraints for all other airports in the schedule. There are 42 flights in our Ultimate Air case study. Each flight has a departure and an arrival. We have two fleet types. Therefore, the total number of constraints for aircraft balance is 168 (42 × 2 × 2).

Fleet Size

This set of constraints is adopted to ensure that the number of aircraft within each fleet does not exceed the available fleet size. To address this, we must count the number of aircraft that are grounded overnight for that fleet type at different airports. Referring to Figure 4.4, the last node, L6 (originating node for wrap-around arc), represents the total number of aircraft in LAX at the end of the day. For this airport, $G_{L6,1}$ represents the total number of grounded B737–800 aircraft in LAX overnight. Similarly, the number of grounded B757–200 aircraft at the last node in LAX is $G_{L6,2}$. The total number of B737–800 aircraft in our network is therefore:

$$G_{L6,1} + G_{S6,1} + G_{B6,1} + G_{O6,1} + G_{A6,1} + G_{I6,1} + G_{M6,1} + G_{J42,1}$$

In the above expression, the integer variables represent the number of aircraft at the last nodes at LAX, SFO, BOS, ORD, ATL, IAD, MIA, and JFK respectively. Note that at JFK, we have 42 daily flights arriving at or departing from this airport. Therefore, the last node is represented as *J*42. Similarly, the total number of B757–200 aircraft in our network is:

$$G_{L6,2} + G_{S6,2} + G_{B6,2} + G_{O6,2} + G_{A6,2} + G_{I6,2} + G_{M6,2} + G_{J42,2}$$

In our case study, Ultimate Air, assume that we have nine and six aircraft in our B737–800 and B757–200 fleets, respectively. We can now incorporate these constraints into our model as follows:

$$G_{L6,1} + G_{S6,1} + G_{B6,1} + G_{O6,1} + G_{A6,1} + G_{I6,1} + G_{M6,1} + G_{J42,1} \leq 9$$

$$G_{L6,2} + G_{S6,2} + G_{B6,2} + G_{O6,2} + G_{A6,2} + G_{I6,2} + G_{M6,2} + G_{J42,2} \leq 6$$

Since there are only two fleet types, there are only two constraints in this set.

Solution to Fleet Assignment Problem

The linear integer program for fleet assignment for Ultimate Air has 252 (84 binary and 168 integer) variables and 212 constraints. Using an optimization software, the solution to this problem generates a minimum daily cost of fleet assignment of \$410,612.57. The following table shows the number of aircraft for each fleet type staying overnight at each airport. These numbers represent the right number of aircraft for each fleet type at the right airport at the right time.

Table 4.5 Optimal number of aircraft grounded overnight at each airport

Airports	737-800 Fleet	757-200 Fleet
Los Angeles (LAX)	2 aircraft	1 aircraft
San Francisco (SFO)	2 aircraft	-
Boston (BOS)	1 aircraft	-
New York (JFK)	3 aircraft	2 aircraft
Chicago (ORD)	1 aircraft	-
Atlanta (ATL)	-	1 aircraft
Washington DC (IAD)	-	-
Miami (MIA)	-	2 aircraft

Table 4.6 presents the assignment of each flight to either one of the two fleet types.

Note that the above solution only shows the assignment of flights to fleet type. It does not show the assignment of flights to any specific aircraft within each fleet. This type of assignment is called aircraft routing, which will be discussed in the next chapter.

Table 4.6 Fleet assignment for Ultimate Air

Flight no.	Origin	Destination	Fleet type
101	LAX	JFK	737-800
104	SFO	JFK	737-800
116	BOS	JFK	737-800
140	JFK	IAD	737-800
125	JFK	SFO	757-200
107	ORD	JFK	737-800
122	JFK	LAX	737-800
137	JFK	BOS	737-800
110	ATL	JFK	757-200
119	IAD	JFK	737-800
113	MIA	JFK	757-200
131	JFK	ATL	757-200
102	LAX	JFK	737-800
105	SFO	JFK	757-200
117	BOS	JFK	737-800
128	JFK	ORD	737-800
134	JFK	MIA	737-800
141	JFK	IAD	737-800
108	ORD	JFK	737-800
138	JFK	BOS	757-200
111	ATL	JFK	757-200
120	IAD	JFK	737-800
114	MIA	JFK	757-200
132	JFK	ATL	737-800
118	BOS	JFK	757-200
129	JFK	ORD	737-800
135	JFK	MIA	757-200

Table 4.6 Fleet assignment for Ultimate Air

Flight no.	Origin	Destination	Fleet type
142	JFK	IAD	737-800
103	LAX	JFK	737-800
106	SFO	JFK	737-800
126	JFK	SFO	737-800
123	JFK	LAX	737-800
109	ORD	JFK	737-800
112	ATL	JFK	737-800
133	JFK	ATL	757-200
136	JFK	MIA	757-200
115	MIA	JFK	737-800
121	IAD	JFK	737-800
124	JFK	LAX	737-800
127	JFK	SFO	737-800
130	JFK	ORD	737-800
139	JFK	BOS	737-800

Scenario Analysis

In this section we address some questions pertaining to the number of aircraft and different fleet combinations.

Case 1

It may be of interest to us to see what is the minimum number of aircraft to cover all flights. In this case, the objective function is modified to minimize the total number of aircraft. Therefore, the fleet size constraints are deleted from the set of constraints and become the objective function as follows:

$$\begin{aligned} Min\ & G_{L6,1} + G_{S6,1} + G_{B6,1} + G_{O6,1} + G_{A6,1} + G_{I6,1} + G_{M6,1} + G_{J42,1} + \\ & G_{L6,2} + G_{S6,2} + G_{B6,2} + G_{O6,2} + G_{A6,2} + G_{I6,2} + G_{M6,2} + G_{J42,2} \end{aligned}$$

Running this integer/linear program results in 13 aircraft of which 9 are 737–800 and 4 are 757–200. According to this result, the minimum number of aircraft that are needed to fly the published Ultimate Air flights is 13. However, as we will discuss in Chapter 5, the number of aircraft needed are more than 13.

Case 2

In this case, we evaluate various combinations of the two fleets. In our Ultimate Air example, we assumed that we have nine 737 and six 757 aircraft. We now change this combination to see its impact on total daily cost. Table 4.7 shows different costs associated with different number of aircraft combinations between the two fleet types.

Table 4.7 Total daily cost for various aircraft combinations

Number of B737-800 aircraft	Number of B757-200 aircraft	Total daily cost
8	7	$411,890
6	9	$416,116
11	4	$409,362
15	0	$413,970
0	15	$446,364

Fleet Assignment Model (FAM)

We now formally present the general mathematical model for the fleet assignment problem. The following model, referred to as the basic fleet assignment model (FAM), is a simplified version of FAM proposed by Hane et al., 1995.

Sets

F = Set of flights
K = Set of fleet types
C = Set of last-nodes, representing all nodes with aircraft grounded overnight at an airport in the network
M = Number of nodes in the network

Index

i = Flight Index
j = Index for fleet
k = Index for nodes

Parameters

$C_{i,j}$ = Cost of assigning fleet type *j* to flight *i*
N_j = Number of available aircraft in fleet type *j*.

$$S_{i,k} = \begin{cases} +1 \text{ if flight i is an arrival at node k} \\ -1 \text{ if flight i is a departure from node k} \end{cases}$$

Decision Variables

$$x_{i,j} = \begin{cases} 1 \text{ if flight } i \text{ is assigned to fleet-type } j \\ 0 \text{ otherwise} \end{cases}$$

$G_{k,j}$ = integer decision variable representing number of aircraft of fleet-type *j* on ground at node *k*

The integer linear programming model is as follows:

$$Min \sum_{j \in K} \sum_{i \in F} c_{i,j} x_{i,j} \qquad (4.1)$$

Subject to

$$\sum_{j \in K} x_{i,j} = 1 \qquad \text{for all } i \in F \qquad (4.2)$$

$$G_{k-1,j} + \sum_{i \in F} S_{i,k} x_{i,j} = G_{k,j} \qquad \text{for all } k \in M \text{ and } j \in K \qquad (4.3)$$

$$\sum_{k \in C} G_{k,j} \leq N_j \qquad \text{for all } j \in K \qquad (4.4)$$

$$x_{i,j} \in \{0,1\} \qquad \text{for all } i \in F \text{ and } j \in K \qquad (4.5)$$

$$G_{k,j} \in Z^+ \qquad \text{for all } k \in M \text{ and } j \in K \qquad (4.6)$$

In the above model, the objective function in (4.1) seeks to minimize the total cost of assigning the various fleet types to all the flights in the schedule. Constraints (4.2) are the flight-cover constraints to ensure that each flight is flown by one type of fleet. Constraints (4.3) are the aircraft balance constraints. The number of aircraft for any fleet type at any node is the number of aircraft of that fleet type just before that node (represented in the model by $G_{k-1,j}$) plus the arrivals (represented by $S_{i,k}$ taking a value +1) minus the departures (represented by $S_{i,k}$ taking a value of -1).

Set of constraint (4.4) represents the fleet size. The number of aircraft in fleet type *j*, should not exceed the available number of aircraft in that fleet (N_j).

Constraints (4.5) and (4.6) represent the binary and integer status of the decision variables. Z^+ is the set of positive integer numbers.

For other mathematical approaches to fleet assignment models see, for example, Jarrah et al. (2000), Ioachim et al. (1999), Barnhart et al. (1998), and Subramanian et. al. (1994).

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Chapter 5
Aircraft Routing

Introduction

The solution obtained from the fleet assignment in the previous chapter identifies the flow of fleet through the network. However, it does not identify which specific aircraft from that fleet is assigned to each flight leg. Aircraft routing is the process of assigning each individual aircraft (referred to as tail number) within each fleet to flight legs. The aircraft routing is also referred to as aircraft rotation, aircraft assignment or tail assignment. The major goal of this assignment problem is to maximize the revenue or minimize operating cost with the following considerations (Clarke et al. 1997, Gopalan and Talluri 1998, Papadakos 2009):

- Flight coverage: each flight leg must be covered by only one aircraft.
- Aircraft load balance: the aircraft must have balanced utilization loads.
- Maintenance requirements: not all the airports that an airline flies to have the capability to perform maintenance checks on all fleet types. The airlines normally have maintenance bases, typically at their hubs, for different fleet types. The maintenance consideration is to ensure that the aircraft are flown through the network in a manner that allows them to receive the required maintenance checks at the right time and at the right base.

Aircraft Tail Number

Aircraft are normally distinguished by their tail registration numbers. A tail number is a unique serial number assigned to each aircraft for each airline in each country. The airlines choose to organize their tail suffix numbering system according to their convenience. In the US, aircraft tail numbers consist of a prefix ‘N’ and five alpha/numeric characters. These characters normally represent the fleet type, the sequence of aircraft in the fleet and the airline. As an example, in N723TZ, N is the country code for USA, 723 is used to designate the particular aircraft, and TZ is the airline code for ATA. For other countries, the tail number typically consists of two characters designating the country, followed by three alpha/numeric characters. For example, a Boeing 747–4H6 for Malaysia Airlines may be assigned the tail number 9M-MPK, where 9M is the country code designator for Malaysia (Airliners.Net 2009).

Maintenance Requirements

Maintenance activities are the backbone of a successful and profitable airline company. In the airline industry, the role of maintenance is to provide safe, airworthy, on-time aircraft every day. An airline generally has a diverse fleet of aircraft. Each fleet type has a predetermined maintenance program established by the aircraft manufacturer and the Federal Aviation Administration (FAA). Aircraft maintenance must be planned and performed according to the prescribed procedures and standards.

The FAA mandates that the airlines perform four types of aircraft maintenance, commonly referred to as A-, B-, C- and D-checks. These checks vary in scope, duration, and frequency. The most common maintenance check is the A-check, which involves a visual inspection of major systems. The FAA mandates that airlines perform the A-checks approximately every 60 flight hours. This is equivalent to four–eight operating days depending on aircraft utilization. If an aircraft does not receive the A-check within this period, it is grounded until such maintenance is performed. B-checks involve a thorough visual inspection and lubricating of all moving parts. This type of maintenance is performed every 300 to 600 hours of flight. C- and D-checks involve taking the aircraft out of service, and are performed every one to four years.

The airline maintenance practices, however, are generally more stringent. They perform A-checks every three to four days. The time required to perform an A-check on an aircraft is about 3 to 10 hours. The A-checks are normally performed between 10 p.m. and 8 a.m. while the aircraft is on the ground. Therefore, the aircraft-routing problem must ensure that the aircraft is at the right base at the right time for this maintenance. Most aircraft-routing models incorporate these A-checks in their formulations since they are routine. Chapter 15 describes aircraft maintenance programs in more details.

Mathematical Approach

The fleet assignment problem for Ultimate Air, solved in Chapter 4, assigned various flight legs to our 737 and 757 fleet types. These flight legs are presented in Tables 5.1 and 5.2 for the 737-800 and 757-200 aircraft types, respectively. This section develops a mathematical model so as to assign specific aircraft within the two fleet-types to each of the flight legs.

There are several approaches to modeling aircraft routing (see for example Papadakos 2009, Sherali 2006, Talluri 1998, Arguello et al. 1997, Bard et al. 2001, Paoletti 1998, Desaulniers 1997, Bartholomew et al. 2003).

The mathematical approach adopted in this chapter is a modified model proposed by Kabbani and Patty (1992) as they applied it to American Airlines. This approach uses a set-partitioning formulation (see Chapter 2 for definition) to determine the daily routing for each aircraft. In this approach, all possible valid aircraft routings are generated. These routings are represented as rows, and the

Table 5.1 B737-800 Fleet Assignment

Flight no.	Origin	Departure time	Destination	Arrival time	(hrs)	Fleet Type
101	LAX	5:00	JFK	13:30	5.5	737-800
104	SFO	5:05	JFK	13:35	5.5	737-800
116	BOS	6:15	JFK	7:45	1.5	737-800
140	JFK	6:20	IAD	7:20	1	737-800
107	ORD	7:30	JFK	10:30	2	737-800
122	JFK	7:35	LAX	10:05	5.5	737-800
137	JFK	7:40	BOS	9:10	1.5	737-800
119	IAD	8:15	JFK	9:15	1	737-800
102	LAX	9:45	JFK	18:15	5.5	737-800
117	BOS	10:00	JFK	11:30	1.5	737-800
128	JFK	10:05	ORD	11:05	2	737-800
134	JFK	10:35	MIA	13:35	3	737-800
141	JFK	12:00	IAD	13:00	1	737-800
108	ORD	12:20	JFK	15:20	2	737-800
120	IAD	14:25	JFK	15:25	1	737-800
132	JFK	14:35	ATL	17:35	2.5	737-800
129	JFK	15:05	ORD	16:05	2	737-800
142	JFK	15:15	IAD	16:15	1	737-800
103	LAX	15:20	JFK	23:50	5.5	737-800
106	SFO	15:25	JFK	23:55	5.5	737-800
126	JFK	15:30	SFO	18:00	5.5	737-800
123	JFK	16:00	LAX	18:30	5.5	737-800
109	ORD	17:10	JFK	20:10	2	737-800
112	ATL	18:00	JFK	20:30	2.5	737-800
115	MIA	18:15	JFK	21:15	3	737-800
121	IAD	18:30	JFK	19:30	1	737-800
124	JFK	19:00	LAX	21:30	5.5	737-800
127	JFK	20:00	SFO	22:30	5.5	737-800
130	JFK	21:00	ORD	22:00	2	737-800
139	JFK	21:30	BOS	23:00	1.5	737-800

Table 5.2 B757-200 Fleet Assignment

Flight no.	Origin	Departure time	Destination	Arrival time	(hrs)	Fleet type
125	JFK	7:25	SFO	9:55	5.5	757-200
110	ATL	8:10	JFK	10:40	2.5	757-200
113	MIA	9:10	JFK	12:10	3	757-200
131	JFK	9:30	ATL	12:00	2.5	757-200
105	SFO	9:50	JFK	18:20	5.5	757-200
138	JFK	12:30	BOS	14:00	1.5	757-200
111	ATL	13:10	JFK	15:40	2.5	757-200
114	MIA	14:30	JFK	17:30	3	757-200
118	BOS	15:00	JFK	16:30	1.5	757-200
135	JFK	15:10	MIA	18:10	3	757-200
133	JFK	18:05	ATL	20:35	2.5	757-200
136	JFK	18:10	MIA	21:10	3	757-200

flights as columns in the set-partition matrix. We then seek to identify the best routes that cover all flights while meeting maintenance opportunities, turn-around time, routing cycles, and so on.

Maintenance Routing

The mathematical approaches to the aircraft-routing problem typically assume that the same schedule is repeated daily over a period of time. A similar approach is adopted for the weekends, when the frequency of flights is lower.

In our Ultimate Air example, we assume that we have the maintenance facilities for the two fleet types only at our hub, that is, JFK. Each aircraft must be routed so that it stays overnight at JFK, at most after three days of operation.

Valid Routings

For a routing to be valid, it needs to incorporate the turn-around time. Turn-around time is the minimum time needed for an aircraft from the time it lands until it is ready to depart again. This time includes the taxi into the gate, unloading passengers and baggage, cleaning, inspections, boarding new passengers, loading new baggage, and so on. The turnaround time varies from 20 minutes to 1 hour among airlines.

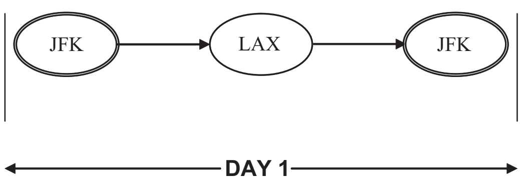
In the Ultimate Air example, we assume that the turn-around time is 45 minutes. According to this turn-around time, a valid routing cannot include flight 113 followed by flight 138 in our 757 fleet. As we see in Table 5.2, flight 113 arrives at JFK at 12:10, while flight 138 departs JFK at 12:30. The turn-around time is 20 minutes, which is less than our minimum of 45 minutes.

Routing Cycles

For Ultimate Air, we assume that only routes with three-day closed cycles are valid. A closed cycle is when an aircraft starts from a city, and at the end of the three-day cycle, ends up at that same city to start another cycle. This requirement is included to better present the process of aircraft routing by reducing the number of potential routings. It should be noted that closed cycles are not typically a requirement for airlines. The airlines usually develop monthly aircraft routing with no closed cycles. That is, an aircraft has the potential to have a totally different routing every day with no pattern or cycles as long as it receives the required maintenance checks.

Figure 5.1 presents a valid sample of a one-day routing. The aircraft stays at JFK every night and repeats the cycle every day. This routing provides a maintenance opportunity for the aircraft every night. It should be noted that each time an aircraft is at a maintenance station, it does not necessarily mean that maintenance is performed on the aircraft.

Flight No.	Origin	Departure Time	Destination	Arrival Time	(Hrs.)	Fleet Type
DAY 1						
122	JFK	7:35	LAX	10:05	5.5	737-800
103	LAX	15:20	JFK	23:50	5.5	737-800



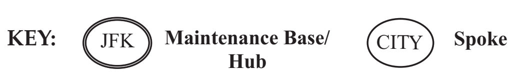


Figure 5.1 B737-800 one-day routing

Figure 5.2 presents a valid sample of a two-day routing. The aircraft starts-off at LAX, spends the first night at JFK, and on the second night is routed back to LAX.

Flight No.	Origin	Departure Time	Destination	Arrival Time	Flight Hrs	Fleet Type
			DAY 1			
101	LAX	5:00	JFK	13:30	5.5	737-800
129	JFK	15:05	ORD	16:05	2	737-800
109	ORD	17:10	JFK	20:10	2	737-800
			DAY 2			
140	JFK	6:20	IAD	7:20	1	737-800
120	IAD	14:25	JFK	15:25	1	737-800
127	JFK	19:00	LAX	21:30	5.5	737-800

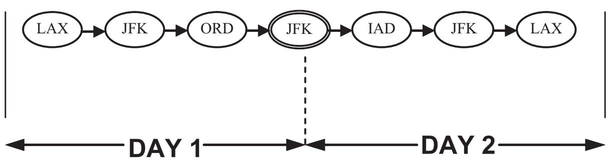


Figure 5.2 B737-800 two-day routing

Similarly, Figure 5.3 presents a valid sample of a three-day routing, where the aircraft is routed to JFK at the end of the second day for maintenance.

Figure 5.4 shows an invalid routing, as it does not provide a maintenance opportunity at JFK after three days of operations.

Note that in our Ultimate Air example, we only selected one, two and three-day routing cycles. The airlines may extend these routings to weekly routings, and so on with a maintenance opportunity every three days.

Route Generators

For the proposed set-portioning mathematical model, we begin by generating all possible valid aircraft routings. It may seem that generating these routes is a very difficult and tedious task. This is certainly the case if we want to enumerate all possible routes manually. Automated systems are used extensively to generate and filter these routes for the airlines in a relatively short time.

Flight No.	Origin	Departure Time	Destination	Arrival Time	Flight Hrs	Fleet Type
			DAY 1			
107	ORD	7:30	JFK	10:30	2	737-800
141	JFK	12:00	IAD	13:00	1	737-800
120	IAD	14:25	JFK	15:25	1	737-800
124	JFK	19:00	LAX	21:30	5.5	737-800
			DAY 2			
101	LAX	5:00	JFK	13:30	5.5	737-800
129	JFK	15:05	ORD	16:05	2	737-800
109	ORD	17:10	JFK	20:10	2	737-800
			DAY 3			
140	JFK	6:20	IAD	7:20	1	737-800
119	IAD	8:15	JFK	9:15	1	737-800
141	JFK	12:00	IAD	13:00	1	737-800
120	IAD	14:25	JFK	15:25	1	737-800
130	JFK	21:00	ORD	22:00	2	737-800

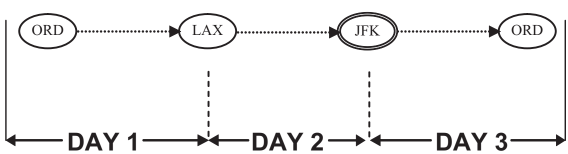


Figure 5.3 B737-800 three-day routing

Recall that in our Ultimate Air example, we are only interested in three-day cycle aircraft routings. That is, after three days the aircraft ends up at the same airport from which it started out on the first day of its cycle, only to repeat another cycle. To provide the maintenance opportunity for the aircraft, the routing must include at least one overnight stay at JFK.

A computer program was developed to generate three-day-cycle aircraft routes. These aircraft are routed through a series of feasible flights. This route then selects at least one overnight stay in JFK at the end of the first and/or second day for maintenance. At the end of the third day, the aircraft is routed back to the airport

Flight No.	Origin	Departure Time	Destination	Arrival Time	Flight Hrs	Fleet Type
			DAY 1			
116	BOS	6:15	JFK	7:45	1.5	757-200
131	JFK	9:30	ATL	12:00	2.5	757-200
111	ATL	13:10	JFK	15:40	2.5	757-200
133	JFK	18:05	ATL	20:35	2.5	757-200
			DAY 2			
110	ATL	8:10	JFK	10:40	2.5	757-200
138	JFK	12:30	BOS	14:00	1.5	757-200
118	BOS	15:00	JFK	16:30	1.5	757-200
139	JFK	21:30	BOS	23:00	1.5	757-200
			DAY 3			
116	BOS	6:15	JFK	7:45	1.5	757-200
131	JFK	9:30	ATL	12:00	2.5	757-200
111	ATL	13:10	JFK	15:40	2.5	757-200
139	JFK	21:30	BOS	23:00	1.5	757-200
			DAY 4			
117	BOS	10:00	JFK	11:30	1.5	757-200
138	JFK	12:30	BOS	14:00	1.5	757-200
118	BOS	15:00	JFK	16:30	1.5	757-200
133	JFK	18:05	ATL	20:35	2.5	757-200
			DAY 5			
110	ATL	8:10	JFK	10:40	2.5	757-200
138	JFK	12:30	BOS	14:00	1.5	757-200
118	BOS	15:00	JFK	16:30	1.5	757-200
136	JFK	21:30	BOS	23:00	1.5	757-200

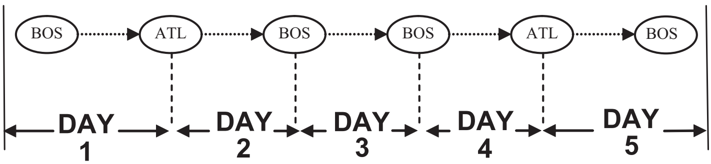


Figure 5.4 B757-200 five-day routing with no opportunity for overnight maintenance at the JFK hub

where it started its three-day cycle. The steps or pseudo-code for this program are as follows:

- Read the flight numbers, departure and arrival cities, as well as departure and arrival times for a set of flights assigned to a specific fleet (identified by fleet routing).
- Create all possible valid one-day routings incorporating turn-around times – place in a file.
- Attach each feasible one day routing of this file to all other one-day routings in this file. Do this step twice to create three-day routings – place in a file.
- Examine each element of this three-day file according to the following criteria:
  - It starts and ends at the same city.
  - Each day, flights start at the city where the aircraft ended the day before.
  - An overnight stay at JFK occurs at least once.
- Add each element that satisfies all the above conditions to a file of potential valid three-day routing candidates.

This program also generates the mathematical model suitable for linear programming software. Running this program generated a total of 6,221 and 455 valid three-day routings for the 737-800 and 757-200 fleet types respectively! Tables 5.3 and 5.4 show samples of five valid three-day routings for each fleet type respectively.

Table 5.3 Sample three-day routing for B757-200 fleet

SAMPLE	DAY 1	DAY 2	DAY 3	Utilization (hrs)
High utilization				
Routing sample #1	**FLT 131**, **FLT 111**, **FLT 133**	**FLT 110**, **FLT 138**, **FLT 118**, **FLT 133**	**FLT 110**, **FLT 138**, **FLT 118**	21
City-pair routing	JFK-ATL, ATL-JFK, JFK-ATL	ATL-JFK, JFK-BOS, BOS-JFK, JFK-ATL	ATL-JFK, JFK-BOS, BOS-JFK	
Routing sample #2	**FLT 110**, **FLT 138**, **FLT 118**, **FLT 136**	**FLT 113**	**FLT 138**, **FLT 118**, **FLT 133**	17
City-pair routing	ATL-JFK, JFK-BOS, BOS-JFK, JFK-MIA	MIA-JFK	JFK-BOS, BOS-JFK, JFK-ATL	
Medium utilization				
Routing sample #3	**FLT 136**	**FLT 113**, **FLT 133**	**FLT 110**, **FLT 138**, **FLT 118**	14
City-pair routing	JFK-MIA	MIA-JFK, JFK-ATL	ATL-JFK, JFK-BOS, BOS-JFK	
Routing sample #4	**FLT 133**	**FLT 111**	**FLT 131**, **FLT 111**	10
City-pair routing	JFK-ATL	ATL-JFK	JFK-ATL, ATL-JFK	
Low utilization				
Routing sample #5	**FLT 138**, **FLT 118**	**FLT 138**	**FLT 118**	6
City-pair routing	JFK-BOS, BOS-JFK	JFK-BOS	BOS-JFK	

Table 5.4 Sample three-day routing for B737-800 fleet

SAMPLE	DAY 1				DAY 2				DAY 3				Utilization (hrs)
High Utilization													
Routing sample #1	**FLT 122**	**FLT 103**			**FLT 122**	**FLT 103**			**FLT 122**	**FLT 103**			33
City Pair Routing	JFK-LAX	LAX-JFK			JFK-LAX	LAX-JFK			JFK-LAX	LAX-JFK			
Routing sample #2	**FLT 137**	**FLT 117**	**FLT 123**		**FLT 101**	**FLT 129**	**FLT 109**	**FLT 139**	**FLT 116**	**FLT 134**	**FLT 115**		27
City Pair Routing	JFK-BOS	BOS-JFK	JFK-LAX		LAX-JFK	JFK-ORD	ORD-JFK	JFK-BOS	BOS-JFK	JFK-MIA	MIA-JFK		
Medium Utilization													
Routing sample #3	**FLT 109**				**FLT 137**	**FLT 117**	**FLT 124**		**FLT 101**	**FLT 142**	**FLT 121**	**FLT 130**	20
City Pair Routing	ORD-JFK				JFK-BOS	BOS-JFK	JFK-LAX		LAX--JFK	JFK-IAD	IAD-JFK	JFK-ORD	
Routing sample #4	**FLT 101**	**FLT 139**			**FLT 116**				**FLT 122**				14
City Pair Routing	LAX-JFK	JFK-BOS			BOS-JFK				JFK-LAX				
Low Utilization													
Routing sample #5	**FLT 116**	**FLT 141**	**FLT 120**	**FLT 139**	**FLT 116**				**FLT 137**				8
City Pair Routing	BOS-JFK	JFK-IAD	IAD-JFK	JFK-BOS	BOS-JFK				JFK-BOS				

Mathematical Model for 757-200 Fleet

Since the 757-200 fleet has a lower number of flights and routing candidates, we start by developing the mathematical model for this fleet. The mathematical model for the 737-700 fleet will follow later on in this chapter.

Decision Variable

The goal of the aircraft-routing problem is to assign routes to individual aircraft within a specific fleet type. In the previous section, we generated all possible valid routings. Each of these routings qualifies as a candidate to be assigned to an aircraft. Among all these candidates, we need to identify those routings that optimize the objective function and satisfy the constraints.

We define the following binary decision variable to find such routings for the 757-200 fleet.

Let:

$$x_j = \begin{cases} 1 \text{ if route } j \text{ is selected}, & j=1,2,..,455 \\ 0 \text{ otherwise} \end{cases}$$

Objective Function

The available mathematical models use different measures for the objective function (see list of references). Some of these measures include:

- *Maximizing through values*. Non-stop flights are the first choice for passengers. In the absence of such point-to-point flights, passengers must take connecting flights. A through flight is a type of connection that uses the same aircraft for the flights involved. This enables the passengers to remain onboard rather than deplaning, searching for and walking to their connecting flight-gate. Through flights are especially attractive in very busy airports. Accordingly, the airlines place higher values on those routes with favorable through flights (Jarrah and Strehler 2000, Clarke et al. 1997).
- *Minimizing cost*. Airlines may assign pseudo-costs to penalize routings which they consider to be unfavorable. These unfavorable routes may include bad connection times and circular routings where aircraft are isolated by flying between a small number of spokes, and so on. (Armacost 2002).
- *Maximizing maintenance opportunities*. Those routings that provide multiple maintenance opportunities for the aircraft are given higher weights.

Assume that in our Ultimate Air case, the objective is to select those routings that maximize maintenance opportunities. To clarify this, let us return to the five sample routings for the 757-200 fleet presented in Table 5.3. The first three samples have only one overnight stay at JFK in their three-day cycles. Accordingly, the coefficients of these variables in the objective function are one. For sample routings four and five, this coefficient is two since they have two overnight stays at JFK in their three-day cycles. We determine these coefficients for every routing candidate for this fleet. Again, a simple computer program can easily generate these coefficients. Thus, the objective function for our 757-200 fleet is as follows:

$$\text{Maximize} \sum_{j=1}^{455} m_j x_j$$

where:

m_j = the number of maintenance opportunities for route j. The values that m_j can take are 1, 2 and 3.

Note that as we discussed earlier, we have 455 valid routings for the flights assigned to 757-200 fleet.

Constraints for 757-200 Fleet

There are two sets of constraints for our aircraft-routing problem: Flight coverage and the number of available aircraft.

Flight Coverage

Each routing candidate covers a certain number of flights in its three-day cycle. Each flight must be covered everyday. For example, sample 1 routing candidate for the 757-200 fleet in Table 5.3 covers flights 131,111 and 133 in day one. In day two it covers flights 110,138, 118 and 133. This routing covers flight 131 in its first day but does not fly this flight in the other two days of its cycle. Accordingly, other routings with flight 131 in their second and third day of cycles must be selected to cover flight 131 in all three days. To cover all flights, we need one constraint for each flight for each day of the three-day cycle.

As an example, searching through all the 455 routing candidates, only six candidates actually cover flight 125 in different days as shown in Table 5.5.

Table 5.5 Routing candidates for flight 125

Routing Candidate Variable	Day 1	Day 2	Day 3
x_1	125	105	131-111
x_2	125	105	138-118
x_3	131-111	125	105
x_4	138-118	125	105
x_5	105	131-111	125
x_6	105	138-118	125

According to the above variable notations, to cover flight 125 in day one, we write the following constraint:

$$x_1 + x_2 = 1$$

This is because flight 125 in day one only appears in x_1 and x_2. Similarly, to cover this flight in the second and third day of the cycle we write the following constraints:

$x_3 + x_4 = 1$ flight 125 in the second day of the cycle

$x_5 + x_6 = 1$ flight 125 in the third day of the cycle

Similarly, we write the constraints for the other 11 flights. The total number of constraints required to cover all daily flights for the 757-200 fleet is 36 (12 flights × 3-day cycles).

Number of Available Aircraft

Each routing candidate is a three-day cycle assigned to one aircraft. Accordingly, the number of selected routes should not exceed the available number of aircraft in the fleet. In Chapter 4, we assumed that we have six 757-200 aircraft. The following constraint ensures that the number of selected routes is limited to the number of aircraft.

$$x_1 + x_2 + \ldots + x_{455} \leq 6$$

Solution for 757-200 Fleet

We used an optimization software to solve this problem. The program reported that there is no feasible solution to this problem! That is, with six aircraft, it is not possible to cover all the flights assigned to the 757-200 fleet. However, our fleet routing in Chapter 4 showed that these six aircraft are capable of flying all our 757-200 flights through the network. So, why do we not get a feasible solution to our aircraft-routing problem? The answer is that the fleet-routing problem does not consider the following constraints that we have imposed on our aircraft routings.

- A 45-minute turn-around time.
- Three-day closed cycles, starting and ending at the same city. This requirement eliminates a large number of potential routes that are perfectly acceptable to the airlines. Note that we introduced this arbitrary requirement to reduce the problem size.
- At least one overnight stay at JFK for maintenance in a three-day period.

These additional constraints in the aircraft-routing problem result in an infeasible solution for our problem.

To search for solutions, we eliminated the constraint on the number of available aircraft to see how many aircraft would be needed to fly the proposed daily schedule of flights assigned to the 757-200 fleet. We ran this model, and the feasible solution now required eight aircraft. The solution for this model with eight aircraft is presented in Table 5.6.

Table 5.6 Feasible eight aircraft solution for the 757-200 fleet

Routing	DAY 1	DAY 2	DAY 3
1	125	105	138-118
2	110	131-111	131-111-133
3	113-135	114	136
4	131-111-136	113-136	114
5	105	138-118	125
6	114	135	113-135
7	138-118	125	105
8	133	110-133	110

It should be noted that the airlines frequently face this problem where the existing aircraft are not enough to fly the proposed schedule. The main reason is that the arriving and departing flights in the proposed schedule are not synchronized.

Let us look at our Ultimate Air schedule and set of constraints. We see that the two flights, 125 and 105, have only two routing candidates each day while other flights have many possibilities (see the constraints for flight 125 in the previous section). Table 5.7 examines these two flights more closely.

Table 5.7 Flights 105 and 125

Flight no.	Origin	Departure time	Destination	Arrival time	(hrs)	Fleet type
125	JFK	7:25	SFO	9:55	5.5	757-200
105	SFO	9:50	JFK	18:20	5.5	757-200

Looking at Table 5.7, we see that flight 125 arrives at SFO at 9:55. The aircraft flying this flight cannot fly flight 105 because it departs at 9:50. Therefore, the aircraft flying flight 125 to SFO is stranded for the entire day, as there are no other flights from SFO for it to connect with. So, one possibility that the operations team at Ultimate Air may consider is to synchronize these two flights. To do this, we need to delay the departure time for flight 105 (or fly flight 125 earlier). If we delay flight 105 by one hour to incorporate our 45-minute turn-around time, then these two flights can be paired. The revised schedule for these two flights is shown in Table 5.8.

With this revised schedule, the two flights, 125 and 105, can be paired. This change is incorporated into the route generator program and the revised three-day

Table 5.8 Revised schedule for flight 105

Flight no.	Origin	Departure time	Destination	Arrival time	(hrs)	Fleet type
125	JFK	07:25	SFO	09:55	5.5	757-200
105	SFO	10:50	JFK	19:20	5.5	757-200

valid routes are generated. The process for developing the linear integer model is repeated, as described earlier. Solving the new model generates multiple optimal solutions with six available aircraft. Table 5.9 presents one of these solutions. As we see, the same routings are repeated every day of the three-day cycle, but in different sequences, which result in multiple optimum solutions.

Table 5.9 One of the optimal solutions with six aircraft

Routing	DAY 1	DAY 2	DAY 3
1	125-105	135	114
2	110-138-118-136	113	131-111-133
3	113	131-111-133	110-138-118-136
4	131-111-133	110-138-118-136	113
5	114	125-105	135
6	135	114	125-105

As this process has shown, changing the departure time for one flight results in a solution with two less aircraft. Furthermore, examining this solution more closely, we notice that the aircraft flying flights 113, 114 and 135 are also stranded at their respective destinations, away from the JFK hub, at the end of the day. Despite the fact that we are covering all our flights with the available six aircraft of the 757-200 fleet type, it is possible to add more flights without needing more aircraft. Further synchronizing the arrival and departure times for these flights will further reduce number of aircraft needed.

The value of the objective function for this solution is nine. This represents the total number of aircraft grounded overnight at JFK over the three-day cycle. The check mark (✔) in Table 5.10 shows the overnight aircraft stays at JFK for the above solution.

According to this solution, each night, three 757-200 aircraft stay at JFK for maintenance. Routes 1, 5, and 6 provide two maintenance opportunities each during their three-day cycles.

This process of changing arrival/departure times is very common among airlines. The initial schedule proposed by the marketing department and schedule-builders

Table 5.10 Overnight stays at JFK for the optimal solution

Routing	Night 1	Night 2	Night 3
1	✔		✔
2		✔	
3	✔		
4			✔
5	✔	✔	
6		✔	✔
Total	**3**	**3**	**3**

(Chapter 3) is submitted to the operations team for feasibility. The operations team provides feedback to the schedule-builders on operational feasibility and possible changes to the schedule. This feedback process continues until all parties are satisfied with the schedule.

Once the airline finds its routings to be feasible and satisfactory, it then assigns each route to a particular aircraft tail number. Note that in the above aircraft-routing process, we are indifferent to the method used for assigning tail numbers to the selected routes. If, however, there are such influencing factors as aircraft age within the fleet, then the airline may use some rule/criteria for assigning specific tail numbers to routes.

Solution for 737-800 Fleet

The same mathematical model approach as described earlier for the 757-200 fleet is adopted for aircraft routing of the 737-800 fleet. Recall that we have nine aircraft in this fleet. Again, there are no feasible solutions to this aircraft-routing problem with only nine aircraft. Relaxing this constraint, results in the solution presented in Table 5.11, which requires 12 aircraft.

Similarly, examining this solution, we notice that flights 102, 106, 126, 112 and 123 are all stranded at their respective destinations at the end of the day. We need one aircraft each day just to fly these flights. Table 5.12 shows the detailed schedule for these five flights.

In an effort to pair the above flights, considering our 45-minute turn-around time, the revised schedule is presented in Table 5.13.

Incorporating these changes, and running the program with this revised schedule, still results in no feasible solution. That is, even with these changes, it is still not possible to fly all flights with nine aircraft in a three-day cyclic routing.

Table 5.11 Solution for aircraft routing of 737-800 fleet with 12 aircraft

Routing	Day 1	Day 2	Day 3
1	101-142-121-139	116-134-115	140-119-128-108-124
2	116-134-115	126	104-142-121-139
3	104-126	106	126
4	140-119-128-108-127	104-132	112
5	102	122-103	123
6	107-141-120-124	102	137-117-129-109-130
7	132	112	122-103
8	106	137-117-142-121-130	107-141-120-127
9	122-103	123	102
10	123	101-129-109-139	116-134-115
11	137-117-129-109-130	107-141-120-127	106
12	112	140-119-128-108-124	101-132

Table 5.12 Flight schedule for B737-800 stranded flights

Flight no.	Origin	Departure time	Destination	Arrival time	(hrs)
102	LAX	09:45	JFK	18:15	5.5
106	SFO	15:25	JFK	23:55	5.5
126	JFK	15:30	SFO	18:00	5.5
123	JFK	16:00	LAX	18:30	5.5
112	ATL	18:00	JFK	20:30	2.5

Table 5.13 Revised flight schedule for B737-800 stranded flights

Flight no.	Origin	Departure time	Destination	Arrival time	(hrs)
102	LAX	07:45	JFK	16:15	5.5
106	SFO	10:25	JFK	18:55	5.5
126	JFK	18:30	SFO	21:00	5.5
123	JFK	19:00	LAX	21:30	5.5
112	ATL	19:00	JFK	21:30	2.5

By relaxing the constraint on the number of aircraft, we see that the minimum number of aircraft required to fly the daily schedule of 737-800 flights is 10. The changes made to the flight schedule for the stranded flights (Table 5.13) reduced the number of aircraft needed from 12 to 10. Table 5.14 shows the routings for this 10-aircraft solution.

Table 5.14 Aircraft routing solution for B737-800 with revised schedule

Routing	Day 1	Day 2	Day 3
1	101-126	104-142-121	141-120-123
2	104-132-112	140-119-128-108-124	102-126
3	116-134-115	129-109-130	107-129-109-139
4	140-119-128-108-127	106-139	116-134-115
5	107-142-121	137-117-127	104-142-121-130
6	122-103	122-103	122-103
7	137-117-129-109-130	107-141-120-126	106
8	102-123	101-132-112	140-119-128-108-124
9	141-120-124,	102-123	101-132-112
10	106-139	116-134-115	137-117-127

Other minor changes to the flight schedule also failed to generate a solution that flies all the above flights with nine aircraft. Again, our routing problem here is more restricted than a typical airline-routing problem because of our closed-cycle requirement.

It is, of course, possible to manually make major changes to the schedule by pairing the flights such that a feasible solution is obtained with nine aircraft. Tables 5.15 (schedule) and 5.16 (routings) represent such a solution with a totally modified schedule. However, it is not clear if this operationally feasible solution is also attractive to the marketing department and passengers.

Mathematical Models

In this section, the above mathematical model as proposed by Kabbani and Patty (1992) is formally presented.

Sets

F = Set of flights
R = Set of feasible routings

Table 5.15 B737-800 fleet schedule with major modifications

Flight no.	Origin	Departure time	Destination	Arrival time	(hrs)
101	LAX	11:00	JFK	19:30	5.5
104	SFO	12:30	JFK	21:00	5.5
116	BOS	09:30	JFK	11:00	1.5
140	JFK	06:20	IAD	07:20	1
107	ORD	10:00	JFK	12:00	2
122	JFK	07:35	LAX	10:05	5.5
137	JFK	07:00	BOS	08:30	1.5
119	IAD	08:15	JFK	09:15	1
102	LAX	12:30	JFK	21:00	5.5
117	BOS	17:00	JFK	18:30	1.5
128	JFK	07:00	ORD	09:00	2
134	JFK	09:00	MIA	12:00	3
141	JFK	12:00	IAD	13:00	1
108	ORD	16:00	JFK	18:00	2
120	IAD	14:25	JFK	15:25	1
132	JFK	17:00	ATL	19:30	2.5
129	JFK	13:00	ORD	15:00	2
142	JFK	16:25	IAD	17:25	1
103	LAX	13:00	JFK	21:30	5.5
106	SFO	13:30	JFK	22:00	5.5
126	JFK	08:30	SFO	11:00	5.5
123	JFK	08:00	LAX	10:30	5.5
109	ORD	21:45	JFK	23:45	2
112	ATL	20:30	JFK	23:00	2.5
115	MIA	13:00	JFK	16:00	3
121	IAD	18:30	JFK	19:30	1
124	JFK	09:00	LAX	11:30	5.5
127	JFK	10:00	SFO	12:30	5.5
130	JFK	19:00	ORD	21:00	2
139	JFK	14:00	BOS	15:30	1.5

Table 5.16 Aircraft routing for B737-800 with nine aircraft

Routing	Day 1	Day 2	Day 3
1	122-101	126-104	123-102
2	126-104	123-102	124-103
3	123-102	124-103	127-106
4	124-103	127-106	122-101
5	127-106	122-101	126-104
6	140-119-141-120-142-121	137-116-139-117	128-107-129-108-130-109
7	137-116-139-117	128-107-129-108-130-109	134-115-132-112
8	128-107-129-108-130-109	134-115-132-112	140-119-141-120-142-121
9	134-115-132-112	140-119-141-120-142-121	137-116-139-117

Indices

j = Route index
i = Flight index

Parameters

c_j = Cost of route j
$a_{i,j}$ = 1 if flight i is covered by route j, and 0 otherwise
N = Total number of aircraft in the fleet

Decision variable

$$x_j = \begin{cases} 1 \text{ if route } j \text{ is selected} \\ 0 \text{ otherwise} \end{cases}$$

Minimize $\sum_{j \in R} c_j x_j$

Subject to:

$$\sum_{j \in R} a_{i,j} x_j = 1 \qquad \text{for all } i \in F \tag{5.1}$$

$$\sum_{j \in R} x_j \leq N \tag{5.2}$$

$$x_j \in \{0,1\} \qquad \text{for all } j \in R$$

In the above integer linear program, the objective function seeks to minimize the total cost of selected routes. If other objectives, such as the ones presented in this chapter, are sought, then the above objective function can accordingly be modified. Constraint (5.1) ensures that each flight is covered by one and only one route. Constraint (5.2) restricts the number of selected routes to the available number of aircraft within a particular fleet type.

Recent work attempts to solve the fleet assignment and aircraft-routing problems simultaneously (Papadakos 2009, Sherali 2006, Barnhart et al. 1996, Ioachim et al. 1999). Papadakos (2009) proposes several integrated models to solve fleet assignment, aircraft routing and crew scheduling (discussed in the next chapter) simultaneously. Sherali et al (2006) review models that integrate fleet assignment with aircraft routing. Barnhart et al. (1996) propose a model based on strings of flights as decision variables. These strings start and end at a maintenance station, with maintenance being performed after the last flight. The departure time of the string is the departure time of the first flight, and arrival time is the arrival time of the last flight in the sequence. Cordeau et al. (2001) propose a simultaneous approach to aircraft routing and crew scheduling. These methods result in a large number of decision variables.

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Chapter 6
Crew Scheduling

Introduction

Crew scheduling involves the process of identifying sequences of flight legs and assigning both the cockpit and cabin crews to these sequences. Crew scheduling, like aircraft routing (Chapter 5), is normally performed after the fleet-assignment process.

Total crew cost, including salaries, benefits, and expenses, is the second largest cost figure, after the cost of fuel, for airlines. Table 6.1 presents the total number of crew, annual crew salaries and benefits, and flight-crew expenses for select US airlines.

The third column in this table represents regular flight-crew salaries and benefits. The fourth column, flight-crew expenses, includes per diems and other expenses incurred for hotels, parking, meals, taxi-cabs, among others, in order for an airline to maintain its crew at a city other than their home base. Note that this cost is in addition to the salaries and benefits that the airlines pay to their flight crew. The last column shows flight-crew expenses as a percentage of salaries and benefits (column 4 divided by column 3).

Table 6.1 Crew cost for US major carriers

Carrier	Number of flight crew[1]	Flight crew expenses[2] (000)	Crew expense/ operating expense[2] (%)
Alaska	1,455	180,845,000	5.57%
AirTran	1,632	157,383,851	6.00%
American	11,166	1,152,808,000	4.48%
Continental	4,867	623,767,000	4.05%
Delta	12,299	802,811,000	3.84%
Southwest	5,915	965,329,000	9.13%
United	6,478	757,020,000	3.44%
US Airways	5,275	482,044,882	3.39%

Source: Airline Pilot Central[1]; BackAviation Form 41 iNET.[2]

Unlike the fuel cost, a large portion of flight-crew expenses are controllable (Anbil, 1991). As Table 6.1 suggests, even a small percentage of savings in flight-crew expenses through better scheduling translates into millions of dollars, which ultimately can determine the survival or demise of an airline. Because of such large anticipated savings, the crew scheduling problem has received considerable attention from both academia and industry.

Crew scheduling is one of the most computationally intensive combinatorial problems (see Ryan 1992, Bixby et al. 1992, Gamache et al. 1998, Klabjan 2001, and Barnhart 2008). Computational complexity will be discussed in detail in Chapter 13. The crew scheduling problem is typically solved in two phases, crew pairing and crew rostering. This is mainly because the two problems are too large to address simultaneously.

Crew Pairing

The first phase in the crew scheduling is to develop crew pairing. Crew pairing is a sequence of flight legs, within the same fleet, that starts and ends at the same crew base. A crew base is the home station or city in which the crew actually lives. Large airlines typically have several crew bases. The sequence of crew pairing must satisfy many constraints such as union, government, and contractual regulations. A crew pairing sequence may typically span from one to five days, depending on the airline. The objective of crew pairing is to find a set of pairings that covers all flights and minimizes the total crew cost. The final crew pairing includes dates and times for each day. A typical assumption in crew pairing is that flight schedules are repeated daily. This assumption may be true for the week-day schedules, but for the weekends, the airlines normally have a lower frequency of flights. The adopted approach is normally to solve the crew pairing problem for a typical weekday, and then make modifications and adjustments for the weekends.

Note that in this phase of crew pairing, we generate pairings of flight legs that are feasible and satisfy the regulations. In this phase, we do not address individual crew members. This phase is also referred to as an impersonal phase. The assignment of each specific crew member to these pairings will be discussed in the second phase, that is, crew rostering, later in this chapter.

The following definitions are used in addressing the crew-pairing problem:

- *Duty*: A working day of a crew may consist of several flight segments. The length of a duty is determined by Federal Aviation Regulations (FAR) in the United States, as well as by individual airline rules. Under the Federal law, airline pilots cannot fly more than 8 hours in a 24-hour period. They also must be able to rest for 8 hours in that same time span.
- *Sit connection*: A connection during duty is called a sit connection. This involves the waiting times, on the part of the crew, for changing planes onto their next leg of duty. Normally, airlines impose minimum and maximum sit connection times, typically between 10 minutes and 3 hours.

- *Rest*: A connection between two duties is referred to as rest, overnight connection or layover.

Figure 6.1 illustrates a sample from Ultimate Air's B757-200 fleet's two-day crew pairing, showing duty periods, sits within duty periods, overnight rests, and sign-in and sign-out times, assuming the crew home-base is at JFK. Based on this figure, a crew pairing is a sequence of duties separated by rest periods.

As Figure 6.1 for our two-day pairing suggests, the crew is staying overnight, away from their home base, and therefore, the airline has to pay for their per diems, transportation, accommodation, food, and so on.

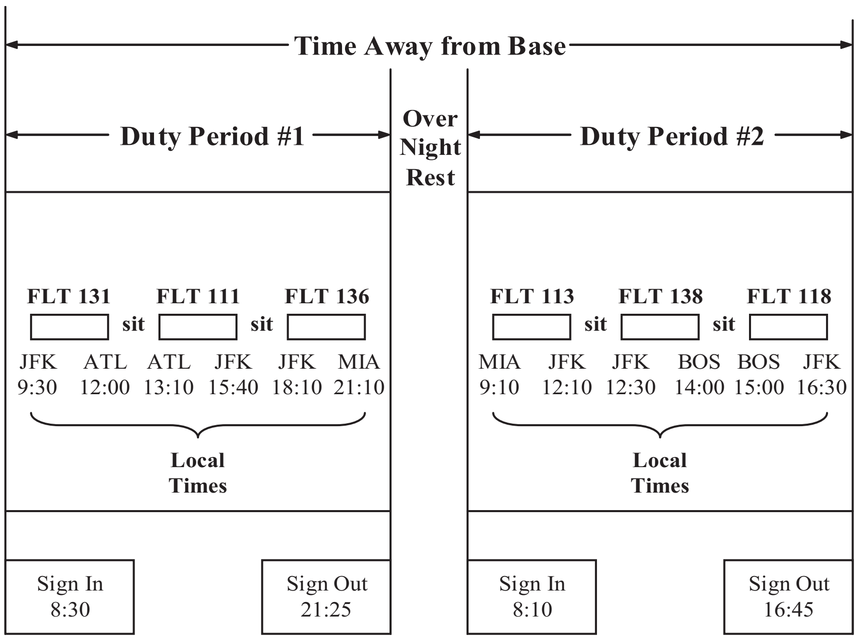

Figure 6.1 A typical pairing with duty periods, sits within duty periods, overnight rests, and sign-in and sign-out times

The objective of the crew pairing problem is to minimize the total cost of assigning crews to flight legs, such that every flight is covered, and making sure that union, government, and airline rules are satisfied. Furthermore, the constraints should also consider the number of available crews at each base. This problem usually seeks pairings that translate into a high utilization of crew flying time, and minimum sit connection times.

The airlines normally attempt to keep the crew with the same aircraft (tail number) on multiple flight legs as much as possible. This way, crew-related problems, such as delays and cancelled connecting flights, will be reduced.

Delayed, cancelled connecting flights, or other difficulties in flight pairings result in deadheading. Deadheading happens when the crew is transported as non-revenue passengers.

It should be noted that the solutions for the aircraft routing (Chapter 5) and crew pairing cannot be the same. First, crew members need more rest. An aircraft can be utilized for 14 hours in one day, but the crew can stay with the aircraft only 8 hours. Second, crew pairing identifies flight legs that start and end at the same crew base (i.e., only JFK to JFK in our case). This is not a constraint for the aircraft routing problem (where, for example SFO to SFO is possible) as long as it stays at a maintenance station overnight every 3–5 days. Third, the crew pairing problem does not consider turn-around times as they may just land with one aircraft and takeoff with another in a very short time.

Similar to aircraft routing discussed in Chapter 5, the crew-pairing problem is typically formulated as a set-partitioning problem (see Chapters 2 and 5) with some side constraints. In this set-partitioning problem, the rows of the matrix represent feasible crew pairings and the columns are scheduled daily flights.

Pairings Generators

The pairings are generated based on rules and regulations. Note that at this stage, these pairings just show the sequence of flights assigned to crew members. It starts with a crew base and adds all the feasible flight legs according to the specified rules. It finally ends up at the same crew base from which it started. A pairing satisfying all the rules and regulations is called a legal pairing. The length of a pairing depends on the airline and union regulations. A pairing may span from one to five days. Some of the rules in generating the feasible pairings include the total daily flight time, and minimum and maximum sit-connection times. All possible feasible pairings are generated during this phase. For large airlines with many daily flights, the number of pairings generated becomes very large (billions of legal pairings!). This is especially very applicable to airlines with large hubs. Each flight leg at this hub can be potentially paired with many departing flights. This combination is compounded if the aircraft is rerouted to the hub several times in a day. In such cases, the generators are normally equipped with some extra rules and filters to identify and select good potential pairings. Barnhart (2008) and Klabjan (2003) provide an overview of these rules and filters to reduce the number of pairings.

The following represents the crew pairing requirements for Ultimate Air:

- Each duty should not exceed 8 hours of flight time.
- A maximum length of two days is allowed for a routing (i.e., two-day pairings).
- The home base for the crew is JFK.
- The minimum and maximum sit-connection times are 10 minutes and 3 hours respectively.

A similar program to route generators, in Chapter 5, was developed to generate the potential crew pairings. The steps for this program are as follows:

- Read the flight numbers, along with their departure and arrival cities and times, for a set of flights assigned to a specific fleet type (as identified by fleet-routing module).
- Create all possible one and two-day pairings – place in a file.
- Examine each pairing in this file so that:
  - the pairing ends up at JFK over the routing cycle;
  - for two-day pairing, the first flight of the second day starts out at the city where it ended up the night before;
  - the duty does not exceed eight hours of flight time in any given day;
  - the sit-connection times are between the allowable minimum and maximum times.
- If a pairing satisfies all of the above conditions, it is added to a file of potential valid pairing candidates.

This program generated a total of 28 and 314 legal pairings for the 757-200 and 737-800 fleet types respectively. Note that the number of crew pairing candidates is much lower than potential aircraft routings (Chapter 5) for both fleet types. The main reason is that for crew pairing we generated only one- and two-day pairings as opposed to three-day routings in Chapter 5. Furthermore, other factors such as a maximum of eight-hour flight blocks per day, and a maximum three-hour sit-connection times, contribute to the lower numbers of possible combinations. Table 6.2, overleaf, presents all 28 legal pairings for the 757-200 fleet. In this table, if the pairings are one-day, then no flights appear in the day two column.

Since the number of combinations for the 737-800 fleet is large, five one-day and two-day pairing samples are presented in Tables 6.3 and 6.4 (see page 89).

Mathematical Model for B757-200 Fleet

Similar to Chapter 5, since the 757-200 fleet has a lower number of crew pairings, we first start developing the mathematical model for this fleet. The mathematical model for the 737-800 will follow later on in this chapter.

Decision Variable

Having generated potential valid crew pairings, the task of the mathematical model is to identify which candidates should be selected. We define the following binary decision variable:

$$x_j = \begin{cases} 1 \text{ if pairing } j \text{ is selected,} & j\text{=1,2,..,28} \\ 0 \text{ otherwise} \end{cases}$$

Table 6.2 All legal crew pairings for B757-200 fleet

Crew pairing index	Day-one flights	Day-two flights	Flight hours
1	125	105	11
2	131	110	5
3	131	111	5
4	131	110-138-118	8
5	133	110	5
6	133	111	5
7	133	110-138-118	8
8	135	113	6
9	135	114	6
10	135	113-138-118	9
11	136	113	6
12	136	114	6
13	136	113-138-118	9
14	138	118	3
15	131-111		5
16	131-111-133	110	10
17	131-111-133	111	10
18	131-111-133	110-138-118	13
19	131-111-136	113	11
20	131-111-136	114	11
21	131-111-136	113-138-118	14
22	138-118		3
23	138-118-133	110	8
24	138-118-133	111	8
25	138-118-133	110-138-118	11
26	138-118-136	113	9
27	138-118-136	114	9
28	138-118-136	113-138-118	12

Table 6.3 Sample one-day crew pairing for B737-800 fleet

SAMPLE	DAY 1				Crew utilization (hrs)
High utilization					
Pairing #1	**FLT 140**	**FLT 119**	**FLT 128**	**FLT 108**	6
City-pairs	JFK-IAD	IAD-JFK	JFK-ORD	ORD-JFK	
Dept-Arr times	6:20-7:20	8:15-9:15	10:05-11:05	12:20-15:20	
Low utilization					
Pairing #2	**FLT 140**	**FLT 119**			2
City-pairs	JFK-IAD	IAD-JFK			
Dept-Arr times	6:20-7:20	8:15-9:15			

Table 6.4 Sample two-day crew pairing for B737-800 fleet

SAMPLE	DAY 1			DAY 2			Crew utilization (hrs)
High utilization							
Pairing #3	**FLT 142**	**FLT 121**	**FLT 127**	**FLT 104**	**FLT 142**	**FLT 121**	15
City pairs	JFK-IAD	IAD-JFK	JFK-SFO	SFO-JFK	JKF-IAD	IAD-JFK	
Dept-Arr times	15:15-16:15	18:30-19:30	20:00-22:30	5:05-13:35	15:15-16:15	18:30-19:30	
Medium utilization							
Pairing #4	FLT 132	FLT 112	FLT 130	FLT 107	FLT 141	FLT 120	11
City pairs	JFK-ATL	ATL-JFK	JFK-ORD	ORD-JFK	JFK-IAD	IAD-JFK	
Dept-Arr times	14:35-17:35	18:00-20:30	21:00-22:00	7:30-10:30	12:00-13:00	14:25-15:25	
Low utilization							
Pairing #5	FLT 140			FLT 119			2
City pairs	JFK-IAD			IAD-JFK			
Dept-Arr times	6:20-7:20			8:15-9:15			

Objective Function

The determination of cost for crew pairings is a complex process (Barnhart 1997). It is based on the sum of all duty cost in the pairing, cost of time away from the base, and minimum guaranteed pay multiplied by the number of duties. The maximum of these three costs determines the above cost function for each pairing.

In our Ultimate Air example, we assume two-day pairings to be three times as costly as one-day pairings. This is because, in two-day pairings, the crew stays away from home base for one night, and hence the airline is responsible for the incurring costs. Therefore, according to Table 6.2, the cost coefficient is one for pairings 15 and 22, and three for all other pairings.

The objective functions for our 757-200 fleet, therefore, is as follows:

$$\text{Minimize} \sum_{j=1}^{28} c_j x_j$$

where:

c_j = the cost of pairing j.

For our Ultimate Air, c_j is designated the value 1 for one-day, and 3 for two-day pairings.

Flight-Coverage Constraints for B757 Fleet

Each pairing candidate covers a certain number of flights. We must ensure that the crew covers each flight exactly once. To write the coverage constraint for flight 125, according to Table 6.2, we write:

$$x_1 = 1$$

This is because flight 125 only appears in crew pairing 1. Referring to Table 6.2 again, flight 114 appears in crew pairings 9, 12, 20, and 27. Therefore to cover this flight we have:

$$x_9 + x_{12} + x_{20} + x_{27} = 1$$

Similarly, referring to Table 6.2, we can write the flight coverage constraints for the other 10 flights with this fleet type.

Note that unlike the three-day aircraft routing in which we had a constraint for each flight for each day, in crew pairing we address each flight only once. This is because we are interested in knowing which flights should be paired rather than the actual assignment of flights to days. A two-day pairing requires two sets of crews with a one-day lag. Each set of crew covers one duty of the pairing. Thus all flights are covered. We will discuss the assignment of pairings to days in the second phase, crew rostering.

Crew-Pairing Solution for B757-200 Fleet

We used an optimization software to solve the above integer linear program model. Four two-day pairings were selected. The objective function is therefore 12. Table 6.5 presents the solution, showing pairings and flights, as well as departure and arrival times.

Table 6.5 Solution to crew pairing for B757-200 Fleet

Solution	**DAY 1**			**DAY 2**		
Pairing #1	**FLT 125**			**FLT 105**		
City pairs	JFK-SFO			SFO-JFK		
Dept-Arr times	7:25-9:55			9:50-18:20		
Pairing #2	**FLT 131**	**FLT 111**	**FLT 136**	**FLT 113**	**FLT 138**	**FLT 118**
City pairs	JFK-ATL	ATL-JFK	JFK-MIA	MIA-JFK	JFK-BOS	BOS-JFK
Dept-Arr times	9:30-12:00	13:10-15:40	18:10-21:10	9:10-12:10	12:30-14:00	15:00-16:30
Pairing #3	**FLT 135**			**FLT 114**		
City pairs	JFK-MIA			MIA-JFK		
Dept-Arr times	15:10-18:10			14:30-17:30		
Pairing #4	**FLT 133**			**FLT 110**		
City pairs	JFK-ATL			ATL-JFK		
Dept-Arr times	18:05-20:35			8:10-10:40		

Crew Pairing Solution for B737-800 Fleet

Similarly, we develop the mathematical model for crew pairing of the 737-800 fleet. Solving this mathematical model generates the following solution, presented in Table 6.6, for this fleet.

Table 6.6 Solution to crew pairing for B737-800 fleet

Solution	DAY 1			DAY 2		
Pairing #1	**FLT 140**	**FLT 119**	**FLT 134**	**FLT 115**		
City pairs	JFK-IAD	IAD-JFK	JFK-MIA	MIA-JFK		
Dept-Arr times	6:20-7:20	8:15-9:15	10:35-13:35	18:25-21:25		
Pairing #2	**FLT 122**			**FLT 103**		
City pairs	JFK-LAX			LAX-JFK		
Dept-Arr times	7:35-10:05			15:20-23:50		
Pairing #3	**FLT 137**	**FLT 117**				
City pairs	JFK-BOS	BOS-JFK				
Dept-Arr times	7:40-9:10	10:00-11:30				
Pairing #4	**FLT 141**	**FLT 120**	**FLT 125**	**FLT 104**	**FLT 142**	**FLT 121**
City pairs	JFK-IAD	IAD-JFK	JFK-SFO	SFO-JFK	JFK-IAD	IAD-JFK
Dept-Arr times	12:00-13:00	14:25-15:25	7:25-9:55	5:05-13:35	15:15-16:15	18:30-19:30
Pairing #5	**FLT 132**	**FLT 112**	**FLT 130**	**FLT 107**		
City pairs	JFK-ATL	ATL-JFK	JFK-ORD	ORD-JFK		
Dept-Arr times	14:35-17:35	18:00-20:30	21:00-22:00	7:30-10:30		
Pairing #6	**FLT 129**	**FLT 109**	**FLT 139**	**FLT 116**	**FLT 128**	**FLT 108**
City pairs	JFK-ORD	ORD-JFK	JFK-BOS	BOS-JFK	JFK-ORD	ORD-JFK
Dept-Arr times	15:05-16:05	17:10-20:10	21:30-23:00	6:15-7:45	10:05-11:05	12:20-15:20
Pairing #7	**FLT 123**			**FLT 102**		
City pairs	JFK-LAX			LAX-JFK		
Dept-Arr times	16:00-18:30			9:45-18:15		
Pairing #8	**FLT 124**			**FLT 101**		
City pairs	JFK-LAX			LAX-JFK		
Dept-Arr times	19:00-21:30			5:00-13:30		
Pairing #9	**FLT 127**			**FLT 106**		
City pairs	JFK-SFO			SFO-JFK		
Dept-Arr times	20:00-22:30			15:25-23:55		

Crew-Pairing Mathematical Model

In this section, the crew pairing model, as was adopted above, is formally presented.

Sets

F = Set of flights
P = Set of feasible pairings
K = Set of crew home-base cities

Indices

j = Pairing index
i = Flight index
k = Crew home-base index

Parameters

= Cost of crew paring j

$$a_{i,j} = \begin{cases} 1 \text{ if flight } i \text{ is covered by pairing } j \\ 0 \text{ otherwise} \end{cases}$$

$$h_{k,j} = \begin{cases} 1 \text{ if home base city (starting and ending flight) for pairing } j \text{ is city } k \\ 0 \text{ otherwise} \end{cases}$$

b_{lower_k} ≡ minimum number of crew to be used at home base city k

b_{upper_k} ≡ maximum number of crew to be used at home base city k

Decision Variable

$$x_j = \begin{cases} 1 \text{ if pairing } j \text{ is part of the solution} \\ 0 \text{ otherwise} \end{cases}$$

The mathematical model is formulated as:

$$\text{Min} \sum_{j \in P} c_j x_j \tag{6.1}$$

Subject to:

$$\sum_{j \in P} a_{i,j} x_j = 1 \quad \text{for all flight legs } i \in F \tag{6.2}$$

$$b_{lower_k} \leq \sum_{j \in P} h_{k,j} x_j \leq b_{upper_k} \quad \text{for all home bases } k \in K \tag{6.3}$$

In this model, the objective function (6.1) attempts to minimize the total cost of flight pairings. Constraint (6.2) guarantees that each flight leg is covered only once. The side constraints (6.3) ensure that the selected flight pairings stay within the available number of crew members at each home base.

Some recent works (see Barnhart 2008 and Klabjan 2003) have attempted to integrate the two problems of crew pairing and aircraft routing. It should be noted that the difficulty in dealing with larger problems than those presented here will be compounded by integrating both these problems.

Crew Rostering

Once the crew pairing problem is solved, the second phase is crew rostering. Crew rostering is the process of assigning individual crew members to crew pairings, usually on a monthly basis.

Some airlines, mainly European, allow their crews to select a number of pairings as identified in the first phase, together with rest periods on specific days to construct their monthly personalized schedule (see Sarra 1998, Giafferri et al. 1982, Hjorring 2000, Konig and Strass 2000). The airline then attempts to grant these schedules if possible. Crew training days, seniority, and other internal regulations are some of the factors that influence the assignment of these schedules to crews.

US Airlines, however, develop their monthly crew schedules based on the solutions generated in the crew-pairing phase, independent of crew desires. This approach is then used to construct the monthly schedule by incorporating employee time off, training, union rules, and other contractual obligations. The airlines then assign crews to these schedules based on their in-house priority system. This method, where the employees bid for pre-constructed rosters, is referred to as a *bid line procedure*. In both rostering systems, the objective is to maximize crew utilization, evenly distributing individual crew workload and rest times.

Since the rules and regulations vary among the airlines, the crew rostering process, and the available literature on this topic, is also diverse. Some of these methods include (Gamache et al. 1999):

- assigning high priority employees to high priority pairings;
- developing monthly rosters for individual crew members based on their requests;
- developing monthly rosters for each day of the month without considering the crew requests.

It should be noted that the processes of assigning cockpit-aircrew members (captain and first officer) and cabin-aircrew members (flight attendants) are typically different. The cockpit aircrew members usually have the required licenses/type ratings to fly only a specific fleet of aircraft, while cabin aircrew members can be assigned to multiple fleet types.

Ultimate Air Rosters

As explained earlier, a roster is a series of crew pairings separated by rest periods and days off. For Ultimate Air, we attempt to develop anonymous rosters on which its employees can bid.

For presentation purposes, and in an effort to keep the rostering problem to a manageable size, we will develop the rosters on a weekly basis, instead of monthly rosters which are more common among airlines. The process of developing monthly rosters is basically the same as that of one done weekly.

The assumptions for the Ultimate Air crew rosters are as follows:

- at least one day off between pairings;
- two pairings per week;
- balanced workload among all rosters – a work week of 20 flight hours is desirable.

Table 6.7 presents all possible combinations on the allocation of pairings to days of the week. This table incorporates the above rules on two pairings per week and at least one day off between pairings. Each (✔) symbol represents a pairing. Note that each pairing spans a two-day period. Therefore, if a crew is assigned to a pairing on Monday, then this crew member will be flying both on Monday and Tuesday. Since we require at least one day rest between pairings, this crew member cannot fly on Wednesday, but can fly on Thursday, Friday, Saturday or Sunday.

Table 6.7 Possible weekly crew roster combinations for Ultimate Air

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
✔			✔			
✔				✔		
	✔			✔		
	✔				✔	
		✔			✔	
		✔				✔
			✔			✔

We assume that the assignment of crew to rosters, in each week, takes into consideration their previous week's rosters. That is to say, if a crew member is assigned to a pairing which starts on Saturday of this week, this crew member cannot be assigned to a roster which starts on Monday, and so on.

Similar to the crew pairing mathematical model in the previous section, a series of set-partitioning approaches is adopted to assign rosters to individual crew members. We use a set-partitioning approach first to identify the anonymous rosters. In this approach, the rows of the set-partitioning matrix represent the valid roster combinations, and the columns are the daily pairings, which span the entire week.

Again, since the 757-200 fleet has a smaller problem size, we develop the crew rosters for this fleet first.

Crew Rosters for B757-200 Fleet

Table 6.5 presented the solution to our crew pairing phase for the 757-200 fleet. Four pairings were identified, which covered all the scheduled 757-200 flights in a day. Let us call these four pairings P1, P2, P3, and P4. Considering these pairing combinations, and assigning these pairings to days in Table 6.7, we get 112 possible valid rosters. Table 6.8 presents three sample valid rosters with corresponding total weekly flight hours.

Table 6.8 Three sample rosters for B757-200 fleet

Sample Rosters	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Flight hrs
1	P1			P2				25
2			P3			P1		17
3		P4			P2			19

Decision Variable

Similar to crew pairing, the task of this mathematical model is to identify which rosters, among the 112 potential candidates, should be selected. We define the following decision variable:

$$x_j = \begin{cases} 1 \text{ if roster } j \text{ is selected,} & j=1,2,..,112 \\ 0 \text{ otherwise} \end{cases}$$

Objective Function

As explained earlier, a major goal of Ultimate Air is to create balanced rosters around 20 weekly flights hours. The objective function is therefore constructed in an attempt to minimize the total deviations of the rosters' weekly flight hours from the target of 20 flight hours. The objective function is therefore represented as:

$$\textit{Minimize} \quad \sum_{j=1}^{112} \left| h_j - 20 \right| \cdot x_j$$

where:

h_j = the total weekly flight hours for roster j.

We use absolute values because the term $h_j - 20$ may be positive, zero, or negative depending on the roster. In this manner, a negative deviation (low weekly flight hours) is treated as bad as a positive deviation (high weekly flight hours). Referring to our sample rosters in Table 6.8, the coefficient for the variable representing sample 1 is |25-20|=5. Similarly, the objective function coefficients for the other two samples are |17-20|=3 and |19-20|=1 respectively.

Pairing Coverage Constraints for B757-200 Fleet

Each roster candidate covers a certain number of pairings in each day. We must ensure that the rosters cover each pairing every day, exactly once. As an example, sample 1 in Table 6.8 covers pairings 1 and 2 on Monday and Thursday respectively. So this sample is a candidate to cover P1 on Monday and P2 on Thursday.

A simple program similar to Chapter 5 can search through our 112 candidates to identify which ones cover which pairings, and on what days. We have four pairings that need to fly every day of the week, which makes a total of (4 × 7) 28 constraints as follows:

$$\sum_{j=1}^{112} a_{i,j} x_j = 1 \qquad \textit{For all } i = 1, 2, .., 28$$

In this set of constraints, index i represents a specific pairing in a given day. As an example, the number 1 represents P1 on Monday, while 2 stands for P2 on Monday,…, and 28 is P4 on Sunday. The parameter $a_{i,j}$ is defined as follows:

$$a_{i,j} = \begin{cases} 1 \text{ if roster } j \text{ covers pairing } i \\ 0 \text{ otherwise} \end{cases}$$

Rostering Solution for B757-200 Fleet

The above integer linear program with 112 binary decision variables and 28 constraints was solved using optimization software. The solution for the objective function is 28 hours, which represents the sum of deviations of all rosters from our target of 20 flight hours. Table 6.9 presents the solution to these weekly rosters. There are 14 disjointed (non-overlapping) rosters, each covering two pairings per day. As we can see from the solution, each pairing is covered exactly once every day. In order to keep the flight hours more balanced, one approach is to rotate the rosters every week among the crew members. This rotation of weekly rosters not only provides a fair and balanced number of flight hours over the whole month for a particular crew member, but is also very desirable for the airlines and crew to stay current with their network of airports.

Table 6.9 Solution to crew rosters for B757-200 fleet

Rosters	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Flight Hours
1	0	P1	0	0	P3	0	0	17
2	0	0	P1	0	0	P3	0	17
3	P1	0	0	P4	0	0	0	16
4	0	0	0	P2	0	0	P3	20
5	P2	0	0	0	P4	0	0	19
6	0	P2	0	0	0	P4	0	19
7	0	0	P2	0	0	0	P4	19
8	0	P3	0	0	P1	0	0	17
9	0	0	P3	0	0	0	P1	17
10	P3	0	0	0	P2	0	0	20
11	0	0	0	P3	0	0	P2	20
12	P4	0	0	P1	0	0	0	16
13	0	P4	0	0	0	P1	0	16
14	0	0	P4	0	0	P2	0	19

According to Table 6.9, we need at least 14 captains and 14 first officers for our 757-200 fleet. The airlines normally have a number of reserve captains and first officers to accommodate unforeseen circumstances. As explained earlier, these are anonymous rosters and can be assigned to any crew member. Once these rosters are constructed, the airline, based on its rules and regulations, assigns them to each individual crew member.

Rostering Solution for B737-800 Fleet

A similar approach is adopted for deriving the solution for the 737-800 fleet. We have nine pairings for this fleet. There is a total of 567 roster candidates and 63 (9 pairings × 7 days/week) constraints. Table 6.10 presents the solution for crew rostering for this fleet. This solution generates a total of 43 hours deviation for 32 rosters.

Table 6.10 Solution to crew rosters for B737-800 fleet

Rosters	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Flight Hours
1	0	0	P1	0	0	0	P7	19
2	P1	0	0	0	P9	0	0	19
3	0	P1	0	0	0	P9	0	19
4	0	0	0	P1	0	0	P9	19
5	P2	0	0	P2	0	0	0	22
6	0	P2	0	0	0	P5	0	20
7	0	0	P2	0	0	0	P5	20
8	P3	0	0	P4	0	0	0	18
9	0	P3	0	0	P4	0	0	18
10	0	0	P3	0	0	P4	0	18
11	0	0	0	P3	0	0	P4	18
12	P4	0	0	0	P3	0	0	18
13	0	P4	0	0	0	P3	0	18
14	0	0	P4	0	0	0	P3	18
15	P5	0	0	0	P6	0	0	20
16	0	0	P5	0	0	0	P6	20
17	0	P5	0	0	0	P8	0	20
18	0	0	0	P5	0	0	P8	20
19	0	0	0	P6	0	0	P1	19

Table 6.10 ***Concluded***

Rosters	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Flight Hours
20	P6	0	0	P8	0	0	0	22
21	0	P6	0	0	P8	0	0	22
22	0	0	P6	0	0	0	P9	22
23	0	0	0	P7	0	0	P2	22
24	0	P7	0	0	0	P6	0	22
25	0	0	P7	0	0	P7	0	22
26	P7	0	0	P9	0	0	0	22
27	P8	0	0	0	P1	0	0	19
28	0	P8	0	0	P2	0	0	22
29	0	0	P8	0	0	P2	0	22
30	0	0	P9	0	0	P1	0	19
31	0	P9	0	0	P5	0	0	20
32	P9	0	0	0	P7	0	0	22

Crew-Rostering Mathematical Model

The mathematical model for crew rostering depends on how we choose to construct the rosters, that is, either individualized or anonymous rosters. The approach that was presented in this chapter was based on developing anonymous rosters. Barnhart and Klabjan provide a review of various rostering problems.

Sets:

P = Set of all pairings over all days of the roster period
R = Set of valid rosters

Indices:

j = Roster index
i = Pairing index

Parameters:

c_j = Deviation of roster j flight time from a target value

$$a_{i,j} = \begin{cases} 1 \text{ if pairing } i \text{ is covered by roster } j \\ 0 \text{ otherwise} \end{cases}$$

Decision Variable:

$$x_j = \begin{cases} 1 \text{ if roster } j \text{ is part of the solution} \\ 0 \text{ otherwise} \end{cases}$$

The mathematical model is formulated as:

$$\text{Minimize } \sum_{j \in R} c_j x_j \qquad (6.4)$$

Subject to:

$$\sum_{j \in R} a_{i,j} x_j = 1 \qquad \text{For all} \quad i \in P \qquad (6.5)$$

In this model, the objective function (6.4) attempts to minimize the total sum of deviations. Constraint (6.5) guarantees that each flight pairing in each day is covered only once.

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Chapter 7

Manpower Planning

Introduction

An airline's product is measured by its timeliness, accuracy, functionality, quality, and price (Yu 1998). The airline employees and equipment are the factors that determine such measures. Manpower planning for airlines represents one of the most important and challenging tasks, covering a wide range spanning from hiring, training, to scheduling of human resources (Yu and Thengvall 2002). The concepts of hiring and training are normally very much dependant on the airline strategic plans (Verbeek 1991). Manpower scheduling refers to the actual work plan including working, non-working days, times, shifts, locations, and leave periods. Scheduling the employees for an airline is an enormous task. There are pilots, flight attendants, ground crew, baggage handlers, reservationists, cooks, janitors, mechanics, administrators, and so on.

The main purpose of manpower scheduling is to derive a cyclic (normally weekly) plan for each employee so that the total manpower costs are minimized, efficiency and utilization are maximized, subject to meeting the requirements and regulations (Brusco and Jacobs 1998).

Chapter 6, on crew scheduling, presented the process of assigning flight crews to flight legs while this chapter introduces mathematical models on manpower planning for ground crews. Simulation models are also used to plan for manpower planning (Chapter 15).

Mathematical Modeling Case Study

We begin the introduction to the mathematical model by applying it to our case study. Table 7.1 presents the weekly manpower requirements for ground operations (check-in counters and baggage handlers) at JFK for our Ultimate Air airline example.

The weekly manpower requirements are normally different at different times of the day and different days of the week. The daily operations are divided into four time blocks with duration of four hours each. According to this table, for example, on Mondays from 6 a.m. – 10 a.m., we need eight employees, and so on. The following contractual issues and airline policies apply:

- Each employee works for eight hours consecutively in a day.

- There are currently three working shifts: shift 1 (6 a.m. – 2 p.m.), shift 2 (10 a.m. – 6 p.m.) and shift 3 (2 p.m. – 10 p.m.).
- Each employee works for five days consecutively followed by two days off.

Table 7.1 Check-in counter agents requirement at JFK for Ultimate Air

Shift/day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
6 a.m. – 10 a.m.	8	8	8	8	10	10	6
10 a.m. – 2 p.m.	12	10	12	10	16	16	8
2 p.m. – 6 p.m.	16	12	16	12	20	20	8
6 p.m. – 10 p.m.	9	8	9	8	12	12	4

The objective is to determine the minimum size for the workforce and their working schedules so that the above manpower requirements and regulations are met.

The mathematical approach discussed in this section is a modified version of the Personnel Scheduling model by Brusco et al. (1995). This method has been used in the development of the automated manpower planning system at United Airlines. For other mathematical approaches to manpower planning see Brusco and Jacobs (1998).

We adopt the following decision variable:

$x_{i,j}$ = number of employees who begin their weekly work in day i adopting shift j

In this decision variable, index i, represents the day that an employee starts his/her five-day work week. Index j, represents the shift that the employee is assigned to. Tables 7.2 and 7.3 show the indices used to represent shifts and days of the week respectively.

Table 7.2 Index for shifts (j)

8-hour shift	Index (j) for shift
6 a.m. – 2 p.m.	1
10 a.m. – 6 p.m.	2
2 p.m. – 10 p.m.	3

Table 7.3 Index for days of the week (i)

Starting day of the working week	Index (i) for day
Mon	1
Tues	2
Wed	3
Thu	4
Fri	5
Sat	6
Sun	7

According to these tables, $x_{1,1}$ represents the number of employees who should start their work week on Monday from 6 a.m. – 2 p.m. shift, and so on.

The objective function is to minimize the total workforce (headcount) as follows:

$$\textit{Minimize } x_{1,1} + x_{1,2} + x_{1,3} + \ldots x_{7,1} + x_{7,2} + x_{7,3}$$

Note that the employees in decision variables are disjoint, meaning no employee appears in two decision variables. As an example, those employees who start their working week on Monday from 6 a.m. – 2 p.m., represented by decision variable $x_{1,1}$, are different from those who start on Monday from 10 a.m. – 6 p.m. represented by $x_{1,2}$. So adding all the decision variables represents the total workforce for this case study, which we wish to minimize.

For the constraints, we should satisfy the manpower requirements for each time block of the day. We have seven days with four time blocks covering the three shifts in each day, resulting in a total of 28 constraints. We classify these constraints in their four respective time blocks.

Constraints

The constraints must cover the manpower requirements for every shift of every day. The following presents the constraints for each time block.

Time Block 6 a.m. – 10 a.m.

The employees working in this time block include only those who start their shift at 6 a.m. (first shift). Those who start their shifts at 10 a.m. or 2 p.m. (second or third shifts) will not be present during this time block. To express this constraint for Monday, 6 a.m. – 10 p.m., we have the following constraint:

$$x_{1,1} + x_{4,1} + x_{5,1} + x_{6,1} + x_{7,1} \geq 8$$

The above constraint specifies that the total number of employees available for work on Monday from 6 a.m. to 10 a.m. includes those who start their working week on Monday ($x_{1,1}$), plus those who start on Thursday ($x_{4,1}$), Friday ($x_{5,1}$), Saturday ($x_{6,1}$), and Sunday ($x_{7,1}$). We require eight employees on Monday in the first time block. This number appears as the right hand side for the constraint. Note that since each employee works five days consequently followed by two days off, those who start their working week on Tuesday or Wednesday will not be present for work on Monday. Similarly we write six more constraints for the first time block of other days within the week.

Constraints for Time Block 10 a.m. – 2 p.m.

Since each employee works for eight hours, then the employees working in this time block include those who start their shifts at 6 a.m. (first shift) and 10 a.m. (second shift). The constraint for this time block for Monday is as follows:

$$x_{1,1} + x_{4,1} + x_{5,1} + x_{6,1} + x_{7,1} + x_{1,2} + x_{4,2} + x_{5,2} + x_{6,2} + x_{7,2} \geq 12$$

The first five terms are the same as the constraint for Monday 6 a.m. – 10 a.m. time block. The second five terms represent those employees who start their shifts at 10 a.m. on different days. The right-hand side represents the number of required employees for Monday's second time block. Similarly six more constraints are added for other days of the week representing this second time block.

Constraints for Time Block 2 p.m. – 6 p.m.

The employees working in this time block include those who start their shifts at 10 a.m. (second shift) and 2 p.m. (third shift). The constraint for this time block for Monday is as follows:

$$x_{1,2} + x_{4,2} + x_{5,2} + x_{6,2} + x_{7,2} + x_{1,3} + x_{4,3} + x_{5,3} + x_{6,3} + x_{7,3} \geq 16$$

The first five terms are the same as the constraint for Monday 10 a.m. – 2 p.m. time block. The second five terms represent those employees who started their shifts at 2 p.m. on different days. The right hand side represents the number of required employees for Monday's third time block. Similarly six more constraints are added for other days of the week.

Constraints for Time Block 6 p.m. – 10 p.m.

The employees working in this time block include only those who start their shifts at 2 p.m. (third shift). Those who have started at 6 a.m. (first shift) or 10 a.m. (second shift) have already finished their 8-hour working day and are not present during this time block. The constraint for Monday's time block is as follows:

$$x_{1,3} + x_{4,3} + x_{5,3} + x_{6,3} + x_{7,3} \geq 9$$

We see that only those employees with the third shift appear in this constraint. Similarly six more constraints are added for other days of the week.

Solution

The above linear integer programming model has 21 integer-decision variables and 28 constraints. Solving this model using a software generates the solution presented in Table 7.4. This table shows the required number of employees who start their working week in different shifts of the day. A total of 36 employees are required to meet the manpower requirement for this case study.

Table 7.4 Solution to manpower planning

Day/shift	Shift 1 (6 a.m. – 2 p.m.)	Shift 2 (10 a.m. – 6 p.m.)	Shift 3 (2 p.m. – 10 p.m.)
Mon	2	1	3
Tue	4	0	7
Wed	0	1	0
Thu	2	4	4
Fri	2	0	0
Sat	2	2	2
Sun	0	0	0

Mathematical Model

The mathematical model proposed by Brusco et al. (1995) addresses both part-time and full-time employees, their limits, numerous combinations of shifts, working days, and weekly rotations. This method has been used in the development of the automated manpower planning system at the United Airlines called Pegasys. This automated system aids the airline in determining the optimal manpower planning system in their 119 domestic airports as well as many international locations. Pegasys uses flight schedules, passenger forecasts, baggage and cargo loads to compute labor requirements. The mathematical model for this automated system utilizes personnel tour scheduling which involves the determination of work and non-work days during the week as well as the associated daily shift starting and finishing times for each employee. The mathematical model is as follows:

Sets

D = Set of days in the weekly planning
S = Set of allowable shifts
T = Set of all time-blocks in the weekly planning

Index

i = Index for day in the weekly planning
J = Index for shift
k = Index for time block

Parameters

$$a_{i,j,k} = \begin{cases} 1 \text{ if time block } k \text{ is work period in shift type } j \text{ which begins in day } i \\ 0 \text{ otherwise} \end{cases}$$

R_k Number of employees required to be present in time block k

Decision Variable

$x_{i,j}$ = Number of employees who begin work in day i adopting shift j

The integer linear program is as follows:

$$\textit{Minimize} \quad \sum_{i \in D} \sum_{j \in S} x_{i,j}$$

Subject to

$$\sum_{i \in D} \sum_{j \in S} a_{i,j,k} \cdot x_{i,j} \geq R_k \qquad \forall\, k \in T$$

$$x_{i,j} \in Z^{+} \qquad \forall\, i \in D,\ \forall\, j \in S$$

In this model, the objective function attempts to minimize the total work force subject to availability of manpower for each time block of the day. Z^+ represents the set of positive integer numbers.

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PART II
Operations and Dispatch Optimization

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Chapter 8
Revenue Management

Introduction

Revenue or yield management represents an important part of daily airline operations. It is concerned with maximizing the revenue or yield. However, yield or revenue management is somewhat misleading. The concept is not to manage yield or revenue, but rather to optimize it through the use of tools and techniques that maximize total revenue.

The concept of yield management is appropriate for business environments offering a product (goods or services) with the following characteristics:

- it is expensive or impossible to store excess inventory;
- future demand is uncertain;
- the firm can differentiate among customer segments (i.e., customers are willing to pay different prices for the same product);
- the fixed cost for offering the product is high, while the marginal cost is low;
- the capacity to offer the product is fixed.

The following industries are examples of business environments that have the above five characteristics:

- car rentals
- broadcasting
- hotels
- cruise lines
- airlines
- trains, buses.

In all these industries, if the product is not sold or rented today, the revenue is lost forever. In the airline industry, the product is the airline seat. If the seat is not sold, and the plane departs, the revenue that could have been generated by selling that seat is lost.

Thus, the main challenge in revenue management is to set the price based on current market conditions, with the question becoming: ‘Do we turn down an existing customer in anticipation of other, more profitable, customers?’

A variety of analytical tools that addresses this question falls under the revenue management topic. Since these tools are used by firms offering perishable products (i.e., either expensive or impossible to store), they are also called *perishable asset revenue management tools*. See McGill and Van Ryzen (1999) for a review of revenue management models for non-airline service sectors.

Airline Revenue Management

The techniques of revenue management are relatively new. After deregulation in 1978, airlines were free to set the price for their seats. This led to heavy competition and new opportunities for revenue management. American and Delta Air Lines credit revenue management techniques for an increase in revenue amounting to $500 million and $300 million per year respectively.

An airline typically offers seats for several origin-destination (OD) itineraries in various fare classes. The seat fares not only differ between the traditional first, business, and economy classes, but are also differentiated within the same class as well.

Considering that the seats offered, and their availability, are the source of revenue for the airline, the concept of revenue management thus primarily translates into a seat-inventory control problem. Accordingly, the airline seat-inventory control system has received a lot of attention from both the airline industry and academia.

Seat-Inventory Control Problem

The seat-inventory control problem is to decide if a seat should be sold at a current booking request, or if it should be saved for a more profitable customer. The mathematical models described in this chapter attempt to determine seat allocations according to the demand pattern at the beginning of the booking periods, and are referred to as *static* seat-inventory control problems. See the list of references at the end of this chapter for an overview of *dynamic* seat-inventory control systems.

Nested and Non-Nested Allocations

Basically, there have been two approaches to the airline seat-allocation problem: nested and non-nested. In non-nested approaches, distinct numbers of seats called buckets are exclusively assigned to each fare class. The sum of these buckets adds up to the total aircraft seat capacity. In nested allocations, each fare class is assigned a booking limit, which is the total number of seats assigned to that fare class plus the sum of all seat allocations to its lower fare classes. To clarify this further, consider an Airbus 320 with 150 seats. The following table shows the seats allocated to each fare class under nested and non-nested assignments:

As an example, for fare class B, under the non-nested approach, 50 seats are allocated, while under the nested approach there are 120 or (30 + 20 + 20 + 50) seats allocated to this class.

Table 8.1 Example of non-nested and nested airline seat allocations

Fare class	Non-nested allocation	Nested allocation
Y	30	150
B	50	120
M	20	70
H	20	50
Q	30	30

Earlier revenue management approaches considered non-nested allocations. However, a major difficulty with non-nested approaches is that if the limit for a fare class is reached, a booking request for that class is denied, while a lower fare bucket remains open. In a nested seat allocation, this booking denial does not happen as the inventories are shared among each fare class and its lower classes. Figure 8.1 shows a depiction of both non-nested and nested approaches for the airline seat-allocations example described in Table 8.1.

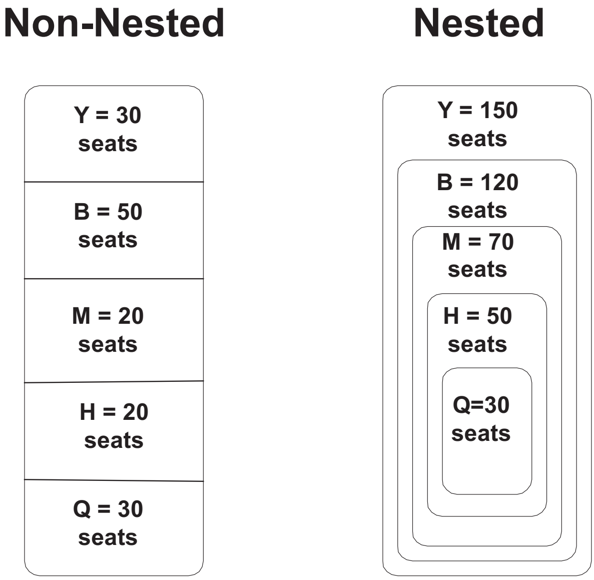

Figure 8.1 Nested and non-nested airline seat-allocations

Within both the nested and non-nested approaches, a further problem concerns the allocation of seats by either single or networked flight legs, referred to as the *single-leg seat-inventory control problem* and the *network (multi-leg) seat-inventory control problem,* respectively. In the following sections, both the nested and non-nested single-leg, and the nested and non-nested networked-legs are discussed. Before addressing these problems, however, we should understand the concept of expected marginal revenue.

Expected Marginal Revenue

At the core of the seat inventory control system is the expected marginal revenue (EMR). The EMR of potentially selling a seat in a fare class is the probability of being able to fill that seat multiplied by the average fare of that class. The concept of probability is introduced here since the demands for different flight legs and fare classes vary (stochastic demand).

In order to sell S seats for fare class i, we should have at least S requests for this fare class. We present this number of seats in fare class i as S_i.

Let r_i be the random variable representing the number of requests, and $p_i(r_i)$ be the probability distribution for r_i for fare class i. Assuming a continuous probability distribution for r_i, the probability of selling S_i seats in fare class i is:

$$P_i\left[r_i \geq S_i\right] = \int_{S_i}^{\infty} p_i\left(r_i \geq S_i\right) dr_i \tag{8.1}$$

As an example, if the probability distribution function is normal, then the above probability is represented by the shaded area in Figure 8.2.

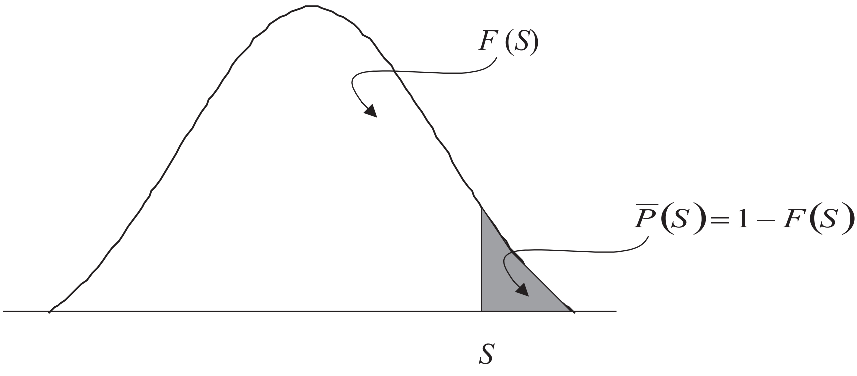

Figure 8.2 Normal probability distribution for demand with shaded area representing demand exceeding a certain level

Referring to Figure 8.2, the above probability (equation 8.1) can be rewritten as:

$$P_i\left[r_i \geq S_i\right] = \int_{S_i}^{\infty} p_i\left(r_i \geq S_i\right) dr_i = 1 - F_i\left(S_i\right) \tag{8.2}$$

where:
$F_i(S_i)$ is the cumulative distribution function of having S_i or lower requests for fare class i.

The literature on revenue management adopts the notation $\overline{P}_i\left(S_i\right)$ to represent the above probability (see Figure 8.2).
Therefore:

$$P_i\left[r_i \geq S_i\right] = \int_{S_i}^{\infty} p_i\left(r_i \geq S_i\right) dr_i = 1 - F_i\left(S_i\right) = \overline{P}_i\left(S_i\right) \tag{8.3}$$

Going back to the definition of expected marginal revenue, the EMR for the S^{th} seat in fare class i, is simply the above probability multiplied by the average fare level in the respective fare class, or:

$$EMR\left(S_i\right) = f_i \cdot \overline{P}_i\left(S_i\right) \tag{8.4}$$

where:
$EMR(S_i)$ = the expected marginal revenue for the S^{th} seat in fare class i,
f_i = the average fare level for class i,
$\overline{P}_i(S_i)$ = the probability of selling S or more seats in fare class i, as defined above.

To clarify this, let us consider the following example. Assume that the demand for class Y for a specific flight is normally distributed with a mean of 10 and a standard deviation of 2. The fare for this class is \$400. The following table shows the EMR for each seat. According to this table, the EMR of selling the first seat in this fare class is \$400. This is because for a normal probability distribution function, with a mean of 10 and a standard deviation of 2, the probability that a first seat is sold is almost 1. This probability reduces to 0.8413 for the 8^{th} seat in this class, and so on. Note that table below can be easily set up using Microsoft EXCEL's *NORMDIST* function.

Single-Leg Seat-Inventory Control Problem

In this problem, every flight leg is independent of other legs and is optimized separately. The problem is to determine how many seats should be allocated to each fare class in an attempt to maximize the total revenue.

Table 8.2 Probability and expected marginal revenue for each seat in the fare class

Seat (S)	$P_i(S_i)$	$EMR(S_i)$
1	1.0000	\$400.00
2	1.0000	\$400.00
3	0.9998	\$399.91
4	0.9987	\$399.46
5	0.9938	\$397.52
6	0.9772	\$390.90
7	0.9332	\$373.28
8	0.8413	\$336.54
9	0.6915	\$276.58
10	0.5000	\$200.00

Note: Probabilities are rounded to 4 decimal places.

Non-Nested Model

Littlewood (1972) was the first to introduce a two-fare non-nested seat-inventory system. He proposed that as long as the expected marginal revenue from a seat for a higher fare passenger is larger than that of a lower fare passenger, then that seat should not be sold at a lower fare. In this model we have two fare levels: *Full fare* and *discount fare*. To express this mathematically, let:

f_1 = Full fare level
f_2 = Discount fare level
$P(r_1 \geq S_1)$ = Probability that the demand for full fare seat (r_1) is equal or exceeds S_1

We want to determine S_i, the number of seats protected for full-fare-paying passengers. Of course, subtracting this number from the total seat capacity determines the number of seats available for discount-fare-paying passengers. According to Littlewood, low-fare passengers should be accepted as long as:

$$f_2 \geq f_1 \, P(r_1 \geq S_1) \tag{8.5}$$

The smallest value of S_i that satisfies the above condition is the protected number of seats for full-fare-paying passengers.

The following example explains how seat protections are determined using Littlewood's model:

We want to determine the number of protected seats for full fare paying passengers on an Airbus 320 with 150 seats. The full and discount fares on a specific flight are \$250 (f_1) and \$100 (f_2) respectively. Historical data shows that the demand for a full-fare class is normally distributed with a mean of 100 and a standard deviation of 15 passengers

Figure 8.3 presents the EMR values for the full fare level (\$250). The EMR values are determined similarly to the process described in Table 8.2. This figure shows EMR for different numbers of seats. According to this figure, the EMR starts declining from the 70th seat until it reaches around the 140th seat, which has almost zero value for EMR.

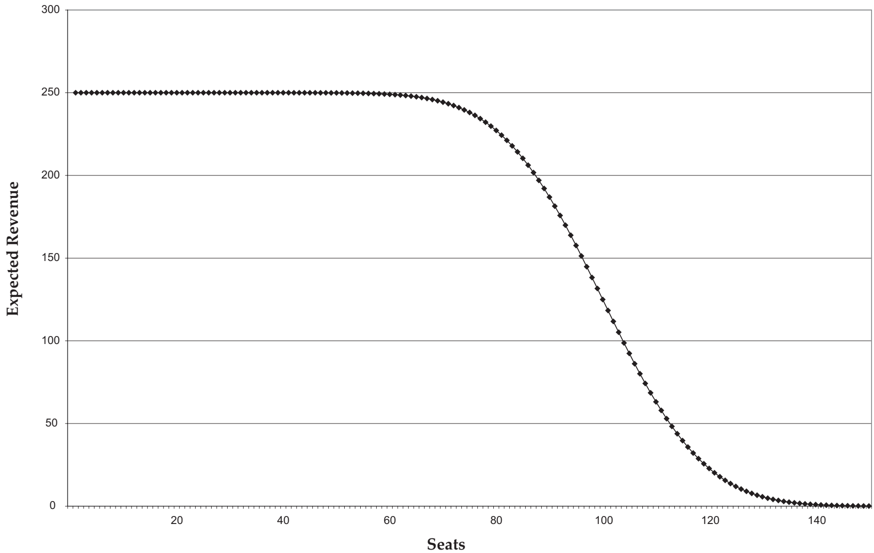

Figure 8.3 Expected marginal revenue for full-fare-paying passengers

According to the inequality in (equation 8.5), the requests for discount fares are accepted if this fare exceeds the EMR for the full-fare level. The EMR for a full fare paying passenger for the 103rd seat is \$105.19, and for the 104th seat is \$98.72. Therefore the smallest value for S_1 is 103. Thus, the airline should protect 103 seats for full fare paying passengers, and the remaining 47 (150-103) seats for the discount-fare-paying passengers.

Nested Model

Belobaba (1987) extended the above two-fare-class rule to multiple nested fare classes by introducing the term *expected marginal seat revenue* (EMSR). This method generates the nested protection level for different class fares. He proposed that in a nested seat allocation, the number of seats which should be protected for fare class *i* over fare class *j* is:

$$EMSR\left(S_j^i\right) = f_i \cdot \overline{P}_i\left(S_j^i\right) = f_j \tag{8.6}$$

In this model, S_j^i is the number of seats that should be protected for higher class *i* over class j, while f_i and f_j are the average fare levels for the two classes of *i* and *j* respectively.

Based on this model, the number of seats protected for the highest fare class (Π_1) is S_2^1 satisfying:

$$EMSR\left(S_2^1\right) = f_1 \cdot \overline{P}_1\left(S_2^1\right) = f_2 \tag{8.7}$$

To capture the nested seat allocation characteristic, the total protection level for the two highest fare classes (Π_2) is the sum of the individual protection levels S_3^1 and S_3^2 satisfying:

$$EMSR\left(S_3^1\right) = f_1 \cdot \overline{P}_1\left(S_3^1\right) = f_3 \tag{8.8}$$

and

$$EMSR\left(S_3^2\right) = f_2 \cdot \overline{P}_2\left(S_3^2\right) = f_3 \tag{8.9}$$

The total protection level for the highest two fares is therefore:

$$\Pi_2 = S_3^1 + S_3^2 \tag{8.10}$$

Applying the same principle, the protected number of seats for the (n-1) fare class is determined by:

$$\Pi_{n-1} = \sum_{i=1}^{n-1} S_n^i \tag{8.11}$$

The booking limit or the number of seats available for each class *i*, represented by BL_i, is determined by subtracting the number of seats protected for the higher fare class, Π_{i-1}, from the total aircraft seat capacity, C. Therefore:

$$BL_i = C - \Pi_{i-1} \tag{8.12}$$

It should be noted, that based on our definition for nested seat allocation, the booking limit for the highest fare class is:

$$BL_1 = C \tag{8.13}$$

BL_i may also be negative (especially for lower-fare classes), in which case the above booking limit becomes:

$$BL_i = Max(0, C - \Pi_{i-1}) \tag{8.14}$$

The nested protection for fare class i is therefore the difference between the booking limits for that fare and its lower-fare class as follows:

$$NP_i = BL_i - BL_{i+1} \tag{8.15}$$

where:
NP_i = the nested seat protection level for fare class i.

Figure 8.4 shows the booking levels and seat protections as described by this model.

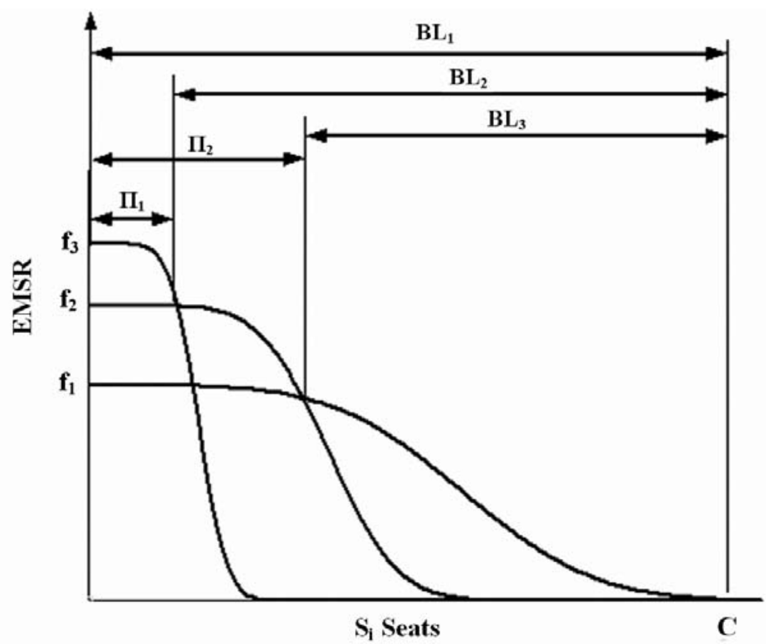

Figure 8.4 Seat protections and booking levels for three fare-classes under the nested seat allocation model

To clarify the above nested seat allocation model, consider an Airbus 320 with 150 seats. The following table shows the distribution of demand for four classes, with the fare levels for each class on a specific flight. All demand for different fare classes follows normal distributions with indicated means and standard deviations. We want to adopt the above nested EMSR approach to determine the seat allocation and booking level for each fare class.

Table 8.3 Fare classes, demand distributions and fare levels for a flight

Fare class	Demand distribution	Fare level
Y	Mean = 25 SD = 5	$580
B	Mean = 54 SD = 12	$480
M	Mean = 84 SD = 23	$350
Q	Mean = 130 SD = 20	$250

To determine the booking limits, an EXCEL spreadsheet may be useful. A table similar to Table 8.2 is constructed for every fare class. Figure 8.5 shows the EMSR for the four classes.

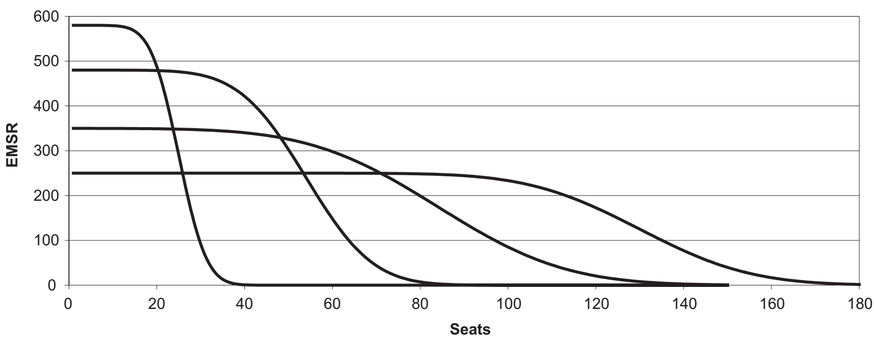

Figure 8.5 EMSR for the four-fare-class example

By comparing different EMSR within different fare classes, we can determine S^i_j. Table 8.4 shows the values of S^i_j or the protected number of seats for each fare class over each of its lower classes. As an example, S^1_2 in this table is 20 seats, which represents the number of protected seats for Y over the B fare class, and so on.

Table 8.4 Protected number of seats for each fare class over lower classes

Fare class/ fare class	B (fare class 2)	M (fare class 3)	Q (fare class 4)
Y (fare class 1)	20	23	25
B (fare class 2)	-	46	53
M (fare class 3)	-	-	71

Using the above definitions, the protection level for each fare class and booking limit is as follows:

$$\Pi_1 = S_2^1 = 20$$

$$\Pi_2 = S_3^1 + S_3^2 = 23 + 46 = 69$$

$$\Pi_3 = S_4^1 + S_4^2 + S_4^3 = 25 + 53 + 71 = 149$$

Therefore 20 seats should be protected for class Y; 69 seats for classes Y and B; and 149 seats for classes Y, B and M. We can therefore determine the booking limits as follows:

$$BL_1 = C = 150$$

$$BL_2 = C - \Pi_1 = 150 - 20 = 130$$

$$BL_3 = C - \Pi_2 = 150 - 69 = 81$$

$$BL_4 = C - \Pi_3 = 150 - 149 = 1$$

$$NP_1 = BL_1 - BL_2 = 150 - 130 = 20$$

$$NP_2 = BL_2 - BL_3 = 130 - 81 = 49$$

$$NP_3 = BL_3 - BL_4 = 81 - 1 = 80$$

$$NP_4 = C - NP_1 - NP_2 - NP_3 = 150 - 20 - 49 - 80 = 1$$

Based on the above values for booking limits, 20 seats should be protected for fare class Y, 49 for class B, 80 for class M, and finally only 1 seat should be allocated to fare class Q. For a nested allocation, these protections result in 1 seat for fare class Q, 81 seats for class M, 130 seats for class B, and finally all 150 seats to class Y.

The above method is very popular owing to its simplicity and ease of implementation. It finds the optimal booking limits between each pair of fare classes. It does not, however, consider the fact that the fare classes are sequentially nested within each other and hence interrelated. In other words, this method does not consider the joint probability distribution among the fare classes. Other researchers (see Talluri and Van Ryzin, 2005, and McGill and Van Ryzin, 1999, for a list of references) have developed optimal booking levels by considering multiple (more than 2) nested fare classes.

Network (Multi-Leg) Seat-Inventory Control Problem

The seat-inventory policy that was described in the previous section considered the revenue generated only on one flight leg. It is very common, however, to see passengers on the same flight having different itineraries owing to the airline hub and spoke systems. We use the term *Origin-Destination* (OD) to represent the starting and ending points of an itinerary. Figure 8.6 shows a simplified network. Passengers flying from Orlando to Chicago or Los Angeles will be on the same flight (F1) departing Orlando for the Atlanta hub. At Atlanta these passengers change flights to their respective destinations.

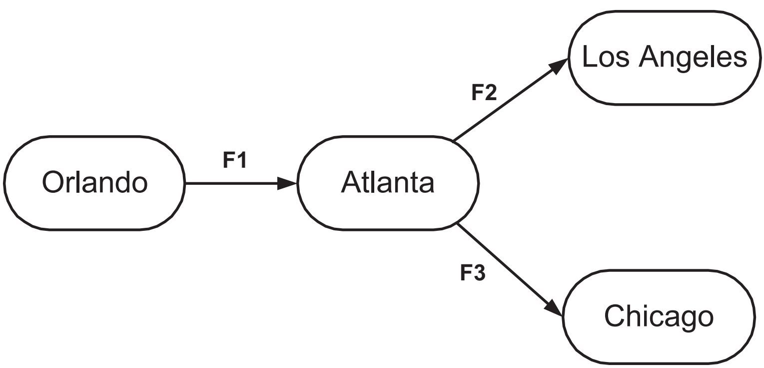

Figure 8.6 A simple network representing passengers with different origin-destination itineraries

In this simplified network, we have passengers with ODs: (Orlando–Atlanta), (Orlando–Los Angeles), (Orlando–Chicago), (Atlanta–Los Angeles) and (Atlanta–Chicago). The network seat-inventory control system attempts to assign seats with different fare classes on each flight leg to different OD passengers so that the total revenue over the entire network is maximized. The following sections study this network revenue management under *deterministic* and *stochastic* demand models.

Network Seat-Inventory Control Model with Deterministic Demand (Non-nested)

In this model we consider that the demand for each fare class and each OD is deterministic, and hence known in advance.

Let us define:

x_{ODF} = Number of protected seats on flight-leg OD (origin-destination) for fare class F.

f_{ODF} = Fare for class F on flight-leg OD.

C_j = Aircraft capacity on flight-leg j.

D_{ODF} = Deterministic demand for OD for fare class F.

The deterministic approach seeks to determine x_{ODF} so that the total revenue generated, from allocating seats to fare classes on every flight leg, is maximized. The following mathematical model attempts to find these seat allocations:

$$Max \sum_{ODF} f_{ODF} \cdot x_{ODF}$$

Subject to:

$$\begin{aligned} &\sum_{ODF} x_{ODF} \le C_j && \text{for all ODFs on flight-leg j, for all flight-legs j} \\ &x_{ODF} \le D_{ODF} && \text{for all ODFs} \\ &x_{ODF} && \text{integer for all ODFs} \end{aligned} \qquad (8.16)$$

The first set of constraints limits the total number of bookings to aircraft capacity on each leg. The second set ensures that the allocated seats on each OD, and for each fare class F, do not exceed the demand. The solution to this integer linear programming model determines the number of each origin–destination, and each fare class, so that the total revenue over the entire network is maximized.

To demonstrate how this model works, let us consider the network presented in Figure 8.7. Node H represents the hub, and the other nodes are spokes. As the network suggests, passengers wishing to go from A to C will have one stop in H. Therefore, origin–destination A to C consists of two flight legs, A to H and H to C.

Table 8.5 presents the deterministic demand and fare level for all ODs in this network. For each flight, we have two fare classes, namely Y and B. All aircraft flying from A and B to H have a capacity of 90 seats, and all aircraft flying from H to C and D have a 142-seat capacity.

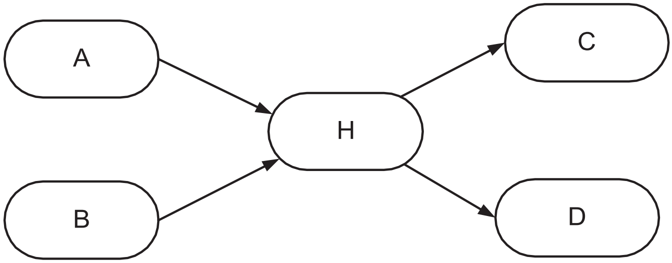

Figure 8.7 Network diagram for the multi-leg example

Table 8.5 Demand and fare levels for the multi-leg example

Flight-leg	Fare class Y	Fare class B
AH	Demand: 38 Fare: 354	Demand: 52 Fare: 181
AC	Demand: 26 Fare: 376	Demand: 43 Fare: 286
AD	Demand: 24 Fare: 283	Demand: 40 Fare: 200
BH	Demand: 25 Fare: 236	Demand: 35 Fare: 133
BC	Demand: 28 Fare: 511	Demand: 38 Fare: 281
BD	Demand: 22 Fare: 500	Demand: 45 Fare: 365
HC	Demand: 35 Fare: 354	Demand: 43 Fare: 191
HD	Demand: 28 Fare: 367	Demand: 40 Fare: 195

To formulate this mathematical model, we adopt the following decision variable:

x_{ODF} = Number of seats allocated to origin O, destination D, and fare class F.

The objective function is to maximize the total revenue generated in the network. Thus,

$$Maximize\ \ 354x_{AHY} + 181x_{AHB} + ... + 367x_{HDY} + 195x_{HDB}$$

The first set of constraints concerns the aircraft capacity on each leg.

$$x_{AHY} + x_{AHB} + x_{ACY} + x_{ACB} + x_{ADY} + x_{ADB} \le 90$$

$$x_{BHY} + x_{BHB} + x_{BCY} + x_{BCB} + x_{BDY} + x_{BDB} \le 90$$

$$x_{HCY} + x_{HCB} + x_{ACY} + x_{ACB} + x_{BCY} + x_{BCB} \le 142$$

$$x_{HDY} + x_{HDB} + x_{ADY} + x_{ADB} + x_{BDY} + x_{BDB} \le 142$$

As an example, in the first constraint we specify that the total number of passengers on flight leg AH, which includes passengers flying from A to H plus those who are flying from A to C plus passengers flying from A to D (in both fare classes), should not exceed the aircraft capacity.

The following set of constraints restricts the number of passengers on each origin–destination and fare class to the corresponding demand:

$$x_{AHY} \le 38$$
$$x_{AHB} \le 52$$
.....
$$x_{HDY} \le 28$$
$$x_{HDB} \le 40$$

This linear integer program has 16 variables and 20 constraints. Solving this model using an optimization software results in a total network revenue of $89,096. The solution to the seat allocations for each ODF is presented in Table 8.6. According to this solution, no seat should be allocated to origin–destination BH. However, 28 seats are allocated to BC. This is because allocating the seat to a passenger with a multiple-leg itinerary (BH + HC), generates more revenue for the airline than a single-leg itinerary from B to H. This is one reason behind the familiar case of a reservations system showing no seats available on a specific flight, while another passenger with a multi-leg OD is successful in making a reservation on the same flight.

Network Seat-Inventory Control Model with Probabilistic Demand (Non-Nested)

A major difficulty with the previous deterministic network model is that the solution is based on certainty of demand. In many real-world cases, the demand is stochastic, and hence varies over time. To capture the variability of demand, the mathematical model in (equation 8.16) is revised to accommodate the probability

Table 8.6 Solution to the deterministic network seat allocation example

Flight-leg	Protected seats for Y class	Protected seats for B class
AH	Demand: 38 Allocation: 38	Demand: 52 Allocation:0
AC	Demand: 26 Allocation: 26	Demand: 43 Allocation: 14
AD	Demand: 24 Allocation: 12	Demand: 40 Allocation:0
BH	Demand: 25 Allocation:0	Demand: 35 Allocation:0
BC	Demand: 28 Allocation: 28	Demand: 38 Allocation:0
BD	Demand: 22 Allocation: 22	Demand: 45 Allocation: 40
HC	Demand: 35 Allocation: 35	Demand: 43 Allocation:0
HD	Demand: 28 Allocation: 28	Demand: 40 Allocation: 40

distribution of demand for different ODFs. In this model, we seek to maximize the expected revenue over the entire network.

Based on expected marginal revenue discussed earlier in this chapter, we have:

$$EMR_{ODF}\left(S_{ODF}\right) = f_{ODF} \cdot \overline{P}_{ODF}\left(S_{ODF}\right) \tag{8.17}$$

where:

$EMR_{ODF}(S_{ODF})$ = Expected marginal revenue from the S^th^ seat in fare class F on OD.

f_{ODF} = Average fare for class F on OD.

$\overline{P}_{ODF}\,(S_{ODF})$ = Probability of selling the S^{th} seat (i.e., demand ≥ S) in fare class F on OD.

To formulate this mathematical model, we define binary decision variables as follows:

$$x_{S,ODF} = \begin{cases} 1 \text{ if the } S^{\text{th}} \text{ aircraft seat is assigned to fare class } F \text{ on origin-destination } OD \\ 0 \text{ otherwise} \end{cases}$$

The objective function is to maximize the total expected revenue through the network.

$$\text{Maximize} \sum_{ODF} \sum_{S=1}^{C_j} EMR_{ODF}\left(S_{ODF}\right) x_{S,ODF}$$

Subject to:

$$\sum_{ODF} \sum_{S=1}^{C_j} x_{S,ODF} \leq C_j \qquad \text{for all flight-legs j}$$

The above constraint states that the total number of allocated seats should not exceed the aircraft capacity on each flight leg.

Let us return to our example for deterministic demand. We assume that all the demand distributions are normal, with the same means as in the deterministic case. Table 8.7 provides the means and standard deviations of demand for each fare class and OD.

Table 8.7 Probabilistic demand for the network seat allocation example

Flight-leg	Fare class Y	Fare class B
AH	Demand: 38 SD: 14 Fare: 354	Demand: 52 SD: 11 Fare: 181
AC	Demand: 26 SD: 6 Fare: 376	Demand: 43 SD: 9 Fare: 286
AD	Demand: 24 SD: 4 Fare: 283	Demand: 40 SD: 8 Fare: 200
BH	Demand: 25 SD: 8 Fare: 236	Demand: 35 SD: 4 Fare: 133
BC	Demand: 28 SD: 6 Fare: 511	Demand: 38 SD: 9 Fare: 281
BD	Demand: 22 SD: 4 Fare: 500	Demand: 45 SD: 8 Fare: 365
HC	Demand: 35 SD: 6 Fare: 354	Demand: 43 SD: 12 Fare: 191
HD	Demand: 28 SD: 6 Fare: 367	Demand: 40 SD: 8 Fare: 195

To construct a mathematical model, we need to compute the EMR for each seat on each ODF. An EXCEL spreadsheet is helpful in generating these EMRs. As an example, Table 8.8 shows the EMR for the first ten seats for origin–destination AH for class Y.

Table 8.8 Expected marginal revenue for the probabilistic network seat allocation example

Seat number	Probability	Fare	EMR
1	0.995889	\$354.00	\$352.54
2	0.994936	\$354.00	\$352.21
3	0.99379	\$354.00	\$351.80
4	0.992421	\$354.00	\$351.32
5	0.990792	\$354.00	\$350.74
6	0.988865	\$354.00	\$350.06
7	0.986595	\$354.00	\$349.25
8	0.983938	\$354.00	\$348.31
9	0.980841	\$354.00	\$347.22
10	0.97725	\$354.00	\$345.95

The mathematical model for this case will be to:

$$\textit{Maximize} \quad 352.54x_{1,AHY} + 352.21x_{2,AHY} + ...$$

Subject to:

$$x_{1,AHY} + x_{2,AHY} + ... + x_{90,ADB} \leq 90$$

$$x_{1,BHY} + x_{2,BHY} + ... + x_{90,BDB} \leq 90$$

$$x_{1,HCY} + x_{2,HCY} + ... + x_{90,BCB} \leq 142$$

$$x_{1,HDY} + x_{2,HDY} + ... + x_{90,BDB} \leq 142$$

This mathematical model has more than 1,200 binary decision variables and 4 constraints. The four constraints represent the capacity on four flight legs. The airlines have automated systems that generate the linear programming model, which it then solves either optimally or using heuristics (see Chapter 13).

Solving the above binary integer programming model results in a total network expected revenue of $100,298. The solution to the seat allocations for each ODF is presented in Table 8.9.

Table 8.9 Solution to the probabilistic network seat allocation example

Flight-leg	Protected for class Y	Protected for class B
AH	Demand: 38 Allocations: 37	Demand: 52 Allocations:0
AC	Demand: 26 Allocations: 21	Demand: 43 Allocations: 16
AD	Demand: 24 Allocations: 16	Demand: 40 Allocations:0
BH	Demand: 25 Allocations: 10	Demand: 35 Allocations:0
BC	Demand: 28 Allocations: 25	Demand: 38 Allocations:0
BD	Demand: 22 Allocations: 20	Demand: 45 Allocations: 35
HC	Demand: 35 Allocations: 38	Demand: 43 Allocations: 42
HD	Demand: 28 Allocations: 31	Demand: 40 Allocations: 40

As we see in this table, for some ODs such as HC and HD, the number of allocated seats for the Y fare class is actually larger than the expected demand. This is due to the higher EMR generated from these seats.

Network Seat-Inventory Control Models (Nested)

There are several different methods for nested network seat-inventory control systems. The common approach for these methods is to cluster the seat allocations derived from non-nested into *virtual nested* allocations. There are many heuristics for such clustering. In this section, we briefly discuss one of these clustering methods, namely *nesting by fare class*. See McGill and Van Ryzin (1999) for an overview of other clustering methods.

Nesting by Fare Class

In this method, for each fare class on a flight leg, the respective solutions for non-nested ODF allocations from either deterministic or probabilistic networks are summed together (Williamson 1992). These total allocations are then used as the protection levels for each fare class. The booking limits for each fare class are determined by subtracting these protection levels from the capacity of the flight leg.

To clarify this, let us return to our non-nested example for the deterministic demand network. Consider the flight leg AH. We want to determine the booking limits for the two fare classes Y and B. Returning to Figure 8.7, the passengers on flight leg AH include those with origin-destinations: AH, AC, and AD. The solutions obtained from the non-nested network for these three OD passengers are shown in Table 8.10.

Table 8.10 Seat allocations on flight leg AH

ODF	Non-nested seat allocations
AHY	38
ACY	26
ADY	24

According to the virtual nesting method described above, we assign the total aircraft capacity to the highest fare class, in this case, fare class Y. The aircraft capacity is 90 seats. Therefore, the booking limit for class Y on flight leg AH is also 90 seats. For the lower-fare class B, the booking limit is simply the booking limit for fare class Y minus the number of seats assigned to fare class Y for all passengers with AH as part of their itinerary. So, the booking limit on flight leg AH for fare class B is 14 seats (90-38-26-24).

Overbooking

Airlines regularly face passengers who cancel their flight reservations at the last minute, or fail to show-up for flights (called *no-shows*). Certainly the seats allocated to such passengers will remain empty. To generate revenue from these anticipated empty seats, the airlines normally overbook their flights by selling more seats than the capacity on a given flight. This process of overbooking has been studied under revenue management techniques. A major issue in overbooking is to balance the anticipated revenue from selling extra seats versus the anticipated cost of not having enough capacity to accommodate all the passengers.

A common approach to addressing this problem is the *single-period inventory control model*.

Let us define the following parameters:

C_0 = Cost of overestimating the number of no-shows. It is the cost of accommodating a passenger with a confirmed reservation when there are no seats available on the flight. This cost is normally referred to as *spillage* cost, and it occurs when the airline sells too many seats, and one or more passengers are denied boarding (referred to as *bumped passengers*). This cost includes finding other arrangements, ticket upgrading, accommodation costs, goodwill costs, and so on.

C_U = Cost of underestimating the number of no-shows. It represents the lost revenue owing to an empty seat. This cost, which is also referred to as cost of *spoilage,* occurs when the airline makes very few seat overbookings, and one or more seats end-up being empty for the flight.

r = Number of overbooked seats.

P(no-shows) = Probability distribution for the number of no-shows. This is the probability distribution for the number of passengers who cancel their reservations at the last minute, or fail to show-up on a given flight. The airlines typically have historical data on no-shows for every flight, from which such probability distributions can be derived.

The optimum level for *r*, the number of overbooked seats, is when we have a balance between the expected *spoilage* and *spillage* costs as follows:

$$C_o \cdot P\left(no-shows \le r\right) = C_u \cdot [1 - P(no-shows \le r)] \qquad (8.18)$$

We have:

$$P\left(no-shows \le r\right) + P\left(no-shows > r\right) = 1$$

Therefore the equation (8.18) can be rewritten as:

$$C_o \cdot P\left(no-shows \le r\right) = C_u \cdot [1 - P(no-shows \le r)]$$

Rearranging this equation results in:

$$P\left(no-shows \le r\right) = \frac{C_u}{C_u + C_o} \qquad (8.19)$$

The solution to this problem is similar to the well-known *newsvendor problem* in operations research. According to this solution, the cumulative probability distribution that meets this threshold determines the optimal value of seats to be overbooked.

Let us consider the following example. According to past data, the number of no-shows on a 150-seat aircraft follows a normal distribution with a mean of 7 and a standard deviation of 2 passengers. The cost of bumping a passenger (C_o) is $750, which includes provisions for accommodation on an alternate flight, board and lodging for an overnight stay, and gift vouchers usable for future flights. On the other hand, the cost of an empty seat (C_u) is $150. According to equation (8.19) we have:

$$P(no-shows \leq r) = \frac{150}{150+750} = .1667$$

Using EXCEL's *Norminv* function, or a normal distribution table, we find r =5.07. Rounding this number results in an optimal number of overbooking of 5 seats. We notice that the optimal number of overbooking (r) is actually less than the expected number of no-shows (7). This is because the cost of *spillage* is larger than the cost of *spoilage*.

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Chapter 9

Fuel Management System

Introduction

The recent surge in fuel prices continues to impose an enormous impact on airlines throughout the world. This impact has resulted in bankruptcies, significant reduction in number of flights, services, and operations among airlines globally.

Since the introduction of jet engines in the 1950s, aircraft manufacturing companies have gradually replaced piston engines with turbine jet engines. The major fuel used for jet engines, Jet A (US market) and Jet A-1 (international market), are kerosene- (oil-) based and are produced according to stringent US and international standards. When the refineries process crude oil, they produce three types of products which are commonly referred to as top, middle and bottom of the barrel by the energy traders (Carter et al. 2004). Lighter products are at the top of the barrel and boil at a much lower temperature. Automobile gasoline is an example of such a product. The middle of the barrel products have a higher boiling temperature and include products such as heating oil and jet fuel. The bottom of the barrel is residual fuel oil and includes products such as heavy oil, which is used as fuel in industries.

Since jet fuels are processed from crude oil, the prices of jet fuel and crude oil are highly correlated. The following figure presents the price of a barrel of crude oil in US dollars over the past four decades. These prices are adjusted for inflation to 2008 prices using the Consumer Price Index (CPI-U) as presented by the Bureau of Labor Statistics. As the figure suggests, crude oil has gone through extensive price fluctuations owing to the political and economic environments of the world in general and of the Middle East in particular.

As indicated before, the crude-oil price fluctuations impact the price of jet fuel directly. The following figure presents the price (inflation adjusted) of jet fuel per gallon over the same time horizon as Figure 9.1.

For the airlines, fuel and crew are the two major components and drivers of operating cost. Figure 9.3 presents the fuel and crew cost as a percentage of total operating cost for all US airlines.

As the figure suggests, fuel cost has been the primary and dominant driver of operating cost for US airlines over the last 30 years. As the price of fuel increases, so does the operating cost. As the figure implies, in 2008, on average, fuel cost was 30% of total airline operating cost.

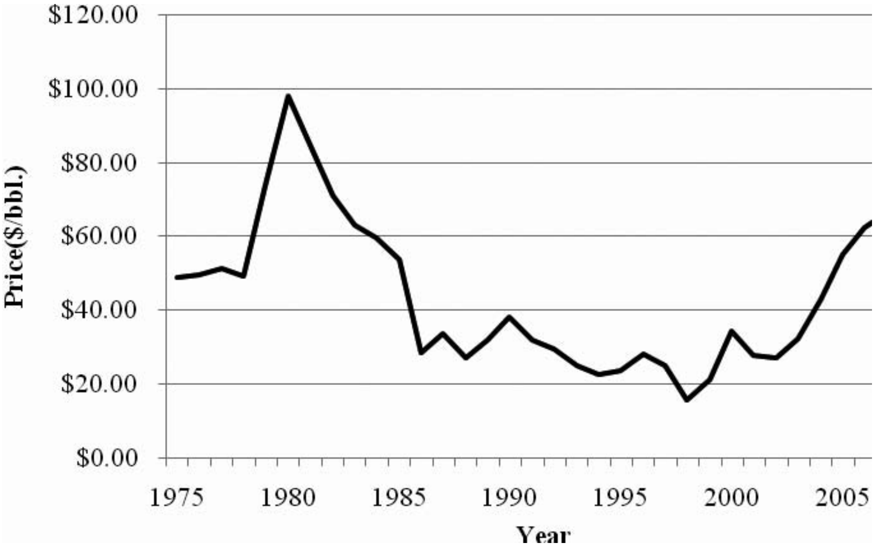

Figure 9.1 Annual average crude oil prices

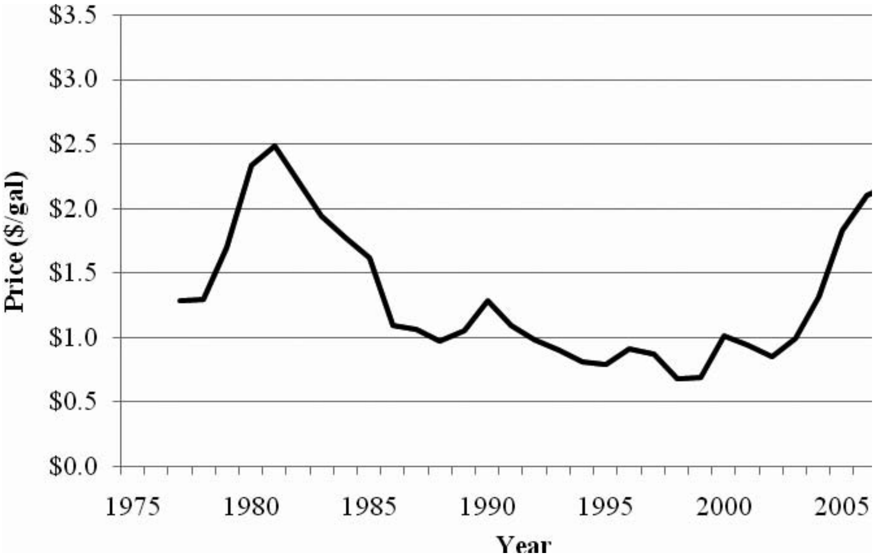

Figure 9.2 Average annual jet fuel prices

Figure 9.4 presents the total volume of jet fuel consumed by all US airlines from 1975 to 2008. The total fuel consumed in 2008 was more than 18 billion gallons. Therefore just a 1% increase in jet fuel consumption translates to an increase of more than 180 million gallons of fuel or $530 million for the US airline industry.

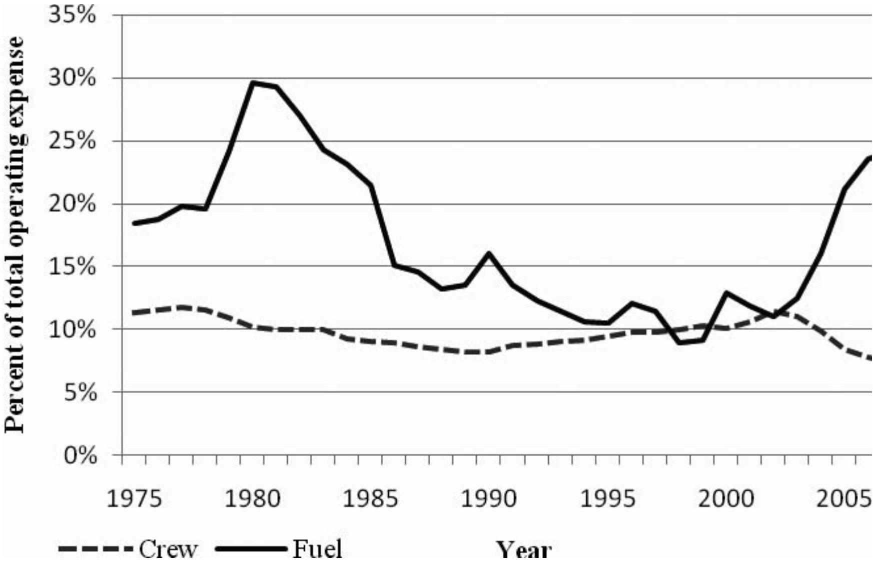

Figure 9.3 Crew and fuel cost as a percentage of total operating cost

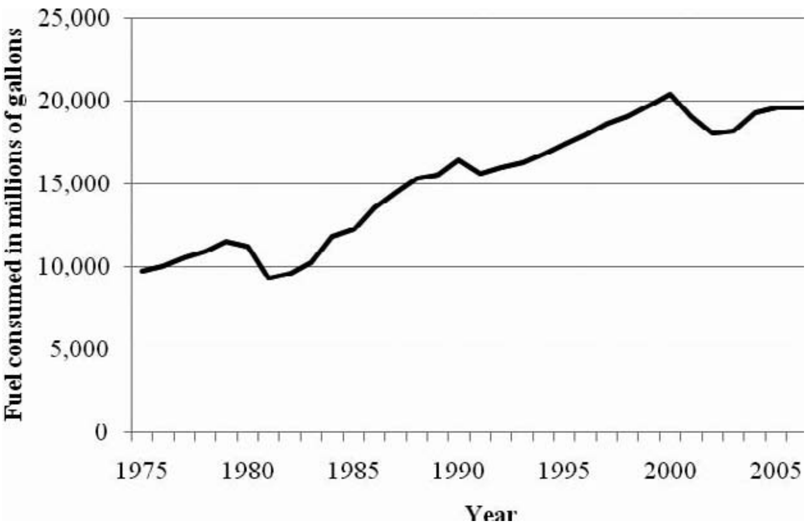

Figure 9.4 Total fuel consumed by all US airlines, in millions of gallons

Fuel Hedging

Hedging is a strategy that typically sellers and/or buyers of commodities adopt in order to protect themselves against risk caused by fluctuations in price. Consider, for example, an airline wishing to purchase 1,000,000 barrels of jet fuel next month. Assume that the current spot price of jet fuel is $50 per barrel. The airline anticipates that the jet fuel prices are likely to increase. Therefore, in an effort to protect against this price hike, the airline enters into an agreement with a seller to buy each barrel of fuel at a future price of $51 delivered next month. This strategy enables the airline to lock the price of jet fuel at $51/barrel for next month. If the airline's forecast is correct and the jet fuel prices increase to more than $51/barrel, then the airline is protected from such price increases and saves money by adopting this hedging strategy. On the other hand if the prices fall to less than $51/barrel then the airline loses money, as it could get jet fuel at a cheaper spot price but adopted the wrong strategy and locked itself to buy at $51/barrel.

Airlines typically adopt fuel hedging to stabilize their major operating cost component (Morrell and Swan 2006). Other operating-cost components, such as crew and maintenance costs, are more predictable. Therefore by hedging fuel, airlines have the flexibility to predict their total cost, cash flows, and profits more accurately. Airlines typically hedge between one- and two-thirds of their fuel cost.

The hedging strategies adopted by airlines can be grouped into two major categories: over-the-counter and exchange-traded contracts. Over-the-counter agreements are contracts between an airline and another party such as an investment bank or a fuel supplier and are not regulated. These types of contracts are subject to risk of default on payment from either party if they go bankrupt. Exchange-traded contracts, on the other hand, are set-up and traded through international exchanges and protect against counter-party risk. The main exchanges are the International Petroleum Exchange (IPE) in London and New York Mercantile Exchange (NYMEX).

The following are some of the most common type of fuel-hedging strategies used by the airlines (Carter et al. 2004 and Chance and Brooks 2009).

Plain-Vanilla Swap

A plain-vanilla energy swap is an over-the-counter agreement between two parties, whereby a future floating price is exchanged for a fixed price. It is called plain vanilla because it is much simpler than other instruments. In this type of an agreement there is no transfer of actual commodity (fuel) and the parties settle the contractual obligation with cash. A vanilla-swap contract specifies the volume of fuel, the duration, the fixed and the floating prices. The users of these contracts are typically financial institutions such as banks, and end users such as airlines. The following example shows how a plain-vanilla swap works.

Example: An airline wants to purchase 1,000,000 gallons of jet fuel per month for a period of one year and the current spot price of jet fuel is $1.30 per gallon. The airline anticipates that the price of jet fuel will rise. To minimize the risk of the anticipated price rise, the airline enters into a plain-vanilla price-swap contract with an investment bank. The contract specifies 1,000,000 gallons of jet fuel per month at a fixed rate of $1.35 for the next 12 months.

At the end of the first month, the price of jet fuel has increased to $1.39 per gallon. At this point, the bank owes the airline the difference between the floating price ($1.39) and the fixed price ($1.35) for each gallon as follows:

$$(\$1.39 - \$1.35) \times 1{,}000{,}000 = \$40{,}000$$

The bank pays $40,000 to the airline. However, the airline needs to buy the fuel at a current price of $1.39 per gallon from the suppliers. Therefore, the airline's effective fuel cost for this month is as follows:

Actual fuel expenses		Hedging gain		Effective fuel cost
$1.39 × 1,000,000	=	$1,390,000 – $40,000	=	$1,350,000

Therefore, the effective price for the airline is $1.35 ($1,350,000/1,000,000) per gallon, which is the fixed rate of the swap contract. Settlements are made in the same manner for each remaining month of the contract.

A major drawback of plain-vanilla swaps is that each party is faced with the risk of default on payment from the other party; since the settlement is not done through the exchanges. That is, if the airline or the investment bank goes bankrupt, then one party is faced with the risk of losing the settlement payment from the other party.

Futures Contract

A futures contract is an agreement to buy or sell a specified quantity and quality of commodity for a certain price at a designated time in the future. These types of contracts are executed through commodity exchanges and thereby eliminate the risk of a party defaulting on payment. The buyers take a long position, which means they will purchase the commodity at the agreed price at a designated time in the future. The sellers take a short position, meaning they agree to deliver the commodity at a designated time. Only a very small percentage of futures contracts result in actual delivery of the commodity. Instead, buyers and sellers of futures contracts make daily cash payments to each other to offset their positions. The exchanges require both buyers and sellers to post a margin which is a small percentage of the initial value of the contract. Losses are drawn from the margin until a maintenance margin is reached. Airlines require a large amount of cash on hand to engage in these futures contracts. It should also be noted that jet-fuel futures contracts do not exist in the United States, so futures on crude or heating oil are used to hedge jet-fuel purchases. The following example shows how a futures contract works.

Example: An airline purchases a long position in a 3-day future contract (standard contracts are much longer). The contract is for 50,000 barrels of crude oil. The future price of crude oil is $45 per barrel. The exchange requires 20% for the initial margin and 10% for maintenance margin. The following table shows the daily transaction over the three-day period.

Table 9.1 Daily futures contract transaction over a three day period

Day	Margin	Margin	Posted to margin	Future price	Gains/loss per barrel	Total Gains/ loss
0	20%	$450,000	$0	$45	0	-
1	16%	$350,000	$0	$43	($2)	($100,000)
2	8%	$150,000	$45,000	$39	($4)	($200,000)
3	13%	$270,000	$0	$41.50	$1.50	$75,000

On day 0, considering the future price is $45/barrel and the contract involves 50,000 barrels, then the total cost of the contract is $2,250,000 (50,000×$45). The airline needs to post at least a 20% initial margin to the exchange. Therefore, $450,000 (20% of the contract) is deposited with the exchange. On day 1, the price of crude oil has dropped to $43. This means the airline has to pay $2 per barrel to the seller of the contract (short position). Therefore, the airline loses $100,000 on day 1. On day 2, the future price of crude oil falls again to $39, meaning the airline needs to pay the seller of the contract (short position) $4 per barrel for a total of $200,000. On this day the airline's margin with the exchange falls to 8% ($150,000/($39×50000)). At this stage, the airline needs to maintain a margin of 10%. Therefore, the airline posts $45,000 with the exchange to keep the 10% maintenance margin. On day 3, the delivery date, the future price of crude oil increases to $41.50. Therefore, the seller of the contract (short position) needs to pay $1.50 to the airline for each barrel, for a total of $75,000. At the end of day 3, the airline has lost $225,000 as follows:

($100,000) + ($200,000) + $75,000 = ($225,000)

At the end of day 3, the airline purchases 50,000 barrels at $41.50 per barrel for a total of $2,075,000. The airline lost $225,000 for daily settlements, making the total cost of fuel $2,300,000. Therefore each barrel of oil is costing the airline $46 per barrel ($2,300,000/50,000).

Forward Contracts

A forward contract is an over–the-counter agreement between two parties which involves a future transaction. In this type of contact, both parties agree upon a price for a commodity that will be paid on a specific future date. Unlike futures contracts which are settled daily, forward contracts are settled at maturity date. These types of contracts are very common in many of our daily transactions. Examples include paying contractors when they finish the job, buying airline tickets, paying utility bills at the end of the month or even ordering a pizza to be delivered in an hour at a specific price. These types of contracts typically involve the actual delivery of the commodity or service. The airlines enter into forward contracts with their fuel supplier to receive a certain quantity of jet fuel on a specific future date at a specific price agreed upon today. The following example demonstrates how forward contracts work.

An airline enters into a forward contract with its supplier for 1,000,000 gallons of jet fuel to be delivered to the airline in one month at a price of $1.40 per gallon. At the maturity time (one month) the airline pays the supplier $1,400,000 ($1.40 × 1,000,000). If at maturity the jet fuel price is more than $1.40 per gallon then the airline has made a saving through this price hedging. If, on the other hand, the price is lower than $1.40 in a month's time, then the airline has lost money by engaging in this forward contact.

There are other types of more complicated hedging strategies that the airlines adopt in order to protect themselves against jet-fuel price fluctuations, including hedging against foreign currency rates, as crude oil is mainly traded in US dollar. Interested readers can refer to books on derivatives and risk management such as Chance and Brooks (2006).

Aircraft Fuel Supply

Airlines distribute and supply fuel to aircraft at different airports based on their network size, hubs, airport infrastructure, hedging strategies, and contracts with their suppliers. Some major airports provide underground piping systems which pipe fuel from tanks inside or outside the airport directly to hydrant systems located at each gate. This is the most convenient and efficient way to supply fuel to aircraft. This type of fuel supply distribution typically occurs in major hubs and airports. However, the most common way of supplying fuel to aircraft is through fuel tankers.

Mathematical Models for Fuel Management Systems

Airlines adopt fuel hedging as discussed before at the planning phase. During the operations phase, airlines can take advantage of other strategies to save on fuel cost. One of these strategies is referred to as fuel ferrying or tankering. In this strategy, airlines take advantage of different fuel prices at different airports. The aircraft may load (ferry or tanker) extra fuel at those airports with lower prices and thus save on overall cost of fuel over the next multiple flights, depending on the capacity and schedule of the aircraft. It should be noted that carrying extra fuel makes the aircraft heavier and thus burns additional fuel to carry this extra load.

Furthermore, carrying extra fuel exerts additional force on the aircraft's landing gear, brakes and tires, thus increasing maintenance cost.

The price of aviation fuel differs at different airports depending on the country they are located in, federal and local regulations, proximity to refineries, piping access to the airport, and volume (Doganis 2001). The following table presents the price of jet fuel in different international markets. Surprisingly the price of jet fuel in the Middle East, a major producer of crude oil, is not much cheaper than the in rest of the world. This is because very few refineries in the world produce jet fuel.

Table 9.2 Price of jet fuel in different international markets during March 2009

	Share in world index	Cents/gallon	$/barrel
Jet fuel price	100%	139.2	58.5
Asia and Oceania	22%	138.1	58.0
Europe and CIS	28%	139.9	58.8
Middle East and Africa	7%	135.1	56.7
North America	39%	139.4	58.6
Latin and Central America	4%	145.4	61.1

Source: International Air Transport Association

Airlines typically purchase fuel for their aircraft from airport fuel suppliers. They negotiate and enter into contracts with the airport fuel-suppliers depending on the number of flights, length of contract, number of competing suppliers at the airport, and daily demand. Some governments may impose a fixed and non-negotiable jet-fuel price for their airports.

In the United States, there are many small refineries supplying jet fuel to airports. Accordingly, many jet fuel suppliers compete at different US airports. Table 9.3 presents the amount of fuel used and the price paid per gallon for different US airlines during March, 2009.

It should be noted that fuel consumption varies significantly depending on type of aircraft, size, number and age of engines, route, and other meteorological conditions (Doganis 2001).

The study of fuel management system dates back to the 1970s, when the price of crude oil increased significantly and airlines started investigating more efficient ways to reduce their fuel cost. Some of these studies include Darnell and Loflin (1977), Stroup and Wollmer (1992), Irrgang (1999, 2005), Zouein et al. (2002) and Abdelghany et al. (2005). These models attempt to provide a strategy to minimize the total fuel cost by ferrying fuel at each airport, for a

Table 9.3 Amount of fuel used and the price paid per gallon for different US airlines during March, 2009

Airport identifier	Airport	Price per gallon
ATL	Hartsfield-Jackson Atlanta International Airport	$1.36
BOS	Boston's Logan International Airport	$1.60
BWI	Baltimore Washington International Airport	$1.39
DEN	Denver International Airport	$1.51
DFW	Dallas Fort Worth International Airport	$1.31
LAS	McCarran International Airport	$1.45
LAX	Los Angeles International Airport	$1.65
LGA	LaGuardia Airport	$1.66
MCO	Orlando International Airport	$1.45
MDW	Chicago Midway Airport	$1.46
PHL	Philadelphia International Airport	$1.48
SFO	San Francisco International Airport	$1.45

given aircraft route, subject to operational and safety constraints. These studies include both linear and non-linear programming models. The model discussed here is a modified version of the linear programming model proposed by Zouein et al. (2002).

Case study

The following case study pertains to an airline operating in the US. The airline requested to stay anonymous. The airline provided us with information on one of their Boeing 737-700 aircraft. The information pertains to 400 flights flown by the aircraft over several months and contains origin/destination, actual flight time, fuel consumed, fuel uplifts, and fuel prices at each airport. Table 9.4 presents data from the last 20 flights of the aircraft. We attempt to apply our mathematical model to these 20 flights to determine the fuel uplift at each airport and evaluate how the solution differs from the actual strategy that the airline implemented. The table presents the fuel price per gallon at each origin airport. Since all the units in this model are in pounds, the fuel prices are converted in pounds (1 gallon = 6.6 pounds).

Table 9.4 Data from the last 20 flights flown by the Boeing 737-700 aircraft

Origin	Destination	Scheduled flight time minutes	Price $/gallon	Price $/lb	Load factor %
1	2	289	1.89	0.29	95%
2	3	126	1.76	0.27	39%
3	4	130	1.84	0.28	61%
4	5	63	1.76	0.27	100%
5	6	102	1.76	0.27	100%
6	7	117	2.08	0.31	95%
7	8	210	1.84	0.28	99%
8	9	214	1.87	0.28	100%
9	10	131	1.76	0.27	97%
10	11	134	2.08	0.31	100%
11	12	130	1.76	0.27	73%
12	13	124	1.84	0.28	63%
13	14	60	1.76	0.27	99%
14	15	111	1.82	0.28	100%
15	16	83	1.81	0.27	100%
16	17	93	1.76	0.27	100%
17	18	117	2.15	0.33	100%
18	19	76	1.76	0.27	79%
19	20	125	1.81	0.27	99%
20	21	91	1.82	0.28	67%

The mathematical model is explained as follows:

Decision Variables

The decision variables pertain to the amount of fuel to be loaded at each airport. We define these variables as:

x_i = Amount of fuel loaded by the aircraft at airport *i*
y_i = Amount of fuel remaining in the aircraft when the aircraft landed at airport *i*

Objective Function

In this model, we attempt to minimize the total fuel cost over the one week routing of the aircraft. Therefore:

$$\sum_{i=1}^{n} c_i x_i = \text{Minimize } .29x_1 + .27x_2 + \quad + .28x_{20}$$

where:
c_i = the cost of 1 pound of fuel at airport *i* and
n = the number of the flight in this model.

Constraints

There are some safety and operational constraints that need to be addressed as follows:

Aircraft Tank Capacity

The amounts of fuel remaining in the aircraft fuel tank and fuel loaded at each airport should not exceed the aircraft's fuel tank capacity. According to Boeing, the aircraft manufacturer, the fuel tank capacity of a 737-700 is 46,063 pounds. Therefore:

$$x_i + y_i \leq 46{,}063 \qquad (9.1)$$

Maximum Takeoff Weight (MTW)

This set of constraints ensures that the weight of the aircraft, including the passengers, bags (payload weight), and fuel, does not exceed a maximum allowable takeoff weight (MTW), specified by the aircraft manufacturer. According to Boeing, the empty weight of a 737-700 aircraft, known as dead operating weight (DOW), is 83,000 pounds and the maximum takeoff weight of the aircraft is 153,000 pounds. The maximum payload weight of the aircraft (that is, the total weight of passengers and bags) is 37,500 pounds. To forecast the payload for each upcoming flight, we used the load factor for that flight. Airlines typically know the load factor of their upcoming flights either based on actual reservation or historical data. We estimate the payload for each flight based on load factor as follows:

$$payload_{i,i+1} = max\_payload \times load\ factor_{i,i+1}$$

where:
$payload_{i,i+1}$ and $load\ factor_{i,i+1}$ represents the payload and load factor on flight from airport i to airport $i+1$ respectively.

As an example, the load factor for the first flight in Table 9.4 is 95% (see the load factor in Table 9.4 for flight 1). Therefore, the payload for this flight is:

$$payload_{1,2} = 37{,}500 \times .95 = 35{,}625$$

Therefore, we can write the maximum takeoff weight constraints for each flight as follows:

$$DOW + payload_{i,i+1} + x_i + y_i \leq MTOW \qquad (9.2)$$

where:
DOW and $MTOW$ are dead operating weight and maximum takeoff weights of the aircraft respectively.

By including the weights of DOW and $MTOW$ for our 737-700 as was explained earlier, we have:

$$83{,}000 + payload_{i,i+1} + x_i + y_i \leq 153{,}000 \qquad (9.3)$$

or:

$$payload_{i,i+1} + x_i + y_i \leq 70{,}000 \qquad (9.4)$$

As an example, for flight 1, we have:

$$35{,}625 + x_i + y_i \leq 70{,}000 \qquad (9.5)$$

or:

$$x_i + y_i \leq 34{,}375 \qquad (9.6)$$

Similarly we can write constraints for the other flights.

Maximum Landing Weight

For safe operations, aircraft manufactures specify a maximum landing weight for their aircraft. The total weight of an aircraft when landing is basically the weight at departure minus the fuel burned during the flight. According to Boeing, the maximum landing weight for a 737-700 is 128,000 pounds.

The fuel consumption for each flight depends on the duration of the flight, weight of the aircraft, tail- and head-wind speeds, taxi in and out of the airport gates, and so on. To forecast the fuel consumption for future upcoming flights, we used a modified regression analysis similar to Zouein et al. (2002). As indicated earlier, we had access to information on 400 flights flown by this particular Boeing 737-700 aircraft. We used data pertaining to the first 380 flights for the regression and applied the model to the last 20 flights, to evaluate its performance. Among the information for each flight, we had access to actual fuel consumed and scheduled flight times. Note that we are using the published scheduled flight times and not the actual flight times. Figure 9.5 presents fuel consumed in pounds against scheduled flight duration in minutes for the 380 flights. A linear regression model was found to be a good fit. According to the regression analysis, the following equation provides an estimate of fuel consumption in pounds based on scheduled flight duration in minutes:

$$F = 81.826t + 937.38$$

where:

F = the fuel consumption in pounds and

t = the scheduled flight-time in minutes.

The correlation coefficient for this linear regression model is R = .9841 or $R^2 = .9685$ as suggested by the figure, which indicates a good fit.

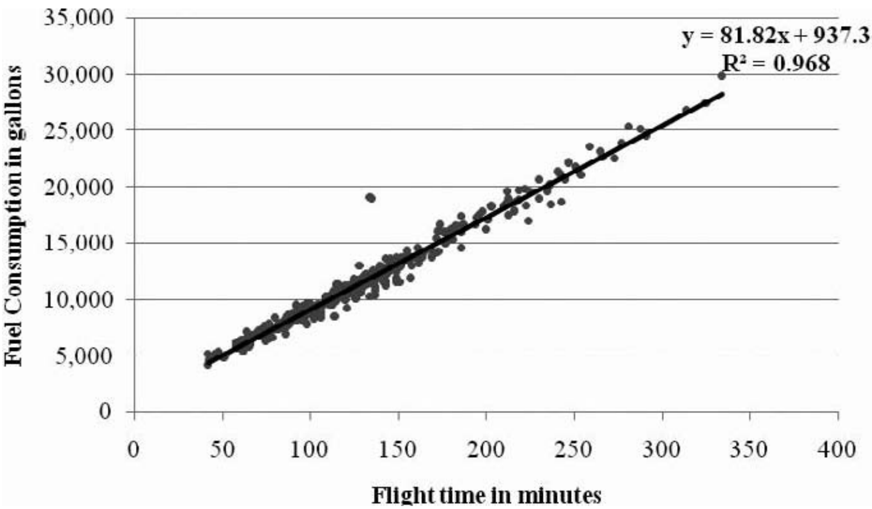

Figure 9.5 Scatter plot of fuel consumption vs. flight time

We use this equation to estimate the fuel consumption for each upcoming flight based on their published flight times. As an example, our forecast for fuel consumption for flight 1 with a scheduled 289 minutes of flight time is:

$F = 81.826 \times 289 + 937.38 = 24{,}585.1$ pounds

Now returning to the constraint for maximum landing weight, we have:

Total weight of the aircraft at takeoff – fuel consumed during the flight ≤ maximum landing weight

or:

$$DOW + payload_{i,i+1} + x_i + y_i - FB_{i,i+1} \leq MLW \qquad (9.7)$$

where the first four terms on the left hand side are the total weight of aircraft at takeoff at airport *i* as described in the previous section on maximum takeoff weight, $FB_{i,i+1}$ is the fuel burned during the current flight and MLW is the maximum landing weight.

Replacing the values for DOW and MLW will results in

$$83{,}000 + payload_{i,i+1} + x_i + y_i - FB_{i,i+1} \leq 128{,}000 \qquad (9.8)$$

or:

$$payload_{i,i+1} + x_i + y_i - FB_{i,i+1} \leq 45{,}000 \qquad (9.9)$$

As an example, for flight 1, where the fuel burn and payloads are estimated to be 24,585.1 and 35,625 pounds respectively, we have:

$$35{,}625 + x_i + y_i - 24{,}585.1 \leq 45{,}000$$

or:

$$x_i + y_i \leq 33{,}960.1$$

Similarly we can write the constraints for other flights.

Safety Fuel at Landing

Airlines are mandated to maintain a minimum level of fuel on the aircraft when landing. This safety fuel should be enough to allow the aircraft to stay in the air while waiting for its turn to land or to go to an alternative airport should the destination airport be closed. Therefore the constraint for the amount of fuel left in the tank when landing is:

$$x_i + y_i - FB_{i,i+1} \geq SF_{i,i+1} \quad (9.10)$$

The first two terms on the left hand side represent the total fuel in the tank at takeoff. *FB* represents the fuel burned during the flight and *SF* is the safety fuel. Airlines employ different strategies to adopt their level of safety fuel. Some airlines consider an extra 5% fuel of the upcoming flight and some indicate 20-30 minutes of flight time as their safety fuel level.

In this case, the airline's policy on safety fuel level is to have at least 6,500 pounds of fuel. Therefore:

$$x_i + y_i - FB_{i,i+1} \geq 6{,}500 \quad (9.11)$$

As an example, for flight 1, we have:

$$x_i + y_i - 24{,}585.1 \geq 6{,}500$$

or:

$$x_i + y_i \geq 31{,}085.1$$

Fuel Balance

This set of constraints provides a balance between the fuel uploads and fuel consumption. Basically, the amount of fuel at take-off minus fuel burned during the flight should be equal to the fuel remaining in the tank at destination. Therefore:

$$x_i + y_i - FB_{i,i+1} = y_{i+1} \quad (9.12)$$

For example, for the first flight, we have the following constraint:

$$x_i + y_i - 24{,}585.1 = y_2 \quad (9.13)$$

Solution

The above linear programming model has 40 decision variables (20 *x* and 20 *y* variables) and 100 constraints. Table 9.5 provides the solution generated by the software for the amount of fuel to be uplifted x_i and the remaining fuel in the tank y_i at every airport.

Table 9.5 Linear programming solution for the case study

x_i	Pounds of fuel uplift	y_i	Pounds of fuel remaining in the tank
x1	26,200.00	y1	7,760.10
x2	27,039.61	y2	9,375.01
x3	0	y3	25,167.16
x4	0	y4	13,592.40
x5	9,283.62	y5	7,499.98
x6	9,511.05	y6	7,499.97
x7	18,120.84	y7	6,500.00
x8	18,448.14	y8	6,500.00
x9	13,781.60	y9	6,500.00
x10	9,777.05	y10	8,625.01
x11	22,658.56	y11	6,500.00
x12	0	y12	17,583.80
x13	7,221.90	y13	6,500.00
x14	8,645.11	y14	7,874.96
x15	7,728.94	y15	6,500.00
x16	9,547.20	y16	6,500.00
x17	9,511.02	y17	7,500.00
x18	7,156.16	y18	6,500.00
x19	12,540.60	y19	6,500.00
x20	7,008.58	y20	7,874.97

This fuel uplift strategy results in a total cost of $62,917 for these 20 flights.

It is of interest to compare and contrast this strategy with the one that the airline adopted for the same 20 flights. We received the data for fuel uplift for the above 20 flights from each of the airports. The total cost associated with the airline's strategy is $64,546. Comparing the optimum cost of $62,917 with this strategy represents a saving of $1,642 or 2.5% on fuel cost. This saving is only for 20 flights on a single aircraft. It can easily translate into large savings annually when this strategy is adopted to all aircraft on all flights. The following describes the general mathematical model adopted in this section.

Fuel-tankering Mathematical Model

The general mathematical model proposed by Zouein et al. (2002) is discussed as follows:

Sets

N = Set of all aircraft in the fleet
n = Set of flights connecting airports in the predetermined horizon

Indices

i = airport index
j = aircraft index

Parameters

C_i = Cost of fuel at airport i
DOW = Dead operating weight of the aircraft
PL_i = Payload weight at departure airport i
$MTOW$ =Maximum allowed weight at takeoff
MLW = Maximum allowed weight at landing
$Tank$ = Maximum tank capacity
$FC_{i,i+1}$ = Amount of fuel consumption by the aircraft between airport i and $i+1$
$SF_{i,i+1}$ = Amount of safety fuel when reaching airport $i+1$ coming from airport i

Decision Variables

x_i = Amount of fuel loaded by the aircraft at airport i
y_i = Amount of fuel remaining in the aircraft when it landed at airport i

$$Minimize \sum_{i=1}^{n} X_i \quad c_i x_i \tag{9.14}$$

$$x_i + y_i \leq Tank \tag{9.15}$$

$$DOW + payload_{i,i+1} + x_i + y_i \leq MTOW \tag{9.16}$$

$$DOW + payload_{i,i+1} + x_i + y_i - FB_{i,i+1} \leq MLW \tag{9.17}$$

$$x_i + y_i - FB_{i,i+1} \geq SF_{i,i+1} \tag{9.18}$$

$$x_i + y_i - FB_{i,i+1} = y_{i+1} \tag{9.19}$$

In this model, the objective function in (9.14) attempts to minimize the total cost of fuel for a set of predetermined flights, for all aircraft in the fleet. The set of constraints in (9.15), (9.16), (9.17) and (9.18) impose operational and safety restrictions as discussed earlier. Equation (9.19) provides the balance between fuel uploads and fuel consumption.

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Chapter 10
Airline Irregular Operations

Introduction

Aircraft mechanical problems, severe weather, crew sickness, airport curfews, and security are among the problems that force an airline to delay or even cancel their regular published flights. On an average day in the United States, approximately 15–20% of all flights experience significant delays (more than 15 minutes) and approximately 1–3% of all flights are cancelled (Yu et al. 2003). The scheduling methodologies described in Chapters 6 and 7 provide an airline with a very efficient plan, high utilization of resources, and very tight aircraft and crew assignments. In many cases a small perturbation in this plan, such as unavailability of an aircraft or crew, results in major disruption to the scheduled flights. The airlines adopt a combination of tactics such as flight delays, flight cancellations, aircraft substitutions, ferry flights (flying an empty aircraft to a point of need), and aircraft diversions to return to their published scheduled flights as soon as possible. Since these activities are not pre-planned and occur only when there is a disruption in the schedule, they are called irregular operations. The recovery time can span from the time the disruption occurs up to the time the airline gets back to its original schedule.

In practice the two problems of aircraft recovery and crew reassignment are handled separately (Jarrah Yu 1993). The airlines that are faced with disruption first attempt to develop a feasible flight rerouting through some of the tactics mentioned above. This new rerouting schedule is checked for crew assignment feasibility. If a feasible crew assignment does not exist, a new rerouting schedule is developed. This process continues until a feasible rerouting and crew reassignment is found. This chapter examines the daily aircraft rerouting schedules for single fleet only. See the list of references for models that examine multi-fleet and crew reassignment. The aircraft-schedule recovery problem is basically defined as:

Given the position of planes at the time of a disruption, the original flight schedule, an estimated length of disruption time, and a time frame for recovery, find the ‘best’ assignment of aircraft to flights so that after the recovery time the airline is capable of operating its regular published flights. Some of the objectives for the ‘best’ assignment include minimizing total passenger delays, minimizing cancellations, honoring curfews and regulations, and minimizing the total cost to the airline.

The following sections provide the analysis for a case study and the development of the mathematical model.

Case Study

We begin by examining a case study involving a break in the regular schedule. This case and the accompanying methodology discussed in this chapter are adapted from Argüello, et al. (1998) with minor modifications.

This case involves three aircraft, twelve flights, and four cities. Table 10.1 presents the routing for each aircraft and the departure/arrival times. All times are eastern standard times. According to this table we have one aircraft available at DAB, ORF, and IAD to start their daily scheduled flights.

Table 10.1 Flight schedule and aircraft routing

Aircraft ID	Flight ID	Origin	Destination	Departure	Arrival
Aircraft 1	11	DAB	ORF	1410	1520
	12	ORF	IAD	1605	1700
	13	IAD	ORF	1740	1840
	14	ORF	DAB	1920	2035
Aircraft 2	21	ORF	DAB	1545	1700
	22	DAB	ORF	1740	1850
	23	ORF	IAD	1930	2030
	24	IAD	ORF	2115	2215
Aircraft 3	31	IAD	ATL	1515	1620
	32	ATL	IAD	1730	1830
	33	IAD	ATL	1910	2020
	34	ATL	IAD	2100	2205

Now let us assume that one of the aircraft becomes unavailable owing to some mechanical problem at an airport. Our objective is to handle all the remaining flights in the network through a series of delays and/or cancellations so that the total cost to the airline is minimized. Thengvall et al. 2000 provides an overview of other objective functions considered by researchers.

Before we present the mathematical model, we need to define the underlying transformation of this problem into a time-band model as was proposed by Argüello et al. (1998). This transformation enables us to use the network structure similar to the time-space network employed in Chapter 4 to represent the mathematical model.

Time-band Approximation Model

The network structure is similar to the time-space network discussed in Chapter 4. The time-space representation of the above case study without any disruption is shown in Figure 10.1.

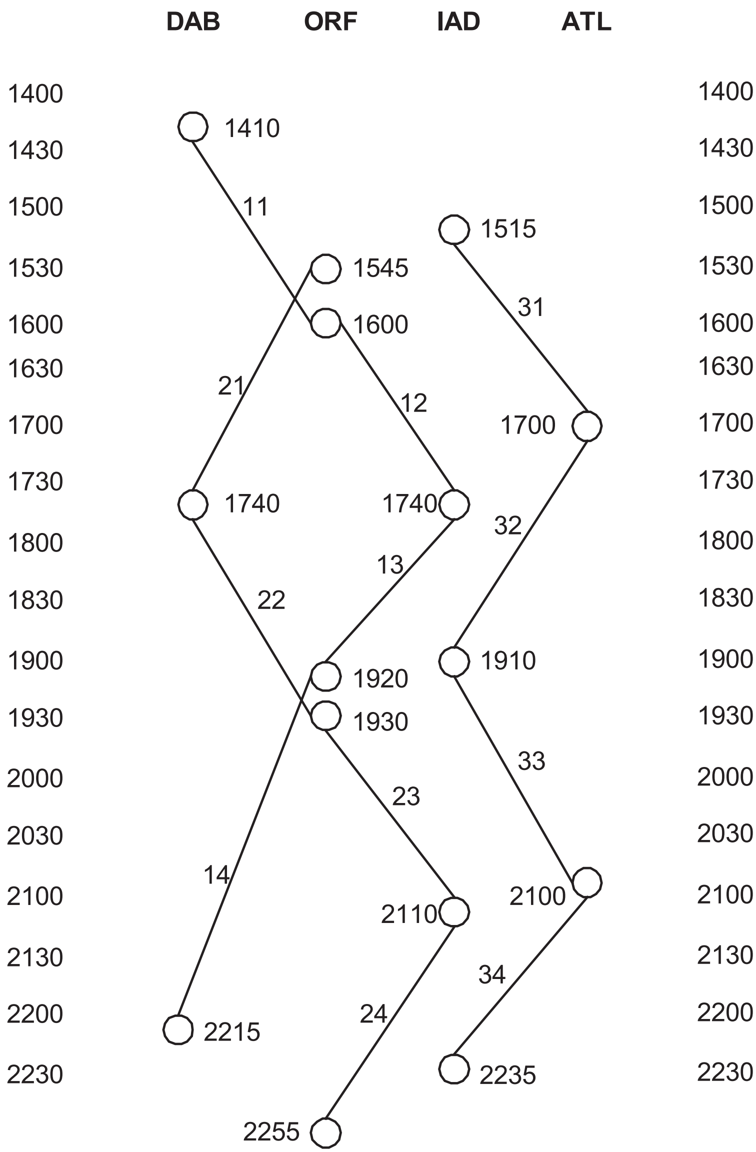

Figure 10.1 Time band network for the case study

In this figure the flight arcs are those that stretch from one airport to another (see for example flight 11). The flight numbers are shown on the flight arcs. The nodes represent an arrival and departure at a specific time. In this network, similar to Chapter 4, the cities and times are represented horizontally and vertically respectively.

In this model the time horizon is partitioned into time bands or discrete intervals of fixed length. Without loss of generality, in our case study as shown in Figure 10.1, this time band is 30 minutes. By partitioning the time horizon into time bands, station activity is aggregated into that time-band node.

We also have the following assumptions for our case study:

- each station requires a minimum of 40 minutes turnaround time;
- midnight arrival/departure curfew (no arrival or departure after midnight);
- each minute of delay on any flight costs the airline $20;
- cancellation cost for each flight leg is as follows.

Table 10.2 Cancellation cost for flight legs

Aircraft ID	Flight ID	Origin	Destination	Cancellation cost
Aircraft 1	11	DAB	ORF	$7,350
	12	ORF	IAD	$10,231
	13	IAD	ORF	$7,434
	14	ORF	DAB	$14,191
Aircraft 2	21	ORF	DAB	$11,189
	22	DAB	ORF	$12,985
	23	ORF	IAD	$11,491
	24	IAD	ORF	$9,581
Aircraft 3	31	IAD	ATL	$9,996
	32	ATL	IAD	$15,180
	33	IAD	ATL	$17,375
	34	ATL	IAD	$15,624

A major assumption and rule in this model is that during the recovery period, any flight arc from any airport (node) can be made available to other feasible airports (nodes).

Considering the time-band intervals, the flight paths, and the fact that flight from every node is available to every other feasible node, results in the following time-band network.

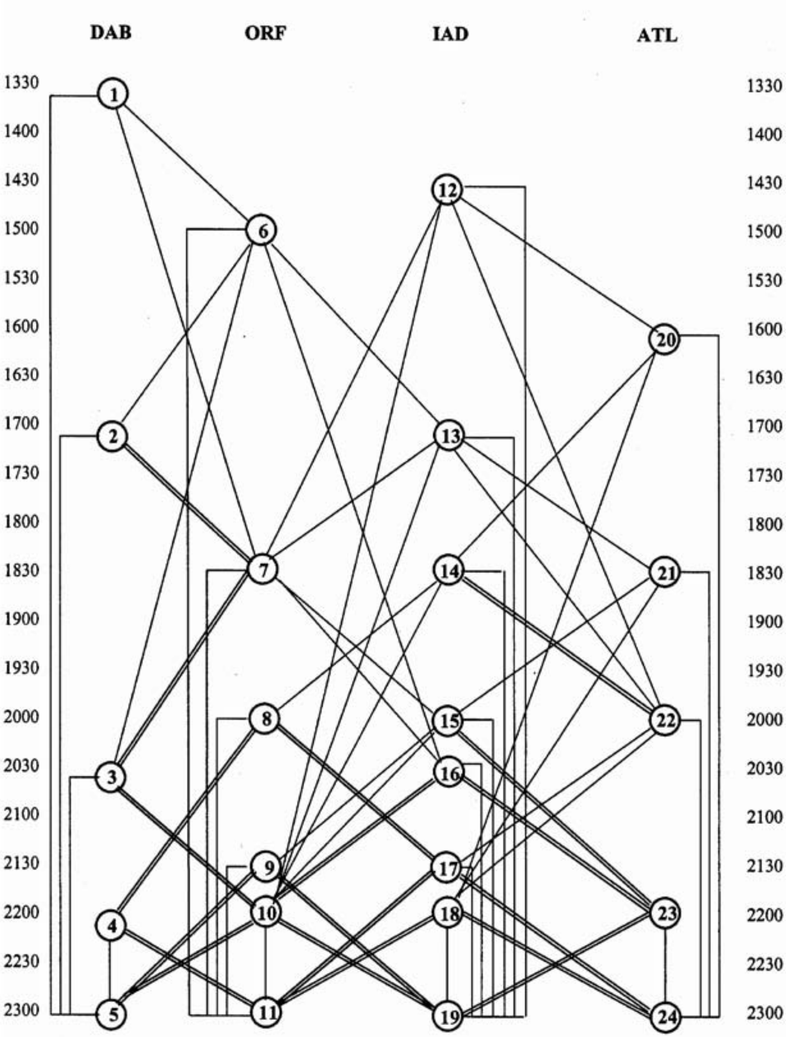

Figure 10.2 Time band approximation network

As the figure suggests, all 30-minute activities within an airport are aggregated in a single node. As an example, node 1 represents all activities from 1:30 p.m. through 1:59 p.m. Argüello et al. (1998) classify the nodes in two groups: transshipment and sink nodes. Transshipment nodes, also referred to as station-time nodes, are those nodes that the aircraft arrives into and leaves. In Figure 10.2 these nodes include 1, 2, 3, 4, 6, 7, and so on. Sink nodes, also referred to as station sink nodes, represent those nodes that the aircraft arrives into but does not leave until the end of recovery time. These nodes are similar to starting nodes for wrap-around arcs discussed in Chapter 4. In Figure 10.2, nodes 5, 11, 19, and 24 are station sink nodes.

In this figure we see that, for example, two arcs are drawn from node 2 to node 7. These two arcs represent flights 11 and 22. For the arc representing flight 11, we have a delay of 210 minutes. This is because flight 11 was scheduled to leave DAB at 1410. If this flight occurs in node 2, we have the departure time of 1700. Considering the nodes are on 30-minute time-bands, this delay spans from 14:00 to 17:30, a total of 210 minutes. Each minute of delay costs the airline $20. So, flight 11 has a delay cost of $4,200 if it departs from node 2. The other arc connecting nodes 2 to 7 represents flight 22. By looking at the departure and arrival times of this flight, there is no delay (within a 30 minute time-band) for this flight. Table 10.3 presents the non-zero delay costs for all flight arcs in Figure 10.2.

Scenario 1

Let us assume that aircraft 2 in airport ORF becomes grounded owing to some mechanical failure at 1400 and is unavailable for the rest of the day. The obvious solution without permitting any rerouting of other aircraft is to cancel flights 21, 22, 23, and 24 which are conducted by this grounded aircraft for the day. These cancellations cost the airline a total of $45,246 (the sum of all cancellation costs for these four cancelled flights). Let us see how this problem is solved through a series of aircraft rerouting and cancellations in an effort to minimize the total cost to the airline.

Table 10.3 Non-zero delay costs

Flight number	Origin node	Destination node	Delay cost
11	2	7	4,200
11	3	10	8,500
11	4	11	10,300
12	7	15	3,900
12	8	17	5,700
12	9	19	7,800
12	10	19	8,100
13	14	8	1,800
13	15	9	3,900
13	16	10	4,200
13	17	11	5,700
13	18	11	6,100
14	8	4	1,800
14	9	5	3,900
14	10	5	4,200

Table 10.3 Non-zero delay costs

Flight number	Origin node	Destination node	Delay cost
21	7	3	4,300
21	8	4	6,100
21	9	5	8,200
21	10	5	8,500
22	3	10	4,300
22	4	11	6,100
23	8	17	1,600
23	9	19	3,700
23	10	19	4,000
24	17	11	1,400
24	18	11	1,800
31	13	21	2,900
31	14	22	4,700
31	15	23	6,800
31	16	23	7,100
31	17	24	8,600
31	18	24	9,000
32	21	15	2,300
32	22	17	4,100
32	23	19	6,200
33	15	23	2,100
33	16	23	2,400
33	17	24	3,900
33	18	24	4,300
34	23	19	2,000

Decision Variables

We define the following decision variables:

$$x_{i,j}^{k} = \begin{cases} 1 \text{ if flight } k \text{ is conducted from station time node } i \text{ to } j \\ 0 \text{ otherwise} \end{cases}$$

$$y_k = \begin{cases} 1 \text{ if flight k is cancelled} \\ 0 \text{ otherwise} \end{cases}$$

z_i = Number of aircraft (integer) terminated at station node *i* (node *i* being a station sink node).

The binary Variable $x_{i,j}^{k}$ is used to identify which flights should be conducted along which routes. For example, $x_{1,6}^{11}$ represents the variable for flight number 11 through nodes 1 to 6 (see Figure 10.2). The binary variable y_k is adopted to identify which flight(s) should be cancelled. For example, y_{11} represents the decision variable for canceling flight number 11. A value of 1 for this variable means that the flight should be cancelled.

The integer variable z_i is used to keep track of aircraft balance and to have aircraft available at the end of the day for the next day's flight schedule. For example, referring to Figure 10.2, z_1 is the number of aircraft in node 1 (DAB) which is not flown through the day and is carried to the station-sink 5.

Objective Function

The objective function consists of two terms, the delayed cost and the cancellation cost for each flight. Referring to Tables 10.2 and 10.3 we have the objective function as:

$$\textit{Minimize } 4200x_{2,7}^{11} + 8500x_{3,10}^{11} + \ldots.. + 7350y_{11} + 10231y_{12} + \ldots + 15624y_{34}$$

Constraints

For this mathematical model, we have three sets of constraints as follows:

Set 1 – Flight Coverage

Each flight must either be flown or be cancelled. As an example to express flight 11's coverage we have:

$$x_{1,6}^{11} + x_{2,7}^{11} + x_{3,10}^{11} + x_{4,11}^{11} + y_{11} = 1$$

The above equation specifies that for flight 11, out of four possible flights (see Table 10.3) and a cancellation, only one must be selected.

We write similar equations for every available flight in Table 10.1, a total of 12 constraints for this set.

Set 2 – Station Time-Node Flow

For this set we need to write the flow of aircraft at each node. There are some nodes that have aircraft available (supply nodes) to start the flow within the network (such as nodes 1, 6, and 12). Most of the station-time nodes are transshipment nodes signifying that the net flow in these nodes is zero. The net flow for a node is determined as follows:

The number of aircraft in a node = number of outgoing aircraft from the node – (minus) incoming aircraft into the node + (plus) the number of aircraft carried over from this node to sink node (same city) for the next day's operation.

For example, for node 1 we have:

$$x_{1,6}^{11} + x_{1,7}^{22} + z_1 = 1$$

Referring to Figure 10.2 and Tables 10.1 and 10.3, we have two outgoing flights from node 1. These are flights 11 represented by arc 1,6 and flight 22, represented by arc 1,7. There are no incoming flights to node 1. z_1 represents the number of aircraft that are carried over to node 5 which is a sink station node. The purpose of z variables is to allow the flexibility to the model to save the aircraft at some specific cities for the next day's operation. The right hand side of the above equation is 1. This is because at node 1, (DAB) we have one aircraft available to start the flights from DAB (see Table 10.1).

Similarly the flow balance for node 2 (transshipment node) is as follows:

$$x_{2,7}^{11} + x_{2,7}^{22} - x_{6,2}^{21} + z_2 = 0$$

We have two outgoing (flights 11 and 22) and one incoming flow (flight 21) in this node. z_2 is the number of aircraft that are grounded in DAB and are carried over to station-sink 5. The right hand side of this constraint is zero since node 2 is a transshipment node. As the aircraft at node 6 is grounded and not available (scenario 1), the right hand side for this constraint is also zero.

In this case we have 20 station-time nodes resulting in 20 constraints for this set.

Set 3 – Station Sink-Node Flow

We include this set of constraints to ensure that there are aircraft available in the designated airports at the end of the day to fly the flights for the next day according to the published schedule. Basically the following rule applies for these sink nodes:

Required number of aircraft at any sink node = Total incoming flight terminating at this sink node + (plus) number of carried over aircraft from previous transshipment nodes at this airport.

In this case study, to be able to fly the published schedule for the next day, we must have one aircraft available in DAB, ORF, and IAD each. Therefore we must ensure that the net flow in station sink nodes for these cities is one. Without this set of constraints, the aircraft may end up at the wrong airports at the end of the day. The following constraint represents the net flow for DAB for station sink node 5 (see Figure 10.2).

$$x_{9,5}^{14} + x_{9,5}^{21} + x_{10,5}^{14} + x_{10,5}^{21} + z_1 + z_2 + z_3 + z_4 = 1$$

There are four arcs coming from other cities to node 5. The first four terms of the above equation represent these four arcs (flights). The other four terms represent the number of aircraft from previous nodes in the same city (DAB) carried over to this sink node. It should be noted that for ORF, we assumed that the aircraft is grounded and not available for the rest of the day. It is assumed, however, that it will be available for the next day.

We have four sink nodes which will result in four constraints for this set.

Solution

The above case study has 64 flight arcs (*x* variables), 12 flight cancellation (*y* variables) and 20 termination nodes (*z* variables), a total of 96 binary/integer variables. It has 36 constraints. The solution to this model is as follows.

Table 10.4 Solution for Scenario 1

Aircraft ID	Flight	Origin	Destination	Origin node	Destination node	Delay cost	Cancellation cost
Aircraft 1	11	DAB	ORF	1	6	-	-
	21	ORF	DAB	6	2	-	-
	22	DAB	ORF	2	7	-	-
	23	ORF	IAD	7	16	-	-
	24	IAD	ORF	16	10	-	-
	14	ORF	DAB	10	5	4,200	-
Cancel	12	ORF	IAD	-	-	-	10,231
	13	IAD	ORF	-	-	-	7,434
Aircraft 3	31	IAD	ATL	12	20	-	-
	32	ATL	IAD	20	14	-	-
	33	IAD	ATL	14	22	-	-
	34	ATL	IAD	22	18	-	-
Total cost						4,200	17,665

The minimum cost solution for this scenario is two cancellations and one delayed flight at a total cost of $21,865 ($4,200+$17,665). Compare this cost with the trivial solution of $45,246 resulting from canceling all flights operated by aircraft 2.

Note that this model was based on aggregating the activities within an airport in a 30-minute time-band into one single node. The above solution is utilized to fine-tune and determine the actual departure/ arrival times and the actual cost for each flight. The detailed solution for each flight with its revised arrival/departure times is presented in Table 10.5.

Table 10.5 Detailed and final solution for Scenario 1

Aircraft ID	Flight	Origin	Destination	Departure time	Arrival time	Delay cost	Cancellation cost
Aircraft 1	11	DAB	ORF	1410	1520	-	-
	21	ORF	DAB	1600	1715	300	-
	22	DAB	ORF	1755	1905	300	-
	23	ORF	IAD	1945	2045	300	-
	24	IAD	ORF	2125	2225	200	-
	14	ORF	DAB	2305	0020	4,500	-
Cancel	12	ORF	IAD	-	-	-	10,231
	13	IAD	ORF	-	-	-	7,434
Aircraft 3	31	IAD	ATL	1515	1620	-	-
	32	ATL	IAD	1730	1830	-	-
	33	IAD	ATL	1910	2020	-	-
	34	ATL	IAD	2100	2205	-	-
Total cost						5,600	17,665

The above departure/arrival times accommodate for 40-minute aircraft turnaround times. Note that the cost for this schedule is higher than the solution presented in Table 10.4. This is because of the 30-minute aggregation in a single node. The actual total cost for the above feasible solution is $23,265, which is still significantly lower than the trivial cost.

Scenario 2

In scenario 1, we assumed that aircraft 2 was grounded. In scenario 2, we assume that both aircraft 1 and 2 are operational for the day but aircraft 3 is grounded in IAD at 14:00 and will be unavailable for the rest of the day. The trivial solution is to cancel all flights conducted by this aircraft, that is, flights 31, 32, 33, and 34 at a total cost of $58,175.

The mathematical model is basically very similar to scenario 1, with the following minor changes:

In set 2 of the constraints, for station-time node 6, the right-hand side becomes one since aircraft 2 is available in city ORF. The right-hand side for node 12, however, becomes zero because aircraft 3 is grounded in IAD and is unavailable.

Similarly, in set 3 of the constraints, the right-hand sides for nodes 11 and 19 become one and zero respectively. The solution to this mathematical model is given in Table 10.6.

Table 10.6 Solution for Scenario 2

Aircraft ID	Flight	Origin	Destination	Origin node	Destination node	Delay cost	Cancellation cost
Aircraft 1	11	DAB	ORF	1	6	-	-
	12	ORF	IAD	6	13	-	-
	33	IAD	ATL	13	22	-	-
	34	ORF	IAD	22	18	-	-
	24	IAD	ORF	18	11	1,800	-
Aircraft 2	21	ORF	DAB	6	2	-	-
	22	DAB	ORF	2	7	-	-
	23	ORF	IAD	7	16	-	-
	13	IAD	ORF	16	10	4,200	-
	14	ORF	DAB	10	5	4,200	
Cancel	31	IAD	ATL	-	-	-	9,996
	32	ATL	IAD	-	-	-	15,180
Total cost						**10,200**	**25,176**

The total cost for this solution is $35,376. Table 10.7, overleaf, shows the conversion of this solution to actual departure and arrival times.

The total cost for this actual flight schedule is also $35,376 which is similar to the approximation time-node solution.

Scenario 3

In this scenario, we assume that aircraft 2 and 3 in cities ORF and IAD are operational all day. Aircraft 1 in DAB, however, must be grounded at 13:00 for four hours. That is, aircraft 1 is unavailable from 13:00 to 17:00. The trivial solution is to cancel flights 11 and 12 which are flown by aircraft 1 during 13:00 to 17:00. The total cost associated with these two cancelled flights is $17,581.

Again with minor modifications to the previous models we can formulate this scenario as shown in Table 10.7 opposite.

In set 2 of the constraints, for station-time node 6 and 12 the right hand side becomes one. The right hand side for node 1 becomes zero because aircraft 1 is grounded in DAB. This aircraft returns back to service after four hours at 17:00.

Table 10.7 Detailed and final solution for Scenario 2

Aircraft ID	Flight	Origin	Destination	Departure time	Arrival time	Delay cost	Cancellation cost
Aircraft 1	11	DAB	ORF	1410	1520	-	-
	12	ORF	IAD	1605	1700	-	-
	33	IAD	ATL	1910	2020	-	-
	34	ORF	IAD	2100	2205	-	-
	24	IAD	ORF	2245	2345	1,800	-
Aircraft 2	21	ORF	DAB	1545	1700	-	-
	22	DAB	ORF	1740	1850	-	-
	23	ORF	IAD	1930	2030	-	-
	13	IAD	ORF	2110	2210	4,200	-
	14	ORF	DAB	2250	0005	4,200	
Cancel	31	IAD	ATL	-	-	-	9,996
	32	ATL	IAD	-	-	-	15,180
Total cost						10,200	25,176

Therefore, the right hand side value for node 2 (representing DAB at 17:00) becomes 1 (see figure 10.2). In set 3 of the constraints, since all the three aircraft are available to station sink nodes, we set the right-hand side values for nodes 5, 11, and 19 equal to 1. The solution to this linear integer programming model is shown in Table 10.8 overleaf.

Based on the above solution no flight is cancelled and the total cost to the airline is $15,800. Table 10.9 shows the actual departure and arrival for each flight derived from Table 10.8.

The total actual cost for the above feasible schedule is $13,500.

Mathematical Model

This section formally introduces the integer linear programming model adapted for the case study. This approach is based on the Time-Band Approximation Model by Argüello et al. 1998.

Indices

i,j = node indices
k = flight index

Table 10.8 Solution for Scenario 3

Aircraft ID	Flight	Origin	Destination	Origin node	Destination node	Delay cost	Cancellation cost
Aircraft 1	11	DAB	ORF	2	7	4,200	-
	12	ORF	IAD	7	15	3,900	-
	33	IAD	ATL	15	23	2,100	-
	34	ATL	IAD	23	19	2,000	-
Aircraft 2	21	ORF	DAB	6	2	-	-
	22	DAB	ORF	2	7	-	-
	23	ORF	IAD	7	16	-	-
	24	IAD	ORF	16	10	-	-
Aircraft 3	31	IAD	ATL	12	20	-	-
	32	ATL	IAD	20	14	-	-
	13	IAD	ORF	14	8	1800	
	14	ORF	DAB	8	4	1800	
Total cost						15,800	-

Table 10.9 Detailed and final solution for Scenario 3

Aircraft ID	Flight	Origin	Destination	Departure time	Arrival time	Actual Delay cost	Cancellation cost
Aircraft 1	11	DAB	ORF	1700	1810	3,400	-
	12	ORF	IAD	1850	1955	3,300	-
	33	IAD	ATL	2035	2145	1,700	-
	34	ATL	IAD	2225	2330	1,700	-
Aircraft 2	21	ORF	DAB	1545	1700	-	-
	22	DAB	ORF	1740	1850	-	-
	23	ORF	IAD	1930	2030	-	-
	24	IAD	ORF	2115	2215	-	-
Aircraft 3	31	IAD	ATL	1515	1620	-	-
	32	ATL	IAD	1730	1830	-	-
	13	IAD	ORF	1910	2010	1,800	
	14	ORF	DAB	2050	2205	1,600	
Total cost						13,500	-

Sets

F = set of flights
$G(i)$ = set of flights originating at station-time node i
$H(k,i)$ = set of destination nodes for flight k originating at station-node i
I = set of station-time nodes
J = set of station-sink nodes
$L(i)$ = set of flights terminating at node i
$M(k,i)$ = set of origination station-time nodes for flight k terminating at node i
$P(k)$ = set of station-time nodes from which flight k originates
$Q(i)$ = set of station-time nodes at airport containing station-sink i

Parameters

a_i = Number of aircraft available at station-time node i
c_k = Cost of canceling flight k
$d^k_{i,j}$ = Delay cost of flight k from station-node i to node j
h_i = Number of aircraft required to terminate at station-sink node j

Decision Variables

$$x^k_{i,j} = \begin{cases} 1 \text{ if flight } k \text{ occurs through station time node } i \text{ to } j \\ 0 \text{ otherwise} \end{cases}$$

$$y_k = \begin{cases} 1 \text{ if flight k is cancelled} \\ 0 \text{ otherwise} \end{cases}$$

z_i = integer number of aircraft terminated at station time node i to station sink node at that airport

Mathematical Formulation

Objective function

$$\text{Minimize} \quad \sum_{k \in F} \sum_{i \in P(k)} \sum_{j \in H(k,i)} d^k_{i,j} x^k_{i,j} + \sum_{k \in F} c_k y_k$$

Subject to

(flight cover) $\quad \sum_{i \in P(k)} \sum_{j \in H(k,i)} x^k_{i,j} + y_k = 1 \quad$ for all $k \in F$

(station-time node flow) $\quad \sum_{k \in G(i)} \sum_{j \in H(k,i)} x^k_{i,j} - \sum_{k \in L(i)} + z_i = a_i \quad$ for all $i \in I$

(station-sink node flow) $\quad \sum_{k \in L(i)} \sum_{j \in M(k,i)} x^k_{i,j} + \sum_{j \in Q(i)} z_j = h_i \quad$ for all $i \in J$

(binary aircraft flow) $\quad x^k_{i,j} \in \{0,1\} \quad$ for all $k \in F$, $i \in P(i)$ and $j \in H(k,i)$

(binary cancellation flow) $\quad y_k \in \{0,1\} \quad$ for all $k \in F$

(integer termination arc flow) $\quad z_i \in Z^+ = \{0,1,2,...\} \quad$ for all $i \in I$

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Chapter 11
Gate Assignment

Introduction

The hub-and-spoke system has resulted in a large volume of baggage and passengers transferring between flights. Assigning arriving flights to airport gates is therefore an important issue in daily operations of an airline. Although the costs of these activities are generally small portions of the overall airline operation costs, they have a major impact on maintaining the efficiency of flight schedules and passenger satisfaction. Some of the factors that impact the assignment of gates to arriving flights include aircraft size, passenger walking distances, baggage transfer, ramp congestion, aircraft rotation, and aircraft service requirements (Gu and Chung 1999).

The problem of finding a suitable gate assignment is usually handled in three levels. In the first level, the ground controllers use the flight schedule to examine the capacity of gates to accommodate these flights. The second level involves the development of daily plans before the actual day of operation. In the third level, because of irregular conditions such as delays, bad weather, mechanical failure and maintenance requirements, these daily plans are updated and revised on the same hour/day of the operation (Bolat 2000).

The problem of gate assignment is well studied in operations research. A common approach in formulating this problem is from the passenger's perspective in a way that the total passenger-walking distance is minimized. The gate assignment problem (GAP) is defined as follows:

Given a set of available gates and flights, the distance matrix between the gates, the passenger transfer matrix between the flights, we seek to assign these flights to the gates so that the total passenger-walking distances are minimized.

The researchers have adopted a variety of problem formulation and solution methods to address the various issues in GAP (see Bolat 2000, Haghani 1998, Gu and Chung 1999, Jo et al. 1997). The model described in this chapter is an integer linear programming model proposed by Bihr 1990.

Mathematical Model for a Case Study

The following case study (not related to Ultimate Air!) involves the assignment of flights to gates. Figure 11.1 shows the C Concourse at San Francisco (SFO) Airport, which has 19 gates (C1-C19). There are already 12 aircraft at the gates (as shown) getting ready for their departures. Within the next 15 minutes seven flights will be arriving in this concourse that should be assigned to the remaining gates. These flights are referred to as F1, F2, F3, F4, F5, F6, and F7. In these seven flights, there are passengers who will connect to other departing flights. Without loss of generality, we assume that any of these arriving flights can be accommodated in any of the seven available gates.

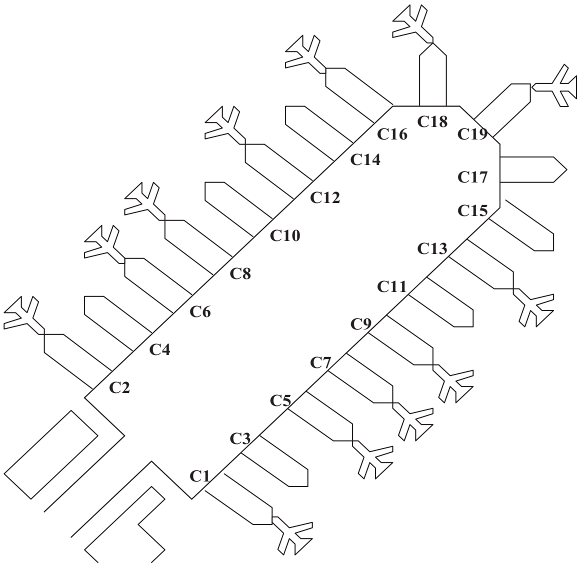

Figure 11.1 C Concourse at SFO

Table 11.1 shows the number of passengers in these flights who will connect to other departing gates. As an example, five passengers from flight F1 should walk to gate 1, and so on. Note that in this model it is assumed that the departing flights are initially, or tentatively have been, designated to gates (Bihr 1990).

Table 11.1 Passenger flow

Flight	Departing gates																		
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
F1	5	5	10	8	15	8	2	10	8	20	5	4	0	9	3	4	1	2	1
F2	5	2	1	4	19	9	4	2	3	2	27	3	8	4	0	2	1	7	2
F3	10	0	4	9	13	4	4	4	3	5	5	8	4	9	11	7	9	4	4
F4	4	8	5	4	10	4	1	0	0	2	4	19	1	2	4	5	5	8	2
F5	4	11	9	9	6	3	1	4	4	2	1	0	3	5	1	2	2	3	4
F6	1	2	42	5	2	7	6	2	4	7	2	3	6	4	10	2	1	0	0
F7	3	3	2	5	9	13	11	2	2	3	7	22	4	0	1	1	2	2	9

The distances in yards between the gates are presented in Table 11.2. Note that in this matrix, only the distances between the candidate arrival gates and other gates are shown.

Table 11.2 Distance matrix (yards)

Gates	Departing Gates																		
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
3	10	40	-	30	10	40	20	50	30	60	40	70	50	80	60	90	70	90	80
4	40	10	30	-	40	10	50	20	60	30	70	40	80	50	90	60	90	70	80
10	70	40	60	30	50	20	40	10	30	-	40	10	50	40	60	30	70	40	50
11	50	80	40	70	30	60	20	50	10	40	-	30	10	40	20	50	30	50	40
14	90	60	80	50	70	40	60	30	50	20	40	10	30	-	40	10	50	20	30
15	70	100	60	90	50	80	40	70	30	60	20	50	10	40	-	30	10	30	20
17	80	100	70	90	60	80	50	70	40	60	30	50	20	40	10	30	-	20	10

Through this model we seek to assign the arriving flights to candidate gates so that the total passengers' walking distance is minimized.

Using the above two tables (Tables 11.1 and 11.2) we can find the total walking distances of passengers on flight i if this flight is assigned to arrival gate j. The walking distance is calculated as follows:

$$\text{Walking distance} = \sum \text{number of passengers} \times \text{distance}$$

For example, if flight F1 is assigned to the candidate arrival gate 3, then the total walking distance for all passengers on this flight assigned to this gate is calculated as follows:

Total walking distance = $5 \times 10 + 5 \times 40 + 10 \times 0 + 8 \times 30 + 15 \times 10 + 8 \times 40 + 2 \times 20 + 10 \times 50 + 8 \times 30 + 20 \times 60 + 5 \times 40 + 4 \times 70 + 0 \times 50 + 9 \times 80 + 3 \times 60 + 4 \times 90 + 1 \times 70 + 2 \times 90 + 1 \times 80 = 5010$ yards

In other words, by assigning flight F1 to gate 3, five passengers on this flight must walk a distance of 10 yards each to gate 1, five passengers must walk 40 yards each to gate 2; and 10 passengers will depart from the same gate as they arrived and therefore not having to walk. We repeat the above calculations for every flight assigned to every candidate gate. Table 11.3 shows the total walking distances for passengers aboard each flight by assigning them to every possible gate.

We define the following binary decision variable:

$$x_{i,j} = \begin{cases} 1 \text{ if flight } i \text{ is assigned to candidate gate } j \\ 0 \text{ otherwise} \end{cases}$$

Table 11.3 Traveling distances (yards)

Flight/gate	3	4	10	11	14	15	17
F1	5,010	4,390	3,820	4,870	5,060	6,650	7,090
F2	4,240	5,290	4,190	3,020	4,650	4,400	4,970
F3	5,610	5,950	4,930	4,270	4,910	4,950	5,320
F4	4,500	3,990	3,280	3,580	3,460	4,320	4,460
F5	2,950	2,720	3,060	,3490	3,620	4,330	4,530
F6	3,060	4,310	4,740	3,900	5,760	5,300	6,020
F7	4,680	4,380	3,290	3,620	3,970	4,960	5,220

The objective function is therefore:

$$\textit{Minimize}\;\; 5010x_{F1,3} + 4390x_{F1,4} + ... + 5220x_{F7,17}$$

For constraints, we should ensure that every flight is assigned to a gate. We have seven gates (3, 4, 10,11,14,15, and 17) available. The constraint for flight F1 is:

$$x_{F1,3} + x_{F1,4} + x_{F1,10} + x_{F1,11} + x_{F1,14} + x_{F1,15} + x_{F1,17} = 1$$

The above constraints ensure that flight F1 is assigned to one and only one gate among the seven available gates. Similarly we write the other six constraints for other flights.

If we run this integer linear program with the above constraints we see that one gate is assigned to two or more flights at the same time; rendering it not feasible. So we must ensure that each gate is also assigned to one flight (aircraft) only. The following additional set of constraints imposes this restriction for gate 3.

$$x_{F1,3} + x_{F2,3} + x_{F3,3} + x_{F4,3} + x_{F5,3} + x_{F6,3} + x_{F7,3} = 1$$

Similarly we write the constraints for other the six gates.

The above integer linear programming model has 49 binary decision variables and 14 constraints. Solving this problem using an optimization software generates the following matching flights to gates solution. The total walking distance for this optimal solution among all passengers is 26,000 yards.

Figure 11.2 shows the allocation of these gates to flights.

Now, we can relax the assumption that any gate can accommodate any aircraft. Let us assume that gates 10 and 14 cannot be used for the aircraft in flight F1. To

Table 11.4 Solution to gate assignment

Flight	Gate assigned to
F1	10
F2	11
F3	15
F4	17
F5	4
F6	3
F7	14

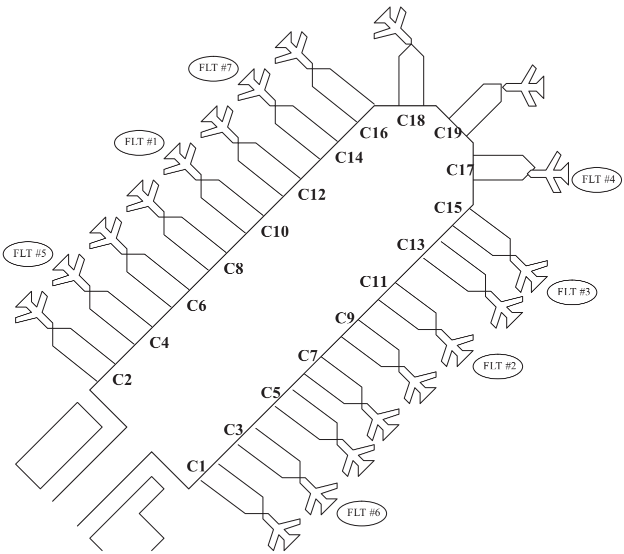

Figure 11.2 Assignment of gates to flights

address this, simply add the following constraints to restrict the assignment of gates 10 and 14 to flight F1.

$$x_{F1,10} = 0$$

$$x_{F1,14} = 0$$

Running the model with these new constraints generates the following solution with a total walking distance of 26,700 yards.

Table 11.5 Revised assignments of gates to flights

Flight	Gate assigned to
F1	4
F2	11
F3	15
F4	14
F5	17
F6	3
F7	10

Baggage Handling

The above model considers only the flow and movement of passengers. The introduction of hub and spoke concept has represented the airlines with challenging and demanding task of baggage handling for transit passengers. Unlike the passengers who can typically walk from one gate to another, the bags actually need to be transported from one gate to another for these transit passengers. The transportation of baggage poses many challenges to airlines, including scheduling the number of baggage handlers, baggage trailers, delays, lost baggage, and missed connections. In fact for major airlines baggage handling for transit passengers seems to be the dominant factor in gate assignment in their major hubs. The airlines normally assign their baggage handlers and trailers according to the ascending order of departure time (Green and Scalise 2007).

The concept of baggage handling has been studied under different scopes. Some of these studies focus on baggage handling for security purposes and detection of explosives. These studies include McLay et al. 2006 and Jacobson et al. 2005. Others study the baggage handling system under gate assignment (see, for example, Haghani and Chen 1998 and Lam, et al. 2002). The new trend of research on baggage handling involves adopting Radio Frequency Identification (RFID) devices to track baggage at airports (see, for example, Maike 2008).

Mathematical Model for Baggage Handling

The mathematical approach presented earlier in this chapter for gate assignment is revised to incorporate baggage handling distances too. In this revised model the objective is to assign gates to arriving flights so that the total traveling distance for transit passengers and bags is minimized.

Referring to the above case study for passenger flow, Tables 11.6 presents the amount of transit bags, mail, and cargo from each of the arriving aircraft to

Table 11.6 Baggage flow from arriving flights to departing gates (units of baggage)

Flight	**Departing gates**																		
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
F1	19	28	11	8	30	25	33	5	49	14	38	38	14	23	17	4	20	44	8
F2	43	40	22	29	4	49	8	6	20	21	17	5	27	29	29	40	42	34	25
F3	22	17	36	45	22	28	17	23	18	44	12	8	41	48	25	11	27	47	28
F4	47	11	4	26	16	21	24	8	45	22	45	20	14	22	32	32	9	39	7
F5	3	24	46	38	48	7	24	33	29	43	7	21	45	47	28	11	17	3	23
F6	9	47	18	3	44	14	4	27	34	38	17	26	2	3	28	40	11	8	46
F7	46	34	48	42	26	12	40	49	18	36	24	6	18	9	2	10	14	47	9

departing gates. Again in this model it is assumed that the departing flights are initially, or tentatively have been, designated to gates (Bihr 1990).

The baggage is normally transported by baggage trailers from gates to gates on the ramp. We assume that the capacity of the trailer is 5 bags per trailer. Therefore we can convert the number of baggage flow in Table 11.6 to the number of trips that the trailers need to make in order to distribute the baggage among the gates. Table 11.7 presents this baggage flow in numbers of trips for trailers. These figures have been all rounded up. As an example, in Table 11.6 we have 19 baggage units to be transported from flight F1 to gate 1. The capacity of the trailer is 5 bags. Therefore, the number of trips that the trailer needs to make to move these bags is given by:

$$\left\lceil \frac{19}{5} \right\rceil = 4$$

where

$\lceil \bullet \rceil$ = the rounded-up integer for •.

The distances in yards between the gates on the airport ramp are presented in Table 11.8. Similar to Table 11.2, in this matrix, only the distances between the candidate arrival gates and other gates are shown.

Similar to the process described for calculating the passenger walking distance, we can calculate the distance traveled by trailers to transport baggage from arriving flights to departing gates, as follows:

Baggage transport distance = $\sum$ number of trips × distance.

The following table presents the total baggage transport distances in yards for each of the arriving flights to each of the candidate gates.

Table 11.7 Baggage flow in number of trips for trailers from arriving flights to departing gates

Flight	Gate																		
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
F1	4	6	3	2	6	5	7	1	10	3	8	8	3	5	4	1	4	9	2
F2	9	8	5	6	1	10	2	2	4	5	4	1	6	6	6	8	9	7	5
F3	5	4	8	9	5	6	4	5	4	9	3	2	9	10	5	3	6	10	6
F4	10	3	1	6	4	5	5	2	9	5	9	4	3	5	7	7	2	8	2
F5	1	5	10	8	10	2	5	7	6	9	2	5	9	10	6	3	4	1	5
F6	2	10	4	1	9	3	1	6	7	8	4	6	1	1	6	8	3	2	10
F7	10	7	10	9	6	3	8	10	4	8	5	2	4	2	1	2	3	10	2

Table 11.8 Distance matrix for baggage trailers on the ramp (yards)

Gate	Departing Gates																		
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
3	15	60	-	45	15	60	30	75	45	90	60	105	75	120	90	135	105	135	120
4	68	17	51	-	68	17	85	34	102	51	119	68	136	85	153	102	153	119	136
10	112	64	96	48	80	32	64	16	48	-	64	16	80	64	96	48	112	64	80
11	65	104	52	91	39	78	26	65	13	52	-	39	13	52	26	65	39	65	52
14	135	90	120	75	105	60	90	45	75	30	60	15	45	-	60	15	75	30	45
15	98	140	84	126	70	112	56	98	42	84	28	70	14	56	-	42	14	42	28
17	112	140	98	126	84	112	70	98	56	84	42	70	28	56	14	42	-	28	14

Table 11.9 Baggage transport distances (yards)

	Gate						
Flight	**3**	**4**	**10**	**11**	**14**	**15**	**17**
F1	6,420	7,820	5,616	4,004	5,685	5,516	5,936
F2	7,965	8,636	6,992	5,707	6,525	6,762	6,986
F3	8,460	9,197	7,136	5,824	6,690	7,224	7,518
F4	7,005	8,296	6,064	4,537	6,015	5,922	6,426
F5	7,320	8,398	6,560	5,174	6,600	7,000	7,546
F6	6,975	7,480	5,328	4,719	5,565	5,978	6,244
F7	6,450	,7327	6,528	5,772	7,590	8,008	8,470

We can now revise our objective function to accommodate for both passenger and baggage traveling and transport distances. The total distance can be represented as:

Total distance = w1(passenger traveling distances) + w2(baggage transport distances)

In this revised total distance, we assign weights w_1 for passenger traveling and w_2 to baggage transportation distances respectively.

As indicated earlier, the airlines probably assign a higher weight to transport of baggage than to passenger traveling distances. For this case study, we assume the weights to be $w_1 = 1$ and $w_2 = 3$ respectively. Therefore the objective function is:

$$\textit{Minimize}\;\; 1(5010x_{F1,3} + 4390x_{F1,4} + ... + 5220x_{F7,17}) + 3(6420x_{F1,3} + 7820x_{F1,4} + ... + 8470x_{F7,17})$$

The constraints discussed earlier for passenger traveling distances remain the same.

Solving this problem using an optimization software generates the following matching flights to gates solution presented in Table 11.10. The total traveling and transport distances for this optimal solution with corresponding weights among all passengers is 161,871 yards. Figure 11.3 overleaf presents the new and revised gate assignment layout.

Table 11.10 Solution to gate assignment for both passenger and baggage transport

Flight	Gate assigned to
F1	11
F2	17
F3	14
F4	15
F5	3
F6	10
F7	4

As we see in Figure 11.3, this gate assignment is different from the case where only passenger movement was the sole objective.

Mathematical Model

The modified mathematical model for the above case study proposed by Bihr (1990) can formally be presented as:

Indices

i = index for arriving flights
j,k = index for gates

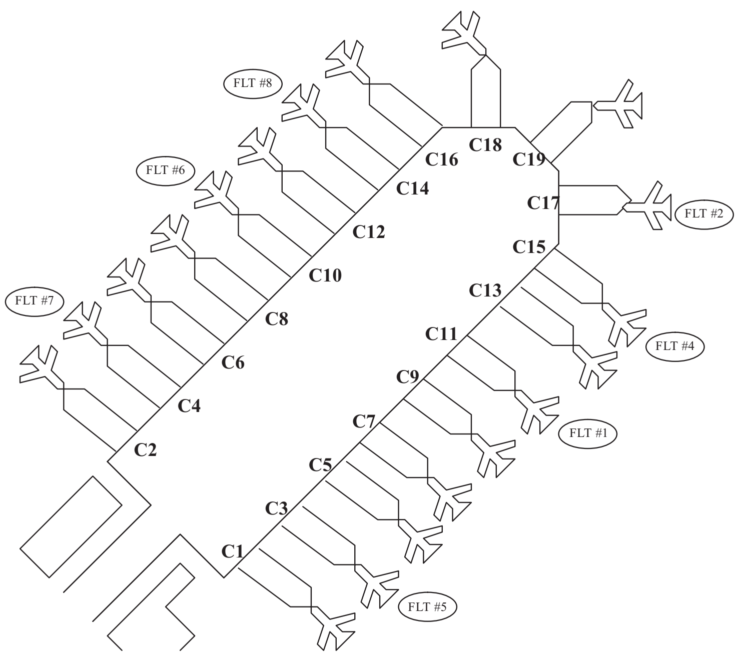

Figure 11.3 Assignment of gates to flights

Sets

F = set of arriving flights
G = set of available gates for arriving flights
K = set of departing gates

Parameters

$p_{i,k}$ = number of passengers arriving on flight i and departing from gate k
$dp_{k,j}$ = distance units (in yards, meters, feet, etc) for passengers from gate k to gate j
$TP_{i,j}$ = Total walking distance for all passengers on flight i assigned to arrival gate j
$t_{i,k}$ = number of trips to transport baggage from flight i to departing gate k
$db_{k,j}$ = distance units (in yards, meters, feet, etc.) to transport baggage on ramp from departing gate k to arriving gate j
$TB_{i,j}$ = Total transport distance for all baggage on flight i assigned to arrival gate j
w_1, w_2 = Weights assigned to total passenger walking and baggage transport distances respectively

$TP_{i,j}$ and $TB_{i,j}$ are calculated as follows:

$$TP_{i,j} = \sum_{k \in K} p_{i,k} \cdot dp_{k,j} \qquad \text{for all } i \in F \text{ and } j \in G$$

$$TB_{i,j} = \sum_{k \in K} t_{i,k} \cdot db_{k,j} \qquad \text{for all } i \in F \text{ and } j \in G$$

Decision Variable

$$x_{i,j} = \begin{cases} 1 \text{ if flight } i \text{ is assigned to gate } j \\ 0 \text{ otherwise} \end{cases}$$

Objective Function

$$\textit{Minimize} \quad \sum_{i \in F} \sum_{j \in G} (w_1 TP_{i,j} x_{i,j} + w_2 TB_{i,j} x_{i,j})$$

Subject to

$$\sum_{j \in G} x_{i,j} = 1 \qquad \text{for all } i \qquad (11.1)$$

$$\sum_{i \in F} x_{i,j} = 1 \qquad \text{for all } j \qquad (11.2)$$

$$x_{i,j} \in \{0,1\} \qquad \text{for all } i \text{ and } j$$

Constraints 11.1 and 11.2 ensure that each flight is assigned to only one gate and each gate is assigned to exactly one flight.

Special Cases

If there are more gates than arriving flights, then constraint 11.2 becomes:

$$\sum_{i \in F} x_{i,j} \le 1 \qquad \text{for all } j$$

The above inequality denotes that an arriving gate can be assigned to a flight by taking a value of 1 or will not be assigned to any flight at all by taking a value of zero.

If, on the other hand, there are more flights than arriving gates then mathematically we can write an inequality for constraint 11.1 similar to the previous special case. However, it will not be realistic. Each flight must land and be accommodated at a gate. If there are no gates available for an arriving flight, as sometimes is experienced in busy airports, then the aircraft has to wait on the ramp or taxiway until a gate becomes available.

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Chapter 12
Aircraft Boarding Strategy

Introduction

The airlines are currently undergoing difficult financial times. The increase in fuel prices, competition from low cost carriers, and operational inefficiencies have resulted in bankruptcies and major losses for airlines around the world. It is therefore extremely important for the airlines to be efficient in areas that they have control over. Airlines generate revenue by utilizing and flying their aircraft; they do not generate any revenue while their aircraft are on the ground. As a result, turnaround time is a major metric for an airline's operations (Van den Briel et al. 2005). The time from the arrival of the aircraft until its next departure constitutes turnaround time. To have high utilization of their aircraft, airlines attempt to minimize the turnaround time. The components in turnaround time include taxi-in and taxi-out, passenger/baggage deplaning, maintenance checks, fueling and passenger/baggage boarding. The typical aircraft turn-around time for short-haul flights is approximately 30–60 minutes (see, for example, Van Landeghem and Beuselink 2002). A major component of turnaround time is the passenger boarding time. Horstmeier and Haan (2001) provide detailed analyses of all the components in an aircraft turn-around time including passenger deplaning, refueling, cleaning, catering, maintenance, and boarding. Because of safety and operational constraints, passenger boarding is the last task in this timeline. Any time saved through efficient boarding directly reduces the turnaround time.

This chapter provides an overview of the strategies adopted by the airlines for their boarding process and examines how optimization models can be implemented to reduce aircraft boarding time.

Common Strategies for Aircraft Boarding Process

Airlines seem to adopt different aircraft boarding strategies based on airline culture and service level. Some airlines do not impose any strategy and let the passengers board randomly. Others arrange passengers into groups, zones or call-offs based on specific boarding strategy adopted by the airline. Each of these groups is then called to board the aircraft in sequence. The following represents some of the popular boarding strategies adopted by many of the airlines:

Back-to-Front

Back-to-front (BF) boarding strategy is widely adopted by many airlines for both narrow and wide-body aircraft. In this strategy, first class, business class, and special-need passengers are boarded first. Then, as the name implies, passengers start filling up the aircraft from back to front. Passengers are called to board the aircraft based either on their seat row numbers or by groups or zones. Each group is then called in sequence to board the aircraft. Figure 12.1 presents a back-to-front boarding strategy where all the passengers on this aircraft are divided into six groups. Group 1 (first class, business class and special need) passengers board first. Then passengers in groups 2 to 6 are called to board the aircraft as shown in Figure 12.1.

Window-Middle-Aisle

Window-middle-aisle boarding strategy (or sometimes called out-in), as the names implies, boards the passengers in window seats, middle seats and finally in the aisle seats. Figure 12.1 presents a window-middle-aisle boarding process. Passengers are usually divided into four groups to follow this boarding strategy. First, business class and special need passengers are assigned to group 1 and board first. Then all the economy class passengers in window, middle, and aisle seats are assigned to groups 2, 3, and 4 respectively and board the aircraft according to their assigned groups as Figure 12.1 suggests. A major disadvantage of this boarding strategy is that the passengers in parties of two or more seated next to each other board the aircraft separately and at different times. This boarding process may not appeal to either passengers and/or airlines.

Random

In random boarding strategy, no specific strategy is used and all passengers board the aircraft in one zone randomly.

Rotating Zone

In rotating zone, passengers are grouped into zones and board the aircraft first in the front, then in the back, then front again, then back in a rotating manner. In this boarding strategy, passengers sitting in the middle of the aircraft are seated last.

Mathematical Model

Most of the studies on aircraft boarding strategies focus on modeling the problem using computer-based simulations (see, for example, Ferrari et al. 2004, Ferrari

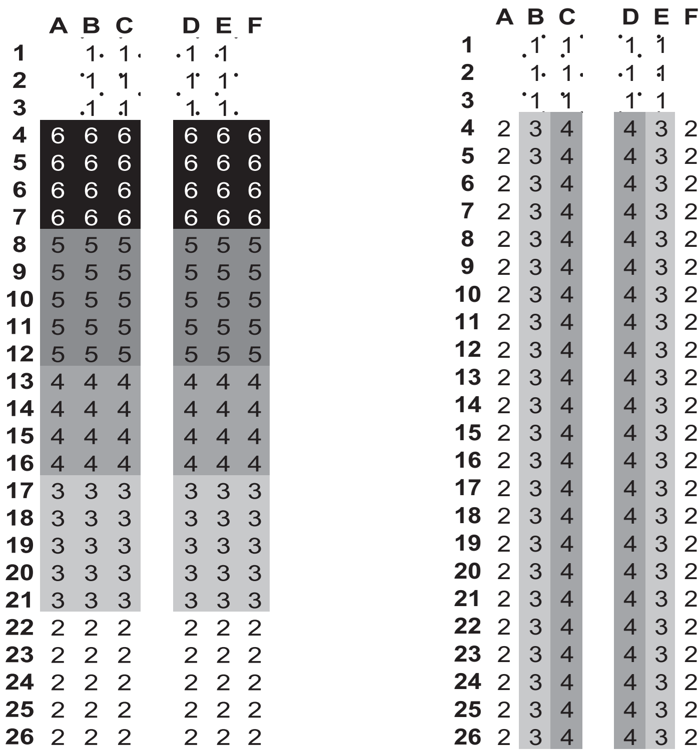

Figure 12.1 Sample of back-to-front and window-middle-aisle boarding process

2005 and Van Landeghem and Beuselink 2002). While these methods provide a good understanding of existing boarding strategies and enable us to evaluate various known strategies and conduct what-if scenarios, they do not help us find the best and other possible unknown alternatives (Van den Briel et al. 2005). Analytical approaches can help achieve these alternatives. Some of the existing analytical models include:

- Van den Briel et al. (2005) proposed a non-linear assignment model with quadratic and cubic terms. The model attempts to minimize the total seat and aisle interferences among passengers (discussed later);
- Bachmat et al. (2006) derived a family of back-to-front boarding policies using stochastic geometry under the assumption of passengers being infinitely thin. The application of this model to a specific aircraft or airline has not been reported.
- Bazargan (2007) adopted a binary/integer linear program to minimize the total seat and aisle interferences.

The analytical model discussed in this chapter is based on a binary/integer linear program introduced by Bazargan (2007). The proposed mathematical model attempts to minimize the total interferences among the passengers, which is a major causes for boarding delays, subject to operational and side constraints.

Interferences

Boarding interferences occur when a passenger blocks another passenger from proceeding to his or her seat. Two types of interferences, seat interferences and aisle interferences, may occur. Seat interferences occur when a passenger blocks another passenger assigned to the same row. Figure 12.2 shows seat interferences for passengers in rows 16, 19, and 22. In all these cases, the blocking passenger(s) need to exit, for the passengers assigned to the middle or window seats to be seated.

Aisle interferences occur when a lower row passenger is in front of the higher row passengers while boarding the aircraft. In this case, the passenger in the lower row will block all the passengers behind him or her to stow baggage in the overhead bin (if any) and be seated (see Figure 12.2). The following sections describe the mathematical models for each type of interference.

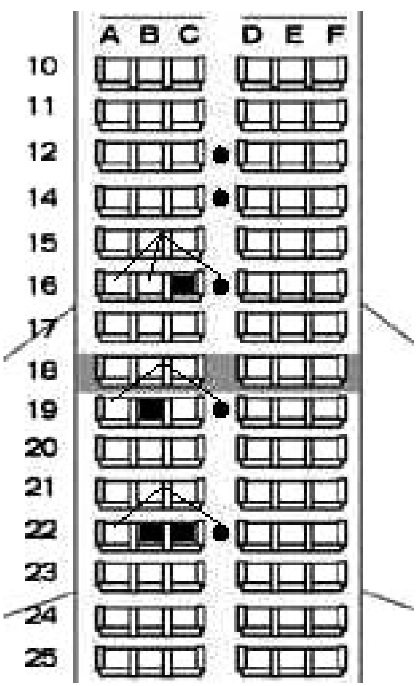

Figure 12.2 Seat and aisle interferences

Model Description

Our focus in this section is to develop a mathematical model which captures the behavior of passengers boarding the aircraft. The objective of this model is to minimize the total number of interferences subject to operational and side constraints. Note that the model assumes a single aisle or narrow-body aircraft such as an Airbus A-320 or Boeing 737 and all passengers board through a single aircraft door.

We assume that each seat in this aircraft is represented by (i,j) where i (i=1,.,N) is the row and j ($j=A,B,.,F$) is the location of the seat within row i as shown in Figure 12.3.

Row *i*	**A Window**	**B Middle**	**C Aisle**	**Aisle**	**D Aisle**	**E Middle**	**F Window**

Figure 12.3 Location of seats within row i

In this model we attempt to assign each seat to a group. Each passenger in seat (i,j) is assigned to a group k (k=1,.,G). Each group is then called in sequence to board the aircraft.

The following binary decision variable is adopted for our integer linear programming model:

$$x_{i,j,k} = \begin{cases} 1 \text{ if seat } j \text{ in row } i \text{ is assigned to group } k \\ 0 \text{ otherwise} \end{cases}$$

Our objective is to assign seats (i,j) to groups ($k \in G$) so that the total number of interferences with penalties attached to them (as described later) is minimized.

Seat Interferences

There are two types of seat interferences: between-groups and within-groups described as follows:

Between-Groups Seat Interferences

This type of seat interference occurs when a passenger from an earlier group blocks another passenger in a later group. For example, in Figure 12.2 if the passenger in seat 16C (aisle seat) boarded in group 2 and passenger in seat 16B (middle seat) boarded in a later group, then the passenger in seat 16C is blocking the passenger in seat 16B. In this case, the passenger in seat 16C needs to exit, to allow the passenger in seat 16B to be seated, thus blocking the flow of passengers in the aisle. More seat interferences occur if the passenger in seat 16A (window seat) boards after passengers are seated in seats 16B and 16C.

Considering all the possible combinations, four types of seat interferences between different groups can occur as follows.

- aisle-seat passenger blocking the middle-seat passenger;
- aisle-seat passenger blocking the window-seat passenger;
- middle-seat passenger blocking the window-seat passenger;
- aisle- and middle-seat passengers blocking the window-seat passenger;

We examine the mathematical model for each case separately.

Aisle-Seat Passenger Blocking Middle-Seat Passenger

First we develop the seat interferences model for seats B and C on the left hand side of the aisle (see figure 12.3) for seats B and C. We define $SB_{i,BC,k}$ as a binary variable representing number of seat interference between the seat C (aisle seat) and seat B (middle seat) in row *i,* who boarded in group *k*. $SB_{i,BC,k}$, takes a value of 1 if an interference occurs or 0 otherwise.

Mathematically, $SB_{i,BC,k}$ can be expressed as the following constraint in our mathematical model:

$$SB_{i,BC,k} \geq x_{i,B,k} + \sum_{l=1}^{k-1} x_{i,C,l} - 1 \quad \forall\ i, \text{ and } k > 1 \tag{12.1}$$

On the right hand side of the constraint, we have $x_{i,B,k}$, which represents if the passenger in the middle seat (seat B) boards in group *k* and

$$\sum_{l=1}^{k-1} x_{i,C,l}$$

indicates if the passenger sitting in the aisle seat (seat C) has boarded in any of the earlier groups (before *k*). The term (-1) in the above constraint is added to set the value of $SB_{i,BC,k}$ equal to 1 if there is an interference or 0 otherwise. Table 12.1 clarifies this further and examines the value of $SB_{i,BC,k}$ for different scenarios on a given row *i*.

Note that in constraint (12.1) we use greater or equal sign (≥). This ensures that $SB_{i,BC,k}$ takes a value of 0 for the last case in the above table if both terms on the right hand side are 0. Similarly we can write the constraints for aisle and middle seat interferences (seats E and D) on the right hand side of the aisle

$$SB_{i,ED,k} \geq x_{i,E,k} + \sum_{l=1}^{k-1} x_{i,D,l} - 1 \quad \forall\ i, \text{ and } k > 1 \tag{12.2}$$

Table 12.1 Examining aisle- and middle-seat interference

$x_{i,B,k}$	$\sum_{l=1}^{k-1} x_{i,C,l}$	$SB_{i,BC,k}$	**Comments**
1 (The passenger in middle seat B is in group k)	1 (The passenger in aisle seat C has already boarded in an earlier group)	1	Interference occurs
0 (The passenger in middle seat B is not in group k)	1 (The passenger in aisle seat C has already boarded in an earlier group)	0	No interference
1 (The passenger in middle seat B is in group k)	0 (The passenger in seat C has not boarded in any of the earlier groups)	0	No interference
0 (The passenger in middle seat B is not in group k)	0 (The passenger in the aisle seat C has not boarded in any of the earlier groups)	-1	No interference

Aisle-Seat Passenger Blocking Window-Seat Passenger

We adopt a similar approach as aisle- and middle-seat interference to expresses the number of seat interferences and its corresponding constraints between aisle- and window-seats among different groups for both sides of the aisle as follows:

$$SB_{i,AC,k} \geq x_{i,A,k} + \sum_{l=1}^{k-1} x_{i,C,l} - 1 \quad \forall\, i, \text{ and } k > 1 \tag{12.3}$$

$$SB_{i,FD,k} \geq x_{i,F,k} + \sum_{l=1}^{k-1} x_{i,D,l} - 1 \quad \forall\, i, \text{ and } k > 1 \tag{12.4}$$

Middle-Seat Passenger Blocking Window-Seat Passenger

The number of seat interference and constraint between window- and middle-seats among different groups for both sides of the aircraft are as follows:

$$SB_{i,AB,k} \geq x_{i,A,k} + \sum_{l=1}^{k-1} x_{i,B,l} - 1 \quad \forall\, i, \text{ and } k > 1 \tag{12.5}$$

$$SB_{i,FE,k} \geq x_{i,F,k} + \sum_{l=1}^{k-1} x_{i,E,l} - 1 \quad \forall\ i, \text{ and } k > 1 \tag{12.6}$$

Aisle- and Middle-Seat Passengers Blocking Window-Seat Passenger

Having established the above seat interferences, we do not need to express a specific set of constraints for a window-seat passenger when both middle- and aisle-seat passengers have already been seated. This type of interference has already been addressed in the form of two separate constraints (interferences) discussed above. These two interferences are window with middle and window with aisle seats. For example, consider a case when a passenger in seat A boards just after both passengers in seats B and C. In this case, according to constraints (12.3 and 12.5) above both $SB_{i,AC,k}$ and $SB_{i,AB,k}$ will take a value of 1 implying that the passenger sitting in seat A (window) will have a total of 2 seat interferences with aisle and middle seats.

Total Seat Interferences among Different Groups

We can now express the total number of seat interferences between different groups by adding all the seat interferences discussed above. Let *TSB* represent the total seat interferences between groups, then:

$$TSB = \sum_{i=1}^{N} \sum_{k=2}^{G} \left(SB_{i,BC,k} + SB_{i,ED,k} + SB_{i,AC,k} + SB_{i,FD,k} + SB_{i,AB,k} + SB_{i,FE,k} \right) \tag{12.7}$$

Within-Groups Seat interferences

This type of interference occurs among passengers boarding in the same group. We assume the sequence in which the passengers within a group board the aircraft is random. For example, passengers in seats 16A and 16B are boarding in the same group (see Figure 12.2). When their group is called, passenger 16A may board first and be in front of 16B in the respective group or vice versa. In the former case when the passenger in seat 16A boards before 16B, no interference occurs. However, in the latter case when the passenger in 16B boards before 16A, there will be a seat interference.

Adopting the same argument as between groups seat interference, we denote the binary variable $SW_{i,BC,k}$ to represent the seat interference between the aisle (seat C) and middle seat (seat B), who board in the same group. We can write the following constraint for this variable:

$$SW_{i,BC,k} \geq x_{i,B,k} + x_{i,C,k} - 1 \quad \forall\ i,k \tag{12.8}$$

Similar to the section on aisle and middle seat interferences, $SW_{i,BC,k}$ can take a value of 1 or 0 depending on values of $x_{i,B,k}$ and $x_{i,C,k}$. If passengers in seats B and C in a row *i* are boarding in the same group *k*, then the constraint (12.8) returns a value of 1 for $SW_{i,BC,k}$, otherwise it will be 0. However, as indicated before the order of these two passengers is random. Therefore the expected number of seat interferences between passengers in seats B and C within the same group is:

$$\frac{1}{2} SW_{i,BC,k} \tag{12.9}$$

Similarly, we can express the constraints for other seat interferences within the same group as follows:

$$SW_{i,AC,k} \geq x_{i,A,k} + x_{i,C,k} - 1 \quad \forall\, i,k \tag{12.10}$$

$$SW_{i,AB,k} \geq x_{i,A,k} + x_{i,B,k} - 1 \quad \forall\, i,k \tag{12.11}$$

$$SW_{i,ED,k} \geq x_{i,E,k} + x_{i,D,k} - 1 \quad \forall\, i,k \tag{12.12}$$

$$SW_{i,FD,k} \geq x_{i,F,k} + x_{i,D,k} - 1 \quad \forall\, i,k \tag{12.13}$$

$$SW_{i,FE,k} \geq x_{i,F,k} + x_{i,E,k} - 1 \quad \forall\, i,k \tag{12.14}$$

Similar to the expression in (12.9), the expected number of seat interferences within each group is ½ each of the above *SW*. Again we do not need to add a new constraint for the case when all three neighboring passengers are in the same group.

The total of seat interferences within the same groups (*TSW*) is therefore obtained by:

$$TSW = \frac{1}{2} \sum_{i=1}^{N} \sum_{k=1}^{G} \left(SW_{i,BC,k} + SW_{i,ED,k} + SW_{i,AC,k} + SW_{i,FD,k} + SW_{i,AB,k} + SW_{i,FE,k} \right) \tag{12.15}$$

Aisle Interferences

In this section, we formulate the aisle interferences. Similar to seat interferences, there are two types of aisle interferences, within groups and between groups.

Within-Groups Aisle Interferences

This common type of aisle interference relates to cases where passengers assigned to the same group block each other. This occurs when a passenger in a lower row blocks other passengers behind him or her in order to be seated. The problem becomes compounded when the passenger has multiple bags to store in the overhead bin (Van Landeghem and Beuselink 2002). We further break down these within-group aisle interferences into interferences with lower rows and interferences with same rows.

Within-Groups Aisle Interferences With Lower Rows

Let the integer variable $AW1_{i,j,k}$ represent the (maximum) number of aisle interferences for the passenger in seat *(i,j)* assigned to group *k* with lower row passengers in the same group. Similar to the previous section, we can write the constraint for $AW1_{i,j,k}$ as follows:

$$AW1_{i,j,k} \geq 6(i-1)x_{i,j,k} + \sum_{u=1}^{i-1} \sum_{v=1}^{6} x_{u,v,k} - 6(i-1) \qquad \forall i > 1, j, k \tag{12.16}$$

On the right-hand side of this constraint, the first term takes a value of *6(i-1)* if the passenger in seat *(i,j)* is assigned to group *k, or* zero otherwise. The second term adds up all the passengers in the same group *k*, who have a lower row seat-assignment than *i*. The third term on the right-hand side is adopted to provide the correct count on aisle interferences for $AW1_{i,j,k}$.

Note that to determine $AW1_{i,j,k}$ we add up all the passengers in rows lower than row *i*. This implies the worst case where the passenger in seat *(i,j)* boards after all passengers in the lower rows in the same group. Therefore, $AW1_{i,j,k}$ represents the maximum number of aisle interferences for the passenger in seat *(i,j)* assigned to group *k.*

The lowest number of aisle interferences for the passenger in seat *(i,j)* assigned to group *k* occurs if the passenger boards before all the passengers in lower rows of that group. In this case since the passenger moves all the way down the aisle to his or her designated row without being blocked by anyone within this group, then there are no within-group aisle interferences. Therefore the expected number of aisle interferences for passenger in seat *(i,j)* assigned to group *k* is:

$$\frac{\min+\max}{2} = \frac{0 + AW1_{i,j,k}}{2} = \frac{1}{2} AW1_{i,j,k} \tag{12.17}$$

The total expected number of aisle interferences for all passengers with their lower row passengers presented by AWL is therefore determined by:

$$AWL = \frac{1}{2} \sum_{i=2}^{N} \sum_{j=1}^{6} \sum_{k=1}^{G} AW1_{i,j,k} \tag{12.18}$$

Within-Groups Aisle Interferences in the Same Row

In the section within-groups aisle interferences with lower rows above, we did not consider possible aisle interferences among passengers in the same row and same group. This section addresses the expected number of aisle interferences for those passengers. We define integer variable $AW2_{i,j,k}$ to represent the (maximum) number of aisle interferences between the passenger in seat *(i,j)* and all other passengers in the same row *i*, boarding in group *k*. We write the following constraint for $AW2_{i,j,k}$.

$$AW2_{i,j,k} \geq 5x_{i,j,k} + \sum_{u=1, u \neq j}^{6} x_{i,u,k} - 5 \quad \forall i,j,k \tag{12.19}$$

The first term on the right hand side takes a value of 5 or zero depending on whether the passenger in seat (i,j) is in group *k* or not. The second term adds up the number of passengers in row *i* and number of passengers in group *k*, except for the passenger sitting in *(i,j)*. The third term is used to provide the right number of counts for aisle interferences. Similar to the previous section, the minimum number of interferences for passenger in seat (*i,j*) boarding in group *k* is 0 and the maximum is $AW2_{i,j,k}$. Therefore the expected number of aisle interferences in the same row within the same group for this passenger is $^1/_2 AW2_{i,j,k}$.

We define *AWS* to represent the total expected number of same-row aisle interferences for all passengers given by:

$$AWS = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{6} \sum_{k=1}^{G} AW2_{i,j,k} \tag{12.20}$$

Between-Groups Aisle Interferences

This type of aisle interference, as many of us have experienced, occurs when a group of passengers are called to board the aircraft while some or all of the passengers in the previous group are still in the jet-way (staircase) or aircraft door waiting to be seated. These interferences occur and get worse as the time between boarding the passengers and groups decreases. We define the integer variable $AB_{i,j,k}$ to represent the maximum number of aisle interferences for passenger in seat (*i,j*) who boards in group *k (k>1)* with all the passengers in the previous group (*k-1*).

We write the following constraint for $AB_{i,j,k}$:

$$AB_{i,j,k} \geq 6(i)x_{i,j,k} + \sum_{i=1}^{i} \sum_{u=1}^{6} x_{i,u,k-1} - 6(i) \quad \forall i,j,k>1 \tag{12.21}$$

The first term on the right hand side of this constraint takes a value of $6i$ if the passenger in seat (i,j) is assigned to group k, or 0 otherwise. The second term adds up all the passengers who boarded in group $(k-1)$ and the third term is used to provide the correct count.

The constraint for $AB_{i,j,k}$ assumes that none of the passengers from the earlier group is seated when the passenger in seat (i,j) assigned to group k boards the aircraft. Of course, the expected number of aisle interferences for this passenger depends on how quickly each group of passengers is called to board the aircraft. We will assume that for any passenger in group k $(k>1)$ boarding the aircraft, there are a fraction of passengers from the previous group $(k-1)$ still in the jet-way trying to reach to their seats. We call this fraction α $(0 \leq \alpha \leq 1)$. Therefore the expected number of aisle interferences between the passenger in seat (i,j) assigned to group k with passengers in group $(k-1)$ is $\alpha AB_{i,j,k}$.

When α is 0, no aisle interferences occur between groups. This occurs when a new group of passengers is called to board the aircraft when all the passengers from the earlier group are fully seated. On the other extreme, when α is equal to 1, the time between calling groups to board is so short that the passengers in each group line up behind the previous group in the aisle or jet-way. In our later analyses, we examine various values for α and its impact on boarding pattern and strategy.

Let ABG represent the total aisle interferences between groups for all passengers boarding in group k $(k>1)$. ABG is therefore determined by:

$$ABG = \alpha \sum_{i=1}^{N} \sum_{j=1}^{6} \sum_{k=2}^{G} AB_{i,j,k} \tag{12.22}$$

To keep the model simple and without loss of generality, we only include aisle interferences between passengers in group k $(k>1)$ with passengers in group $(k-1)$. It is, of course, possible to mathematically include the aisle interferences between passengers in group k and groups $(k-2$, for $k>2)$ or $(k-3$ for $k>3)$, and so on. Our simulation study, discussed later, also confirmed that for realistic times between passengers to board the aircraft, these second or third level interferences are relatively very low compared to the first level that is considered in the model.

Mathematical Model

In this mathematical model, we attempt to minimize all the seat and aisle interferences that were examined in section 4 as follows:

$$Minimize\ p_1TSB + p_2TSW + p_3AWL + p_4AWS + p_5ABG \quad (12.23)$$

Subject to:

$$SB_{i,BC,k} \geq x_{i,B,k} + \sum_{l=1}^{k-1} x_{i,C,l} - 1 \quad \forall\ i, \text{ and } k > 1 \quad (12.24)$$

$$SB_{i,ED,k} \geq x_{i,E,k} + \sum_{l=1}^{k-1} x_{i,D,l} - 1 \quad \forall\ i, \text{ and } k > 1 \quad (12.25)$$

$$SB_{i,AC,k} \geq x_{i,A,k} + \sum_{l=1}^{k-1} x_{i,C,l} - 1 \quad \forall\ i, \text{ and } k > 1 \quad (12.26)$$

$$SB_{i,FD,k} \geq x_{i,F,k} + \sum_{l=1}^{k-1} x_{i,D,l} - 1 \quad \forall\ i, \text{ and } k > 1 \quad (12.27)$$

$$SB_{i,AB,k} \geq x_{i,A,k} + \sum_{l=1}^{k-1} x_{i,B,l} - 1 \quad \forall\ i, \text{ and } k > 1 \quad (12.28)$$

$$SB_{i,FE,k} \geq x_{i,F,k} + \sum_{l=1}^{k-1} x_{i,E,l} - 1 \quad \forall\ i, \text{ and } k > 1 \quad (12.29)$$

$$SW_{i,BC,k} \geq x_{i,B,k} + x_{i,C,k} - 1 \quad \forall\ i,k \quad (12.30)$$

$$SW_{i,AC,k} \geq x_{i,A,k} + x_{i,C,k} - 1 \quad \forall\ i,k \quad (12.31)$$

$$SW_{i,AB,k} \geq x_{i,A,k} + x_{i,B,k} - 1 \quad \forall\ i,k \quad (12.32)$$

$$SW_{i,ED,k} \geq x_{i,E,k} + x_{i,D,k} - 1 \quad \forall\ i,k \quad (12.33)$$

$$SW_{i,FD,k} \geq x_{i,F,k} + x_{i,D,k} - 1 \quad \forall\ i,k \tag{12.34}$$

$$SW_{i,FE,k} \geq x_{i,F,k} + x_{i,E,k} - 1 \quad \forall\ i,k \tag{12.35}$$

$$AW1_{i,j,k} \geq 6(i-1)x_{i,j,k} + \sum_{u=1}^{i-1} \sum_{v=1}^{6} x_{u,v,k} - 6(i-1) \quad \forall i > 1, j,k \tag{12.36}$$

$$AW2_{i,j,k} \geq 5x_{i,j,k} + \sum_{u=1, u \neq j}^{6} x_{i,u,k} - 5 \quad \forall i,j,k \tag{12.37}$$

$$AB_{i,j,k} \geq 6(i)x_{i,j,k} + \sum_{i=1}^{i} \sum_{u=1}^{6} x_{i,u,k-1} - 6(i) \quad \forall i,j,k > 1 \tag{12.38}$$

$$TSB = \sum_{i=1}^{N} \sum_{k=2}^{G} \left(SB_{i,BC,k} + SB_{i,ED,k} + SB_{i,AC,k} + SB_{i,FD,k} + SB_{i,AB,k} + SB_{i,FE,k} \right) \tag{12.39}$$

$$TSW = \frac{1}{2} \sum_{i=1}^{N} \sum_{k=1}^{G} \left(SW_{i,BC,k} + SW_{i,ED,k} + SW_{i,AC,k} + SW_{i,FD,k} + SW_{i,AB,k} + SW_{i,FE,k} \right) \tag{12.40}$$

$$AWS = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{6} \sum_{k=1}^{G} AW2_{i,j,k} \tag{12.41}$$

$$AWL = \frac{1}{2} \sum_{i=2}^{N} \sum_{j=1}^{6} \sum_{k=1}^{G} AW1_{i,j,k} \tag{12.42}$$

$$ABG = \alpha \sum_{i=1}^{N} \sum_{j=1}^{6} \sum_{k=2}^{G} AB_{i,j,k} \tag{12.43}$$

$$\sum_{k=1}^{G} x_{i,j,k} = 1 \quad \forall i,j \tag{12.44}$$

$$\sum_{i=1}^{N} \sum_{j=1}^{6} x_{i,j,k} \geq \min\_pax \quad \forall k \tag{12.45}$$

$$\sum_{i=1}^{N} \sum_{j=1}^{6} x_{i,j,k} \leq \max\_pax \quad \forall k \tag{12.46}$$

$$x_{i,j,k} \in \{0,1\} \quad \forall i,j,k \tag{12.47}$$

The objective function (12.23) attempts to minimize the total expected number of seat and aisle interferences. p_1, p_2,...,p_5 represent the penalties assigned to different types of interferences. The value of these penalties will be discussed later. Expressions (12.24) to (12.43) were explained in the previous section. The set of constraints in (12.44) ensures that each seat in the aircraft is assigned to one and only one group. Typically the airlines favor a balanced number of passengers among different groups. The two sets of constraints (12.45) and (12.46) ensure that the number of passengers assigned to each group is not less than *min_pax* and not more than *max_pax*. Finally, expression (12.47) indicates that the decision variables are binary.

Model Parameters

As indicated before, p_1, p_2,...,p_5 are adopted to assign weights to different seat and aisle interferences. The literature adopting simulation models for boarding strategies mainly uses triangular distributions (Kelton, et al. 2009) to model the times for seat and aisle interferences. Van Landeghem and Beuselink (2002) use triangular distributions (3, 3.6, 4.2) and (1.8, 2.4, 3) seconds in their models for seat and aisle interferences respectively. Similar time parameters are used in the simulation study by Ferrari and Nagel (2004). We adopted the mean of these distributions to represent the penalties in this model. These distributions have a mean of 3.6 for seat interference and 2.4 for aisle interferences. Without loss of generality, we assign the same weight to seat (*TSB* and *TSW*) and same weight to aisle (*AWL, AWS, ABG*) interferences as follows:

$$p_1 = p_2 = 3.6$$

$$p_3 = p_4 = p_5 = 2.4$$

In the between-groups aisle interference section, we assumed that for passengers in group k (k>1) boarding the aircraft, there is a fraction of passengers from the previous group (k-1) still in the jet-way trying to reach their seats. We called this fraction α (0≤α≤1). The two extreme cases, when α is 0 or 1 represent situations when there is 0% or 100% between group interferences, were discussed earlier. To identify the impact of α on boarding strategy, we considered various values for this parameter. We solved the above mathematical model for the following values of α:

$$\alpha \in \{0,.1,.3,.5,.7,.9,1\}$$

Airlines typically assign 4, 5, or 6 groups to board their passengers on a single-aisle aircraft (Van der Briel et al. 2005). To provide a better understanding of boarding strategy as the number of groups change, we solved our integer programming model with 4, 5, and 6 groups, that is:

$$G \in \{4,5,6\}$$

In our model, we set *min_pax* and *max_pax*, to allow a maximum of 20% fluctuations around the mean as follows:

$$\min\_pax = \left\lceil \frac{N}{G} \times .8 \right\rceil$$

$$\max\_pax = \left\lceil \frac{N}{G} \times 1.2 \right\rceil$$

where

[•] denotes the integer value of •. *N* represents the number of rows.

To evaluate the performance of the model, we applied it to an Airbus A-320 aircraft with 26 rows. The first three rows (with 4 seats in each row) are assigned to first and business class passengers (group 1), who always board first. In our model we study all other passengers who are assigned to the other 23 rows (*N*=23), with six seats in each row, who must be allocated to different groups.

Computation and Implementation

Considering the range of possible parameters, we have 21 integer linear programming models (3 groups (G) and 7 values of α for each group). The 4, 5, and 6 group linear integer models have 414, 552 and 690 binary decision variables and 2,131, and 2,886, and 3,641 constraints respectively. These models were solved using a software.

Figure 12.4 presents a sample solution for 6 groups (G=6) based on different values of α. The solutions for different groups and their performances compared with other models are presented in Bazargan (2007). It is of interest to see how various values of α impact the boarding pattern. As the figure implies, the patterns shift from *window-middle-aisle* to *back-to-front* (*BF*) as α increases. Specifically, we see that for $\alpha \geq .5$ the pattern rapidly starts to converge to *back-to-front* strategy. This occurs when the time between boarding different groups of passengers is so short that each group lines up behind 50% of passengers from the previous group.

Table 12.2 presents the solution for the above 6-group boarding process (see Figure 12.4) and the number of seat and aisle interferences for different values of α.

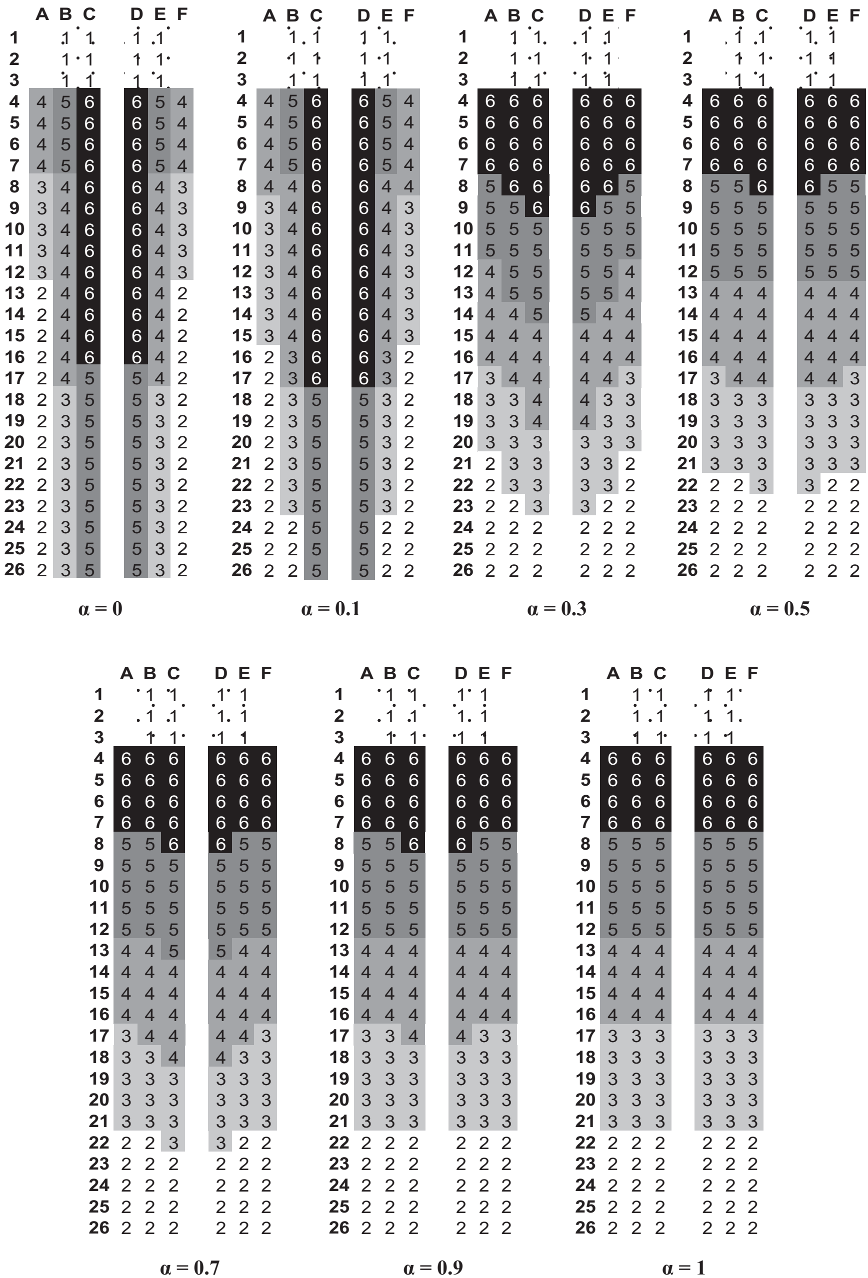

Figure 12.4 Solution for boarding patterns based on 6 groups and different values of α

Table 12.2 Seat, aisle and total interferences for solution to 6-groups boarding process

α	Interference	Solution	Sum	Solution
0	TSB	0	Total seat interferences	0
	TSW	0		
	AWL	884	Total aisle interferences	953
	AWS	69		
	ABG	0		
	Obj. Function	2287.2	Total interferences	953
0.1	TSB	0	Total seat interferences	4
	TSW	4		
	AWL	878	Total aisle interferences	1059.4
	AWS	85		
	ABG	96.4		
	Obj. Function	2556.96	Total interferences	1063.4
0.3	TSB	0	Total seat interferences	47
	TSW	47		
	AWL	792	Total aisle interferences	1091
	AWS	257		
	ABG	42		
	Obj. Function	2787.6	Total interferences	1138

Simulation Model

In order to study and determine the values of α for different boarding times, a simulation model in Arena Simulation Modeling Software was developed (Kelton et al. 2009). The main focus of this simulation study was to identify the number of passengers from an earlier group who will be in front of a latter group, thus providing some guideline for realistic values of α.

The simulation model is similar to those reported by Van Landeghem and Beuselink (2002) and Ferrari and Nagel (2004). Similar aircraft load-factor and time distributions for aisle and seat interferences were adopted. The details of these simulation models are not duplicated here and refer the interested readers to these papers.

We ran the simulation models with inter-arrival times of passengers for boarding changing from 3 seconds to 10 seconds or 20 to 6 passengers per minute. Our main task of measuring performance in this study was to identify the number of passengers at the door, when new passengers will line up behind them for different times between passengers boarding. Table 12.3 presents the result of the simulation model for 6 groups. The table shows the number of passengers at the aircraft door (#pax) for passenger inter-arrival times ranging from 3 to 10 seconds. The table also presents values of α based on the number of passengers at the door (#pax). On average, for an Airbus A-320 with 6 groups, there are 28 passengers per each economy group. Therefore α is determined by dividing #pax by 28.

Table 12.3 Expected number of passengers and values of α for boarding based on varying inter-arrival times

Time between arrivals (sec)		3	4	5	6	7	8	9	10
6 groups	# Pax	25.9	15.8	9	5.7	3.2	0	0	0
	α	0.9	0.6	0.3	0.2	0.1	0.0	0.0	0.0

For average passenger arrival times, Van den Briel et al (2005) considered 7 seconds with 1 gate agent and 5 seconds with 2 gate agents (rounded to the nearest second) and Van Landeghem and Beuslinck (2002) considered 6–7 seconds in their simulation models. These times are based on actual observations of passenger boarding times at different airlines and at different airports. Using these inter-arrival times and based on Table 12.3, the realistic values for α range from 0.3 for 5 seconds to 0.1 for 7 seconds inter-arrival times. Therefore the solutions presented in Figure 12.4 for α taking values 0.1 and 0.3 seem to be appropriate for boarding strategies depending on 1 or 2 gate agents. These solutions indicate that *back-to-front boarding strategy*, as adopted by many airlines, is not necessarily an efficient process.

Conclusion

This chapter introduced an integer linear program to minimize the total number of passenger interferences, which causes delay in aircraft boarding. The operational and side constraints for this mathematical model were examined. The model was applied to an Airbus A-320 aircraft which is commonly used by many airlines. Alternative *efficient solutions* were generated, based on the speed of boarding the passengers. A simulation model was adopted to identify appropriate boarding patterns as the speed of boarding the passengers changes.

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PART III
Computational Complexities and Simulation

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Chapter 13
Computational Complexity, Heuristics, and Software

Introduction

The case studies and examples presented in the previous chapters where integer linear programming models were adopted could be easily solved using optimization software. This was mainly due to a relatively small number of decision variables and/or constraints. The objective of the previous cases and examples was to introduce the development of mathematical models rather than solve the problems. We assumed that once the problem is formulated, we could use a software to obtain the optimal solution. Unfortunately, the problems that many airlines face involve millions or even billions of decision variables. These huge models cannot be solved using the standard software package. Consider the following example.

A cargo airline serves 30 cities within its network with a single fleet-type aircraft. Each route on average consists of 7 flights per day. Referring to Chapter 5 (Aircraft Routing), in order to find the optimum solution we need to list all possible routes. The total number of possible routes will be determined by the following permutation formula:

$$ {}^{30}P_{7} = \frac{30!}{(30-7)!} = 10{,}260{,}432{,}000 $$

In Chapters 5 and 6 we imposed many restrictions to keep the number of decision variables small. Even for the small Ultimate Air case we had more than 6,000 decision variables for aircraft routing for the 737-800 fleet!

Now compare and contrast our small size Ultimate Air case study with Table 13.1 representing actual crew, equipment sizes, and daily number of flights for a select number of airlines.

Complexity Theory

Since the emergence of real-world problems and their solution methodologies, there has been a constant need to classify and compare them on the basis of the computational tractability. Specifically, we are interested in knowing how the computational times increase as the size of the problem grows. As an example, if it takes 5 seconds to solve a linear program with 100 variables and 50 constraints using a specific software package, how long would it take to solve a similar model

Table 13.1 Network and crew size for select airlines

Airline	Daily flights	Pilots[1]	Destinations	Number of aircraft in service
American Airlines	3,400	8,306	250	646
Continental Airlines	2,663	4,578	265	357
Delta /Northwest	5,940	10,736	567	765
Southwest	3,300	5,588	67	533
United Airlines	3,300	6,366	200	444
US Airways	3,096	4,234	210	358

Source: Form41 iNet and Airline Pilot Central[1]

with 1,000 variables and 500 constraints on the same computer and using the same software? Will it take ten times as much time to solve the problem or more? We should note that the computational times are very dependent on the computer processor and programming language adopted.

Therefore, instead of making computational time dependent on the computer, we would like to use other algorithms that present a general overview of the complexity of the problem, as the size of the problem grows. Researchers in the past have focused on *number of steps*. That is, for a given algorithm how the number of steps to solve a problem increases as the size of the problem increases. Consider the following example adapted from Winston and Venkataramanan (2003). Assume that you have a sequence of n numbers to sort from smallest to largest. One method (algorithm) for sorting is by comparing two neighboring numbers. If they are in the wrong order, they are reordered small to large. This process is repeated until all the numbers are in sequence. This method is called bubble sort. The best scenario is that all the numbers are already in the right order (of course, we do not know this in advance!). In this case after n comparisons (steps) the algorithm tells us that the numbers are in order. In the worst possible scenario (again not knowing in advance), where all numbers are sorted from largest to smallest, we have to repeat this comparison n^2 times since every number is in the wrong order. We are normally interested in the worst-case scenario since it will serve as the upper bound for the number of steps to generate the solution. According to this example the bubble sort has a complexity of order of n^2. It is shown as $O(n^2)$ where O stands for the *order of time*. According to this complexity, if the size of the problem (in this example the numbers to be sorted) is doubled then the number of steps (comparisons) is bounded by four times the number of steps of the original problem. This order of complexity provides some guidelines on how the computational complexity will increase for large-scale problems, as compared with lower-scale ones.

During the 1970s, many researchers in computer science and operations research studied various algorithms in terms of their computational complexity (Daskin 1995). They classified the algorithms based on their computation tractability into two groups, polynomial (P) and non-deterministic polynomial (NP) time algorithms. For polynomial time algorithms, the solution times are bounded by a polynomial order. As an example, our bubble sort algorithm is classified as a polynomial time of order of n^2. Polynomial time algorithms are typically considered 'good'. This is because these algorithms can solve large instances of the problem in reasonable steps and time. For non-deterministic polynomial (NP) time algorithms on the other hand, as the name implies, the number of steps to solve the problem grows exponentially with the problem size. As an example, consider that an algorithm is classified as non-deterministic polynomial $O(2^n)$ order. If the problem size doubles, then the steps (times) that it takes to solve this problem will be the number of steps for the original problem to power two or $O(2^{2n}) = O(2^n)^2$. These algorithms are considered 'bad' in the sense that as the problem size increases, the computational times grows very large. Many algorithms that are adopted to solve the combinatorial-type problems such as the traveling salesman (see Chapter 2) have an order of complexity of $O(n!)$. These problems represent the most challenging in terms of computational complexity. As the order of time implies, in these algorithms the computational complexity grows exponentially with the size of the problem. For more technical details on polynomial and non-polynomial time algorithms, the interested reader is referred to the list of books referenced in this chapter.

Unfortunately the algorithms and methodologies that are adopted to solve integer linear programming models discussed in the previous chapters all belong to this class of NP. For this reason, it is almost impossible for the airlines cited in Table 13.1 to get the optimal solutions for their crew scheduling or aircraft routing in a reasonable computational time.

Note that the computational complexity lies with the solution algorithms, that is, the way the problems are solved, and not the mathematical models or the way they have been formulated.

Heuristic Procedures

Not being able to obtain the optimum solutions in a timely fashion for NP type algorithms prompted the researchers to develop other alternatives. These alternative solution methods are generally classified as heuristic methods. Heuristic methods are techniques that do not guarantee or promise the optimum solutions but attempt to provide a 'good' and sometimes 'near optimum' solution in a minimal amount of time.

There are many technical papers in the operations research literature that describe different heuristics and their applications to the airline industry. These heuristics and their technical descriptions are, however, beyond the scope of this book and we refer interested readers to the papers referenced in previous chapters.

As for the airlines, they either develop their in-house customized heuristics to solve their mathematical models in their operations research departments or outsource the service.

Airline Information Technology (IT) Solutions

Airlines that do not develop their in-house customized software outsource their IT needs to Airline IT solutions providers such as Sabre, SITA, Lufthansa Systems, Jeppesen, and EDS. According to Lufthansa, the global market for airline IT was estimated to have a volume of $11.47 billion during 2008, and around 40% of this volume is outsourced to airline IT-solution providers. Some of the main software solutions offered by the airline IT companies are in the areas of crew scheduling, fleet operations, revenue management, ticket distribution, and aircraft maintenance as follows.

Crew Scheduling

Companies like Sabre, Jeppesen, Lufthansa Systems, and SITA offer crew scheduling softwares that are suitable for large airlines. Most solutions are provided on a modular basis and some are customized to integrate with the airlines in-house software. Some of the main features provided in crew scheduling solutions are planning, monitoring, pairing optimizer, and crew access. Planning modules help managers generate long-term crew requirements for up to two years; taking into account factors such as reserve needs, training, and vacation. Monitoring modules help managers with the daily operations and allow tracking of crew and flight operations on a real-time operational basis. Input to generate the crew roster is obtained from the crew through the crew access module. The crew access module also gives crew the ability to directly access their roster through the internet and swap or trade duty shifts. The input from the crew is then optimized using the crew pairing optimizer taking into consideration the flight schedule, cost, and crew needs. Table 13.2 opposite provides a list of major crew-scheduling solutions providers and their website.

Flight Operations

Flight-operation solution contains a range of modules that assist from flight planning to daily tracking and managing of flight operations. Flight-operation solutions typically contain modules that assist with flight planning, flight dispatch, operations monitoring, fleet optimization, load planning, and critical decision support. Lufthansa's NetLine/Plan Route Optimizer, for instance, allows managers to create the optimal flight routing based on simulation of new connections,

Table 13.2 List of airline IT-solution providers offering crew scheduling solutions

Company	Product	Website
AOS	Integrated Crew Planning System	www.aos.us
Jeppesen	Carmen Crew Management System	www.jeppesen.com
Lufthansa Systems	NetLine/Crew	www.lhsystems.com
Navitaire	Geneva Operations Control & Management Suite	www.navitaire.com
Ortec	Integrated aircraft and Crew planning	www.ortec.com
Sabre	AirCentre Crew	www.sabreairlinesolutions.com
SITA	CrewWatch	www.sita.aero

forecast passenger flows, costs, revenues, and identified strengths and weaknesses of the network. The optimized flight routing is then used to assign the available aircraft to achieve maximum overall profitability. Day-to-day operations and flight monitoring are achieved through the operations monitoring module. SITA's FleetWatch, for instance, provides managers with pertinent information about current operations, maintenance events, and helps to evaluate problems and determine the most cost-effective solution. Critical-decision support tools are also provided and integrated with flight-operations solution to help plan for disruption events and create optimized recovery solutions.

Table 13.3, overleaf, provides a list of major flight-operation solution providers and their website.

Revenue Management

Revenue-management tools help airlines increase profitability though better yield and better price structure. The revenue-management solutions contain modules that help managers forecast demand, allocate seats, calculate optimal ticket price, distribute fares to Global Distribution System (GDS), monitor competitor price, manage over-bookings, and cancel multiple bookings. Revenue management tools utilize historical and current data to forecast booking activities and make informed revenue-management decisions. Revenue-management solutions have been continually evolving and one of the recent changes in revenue-management solution is the migration from leg-based revenue-management system to Passenger Name Record (PNR) based Origin and Destination (O&D) revenue-management system. PNR based O&D revenue management system allows managers to tap

Table 13.3 List of major flight-operation solution-providers

Company	Product	Website
Jeppesen	Carmen Integrated Operations Control	www.jeppesen.com
Lufthansa Systems	Integated Operations Control Center	www.lhsystems.com
Navitaire	Geneva Operations Control & Management Suite	www.navitaire.com
Ortec	Integrated aircraft and Crew planning	www.ortec.com
Sabre	AirCentre Flight	www.sabreairlinesolutions.com
SITA	FleetWatch, FleetPlan, Flight Planning	www.sita.aero

into a vast amount of information, develop better forecasts and have a better insight into true traffic flow within the network. Table 13.4 provides a list of major revenue-management solution providers and their websites.

Table 13.4 List of major revenue-management solution-providers

Company	Product	Website
Amadeus	Altéa Revenue Management	www.amadeus.com
Lufthansa Systems	ProfitLine	www.lhsystems.com
Navitaire	RMS Host Revenue Management System	www.navitaire.com
PROS	PROS O&B, PROS NPRS	www.prospricing.com
Sabre	AirMax Suite	www.sabreairlinesolutions.com
SITA	Airfare Price	www.sita.aero

Ticket Distribution

Distribution and ticket sales is another area where airline solution providers play a major role. Owing to the rapid growth of the internet, airlines have begun phasing out travel agents and have led to the growth of computer reservation systems (CRS) and global distribution systems (GDS). Airlines employ ticket-distribution modules to seamlessly transfer fares from their system to the GDS. Table 13.5 provides a list of ticket-distribution solution providers. Ticket-distribution solutions providers sell solutions that include booking engines, business process management, channel distribution, customer relationship management, reservations, customer data, and analysis and ticketing. These systems work together to maximize revenue through maximum distribution of tickets through all available channels.

Table 13.5 List of major ticket-distribution solution-providers

Company	Product	Website
Abacus	FareX	www.abacus.com.sg
Amadeus	Amadeus Airline Retailing Platform	www.amadeus.com
Galileo	Galileo Agency Private Fares	www.travelport.com
KIU	KIU System Solutions	www.kiusys.com
Lufthansa Systems	Passenger Core Systems	www.lhsystems.com
Navitaire	New Skies	www.navitaire.com
Patheo	PAL, FareMate	www.patheo.com
Sabre	SabreSonic	www.sabreairlinesolutions.com
SITA	Horizon	www.sita.aero
Worldspan	Worldspan FareSource	www.worldspan.com

Supplementary Airline IT Solutions

In addition to the above solutions, airline IT providers develop other IT solutions to improve efficiency and effectiveness in areas such as maintenance, repair and overhaul, air-cargo handling, administration, finance, flight navigation, flight planning, gate assignment, aircraft load management, ground handling integration, check-in systems, and so on. For more information please refer to the IT solution provider websites listed above.

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Chapter 14
Start-up Airline Case Study

Introduction

It was explained in Chapter 1 that this book is the result of developing a course on airline operation and scheduling. The following case is a real-world case study referred to our class by an entrepreneur to determine the viability of operations so that a business plan can be developed.

A start-up airline plans to operate a fleet of amphibian (capable of landing both on ground and water) aircraft which would be used to fly leisure passengers between the United States and the Caribbean. The aircraft is the Russian-made Beriev Be-200. It has a capacity of 72 passengers, requires 2 pilots and 2 service personnel, and has a range of 1,200 miles.

The airline plans to start up its operations initially with 4 amphibian aircraft. The proposed flight network for this airline is presented in Figure 14.1. According to this figure the airline flies to 11 cities as presented in Table 14.1.

The requirements for this airline are as follows:

- Only use four airplanes.
- An average load-factor of 65% should be assumed for all flights.
- Each aircraft can be utilized for at most 16 hours per day.
- Aircraft turn-around time is 43 minutes for all flights.
- At the end of each operation day, the aircraft are to be parked in Miami or Nassau for maintenance.

Table 14.2 provides information on the airline's network routes as follows:

- Columns 1 and 2 represent the sequence and the proposed routes.
- Columns 3 and 4 show the distance (miles) and flight blocks (minutes) between city pairs.
- Column 5 shows the total time. This time is obtained by adding the flight block + aircraft turn-around time (43 minutes).
- Column 6 presents the frequency of flights between city pairs. These frequencies are determined by daily forecasted demand and the required load factor as explained in Chapter 3.
- Columns 7 and 8 show average one-way fare and total expected revenue on each flight. Column 8 is calculated by multiplying column 7 × 72 (aircraft capacity) × .65 (load factor).

Table 14.1 List of airports and their codes for case study

Airport	Code
Atlanta International Airport	ATL
Nassau International Airport	NAS
Orlando International Airport	MCO
Tampa International Airport	TPA
Palm Beach International Airport	PBI
Fort Lauderdale/Hollywood International Airport	FLL
Miami International Airport	MIA
Key West International Airport	EYW
Providenciales International Airport	PLS
Norman Manley International Airport	KIN
Claremore Regional Airport	GCM

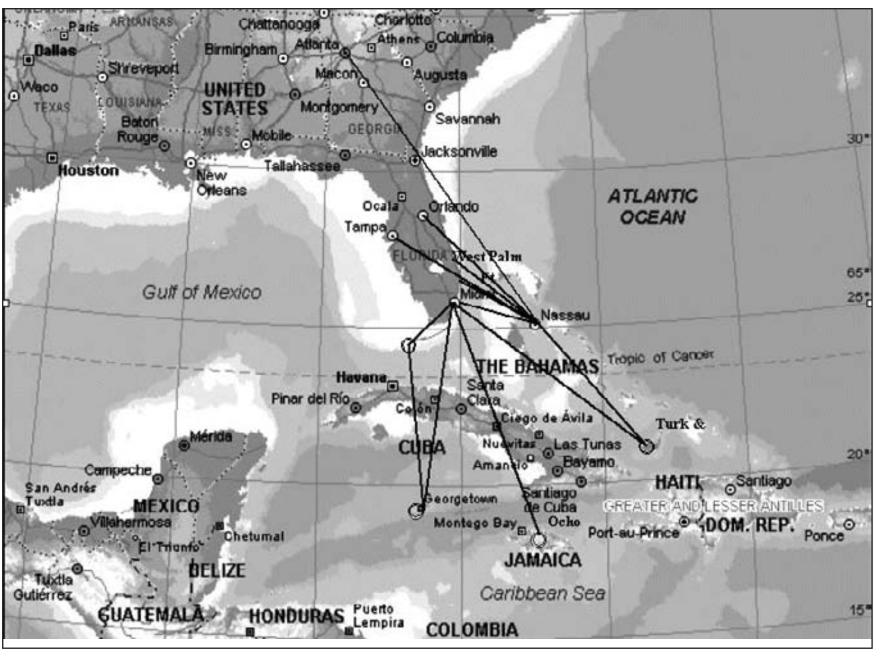

Figure 14.1 Flight network for the start-up airline

The airline realizes that it cannot fly all the flight frequencies in Table 14.2 with its four aircraft in its initial operations. The objective of this case is then to identify and determine valid daily routes for four aircraft so that the total revenue generated through these flights is maximized. Note that in this case we do not have the complete flight schedule (arrival and departure times) as we do not know which flights will be selected, and which frequency they will have. Once these flights are identified, appropriate departure and arrival times will be assigned to them.

Table 14.2 Proposed routes and their frequencies

Sequence	Route	Distance (miles)	Block time (mins)	Total time (mins)	Daily flights	Average fare	Revenue per flight
1	ATL-NAS	725	159	202	2	$436	$20,561
2	MCO-NAS	333	87	130	2	$217	$10,233
3	TPA-NAS	373	95	138	1	$186	$8,771
4	PBI-NAS	199	63	106	1	$121	$5,706
5	FLL-NAS	182	60	108	4	$154	$7,262
6	MIA-NAS	183	60	103	7	$149	$7,026
7	MIA-EYW	109	35	78	1	$222	$10,469
8	MIA-PLS	578	122	165	1	$348	$16,411
9	MIA-KIN	585	123	166	3	$342	$16,128
10	MIA-GCM	452	112	155	2	$236	$11,129
11	NAS-ATL	725	159	202	1	$416	$19,618
12	NAS-MCO	333	87	130	2	$218	$10,280
13	NAS-TPA	373	95	138	1	$169	$7,970
14	NAS-PBI	199	63	106	1	$111	$5,234
15	NAS-FLL	182	60	108	6	$167	$7,875
16	NAS-MIA	183	60	103	12	$153	$7,215
17	NAS-PLS	400	98	141	1	$328	$15,468
18	EYW-MIA	109	35	78	1	$214	$10,092
19	EYW-GCM	363	92	135	1	$369	$17,402
20	PLS-MIA	578	122	165	1	$329	$15,515
21	PLS-NAS	400	98	141	1	$309	$14,572
22	KIN-MIA	585	123	166	5	$367	$17,307
23	GCM-MIA	452	112	155	2	$229	$10,799
24	GCM-EYW	363	92	135	0	$339	$15,987

Solution Approach

A modified set-partitioning approach (Chapter 2) with side constraint as discussed in Chapters 5 and 6 is adopted to determine the efficient aircraft routings. The following binary decision variable is used to formulate the problem.

$$R_i = \begin{cases} 1 \text{ if route } i \text{ is selected} \\ 0 \text{ otherwise} \end{cases}$$

By route we mean complete daily aircraft routing. A route-generator program (similar to the one in Chapter 5) was developed to generate daily routes with the following characteristics:

- maximum daily aircraft utilization of 16 hours (16* 60=960 minutes);
- an overnight stay in Miami or Nassau for maintenance checks;
- since all aircraft stay overnight at Miami or Nassau, they should start and end their daily routings from these two locations.

The program not only generates valid routes but also determines its total flight times and revenue generated over all flights in that route. This program generated more than 1,200 routes. Table 14.3 presents three sample valid routes.

Table 14.3 Three sample routes

Route #	Routing	Total time (min)	Revenue
R_1	NAS-PLS-NAS-MIA-NAS-PBI-NAS-MIA-NAS	906	$69,462
R_2	MIA-EYW-GCM-MIA-NAS-MIA-GCM-MIA	884	$74,839
R_3	MIA-EYW-MIA-NAS-MCO-NAS-MIA-PLS-MIA	952	$87,241

Objective Function

In the proposed set-partition model, the routes are represented as rows, and columns are the daily flights. The mathematical model attempts to maximize the total revenue subject to flight cover and number of available aircraft. The objective function (considering the above routes to be R_1, R_2 and R_3) is as follows:

$$\text{Maximize } 69462R_1 + 74839R_2 + 87241R_3 + \ldots$$

There are two sets of constraints in this problem: flight frequency and aircraft availability.

Flight Frequency Cover

The frequency for each flight must not exceed the daily required frequency of flights (see Table 14.2). As an example we should have at most seven daily flights between MIA and NAS. By looking at the above three sample routes we see MIA-NAS is repeated twice in R_1, once in R_2 and once in R_3. To automate the search for flights, a simple program can identify how many times each flight is repeated in each route. When we have all the frequencies on each route then we can write the constraint for that flight leg. As an example, to address the frequency on MIA-NAS we write:

$$2R_1 + R_2 + R_3 + \ldots \leq 7$$

In general if a flight leg has f_i frequency, then the constraint to cover at most N of these frequencies is as follows:

$$\sum_i f_i R_i \leq N$$

where:
f_i = the number of times that the specific flight leg is repeated in route i and
R_i = the ith route as explained above.

Aircraft Availability

We have a total of four aircraft. Therefore the total number of routes assigned to these aircraft must equal four. Therefore:

$$R_1 + R_2 + R_3 + \ldots\ldots = 4$$

Solution

We used an optimization software to solve this problem. The solution for this problem with four aircraft is presented in Table 14.4. The total daily revenue is $336,706.

Table 14.4 Solution for the case

Aircraft	Routing	Total flight time (mins)	Daily revenue
1	NAS-MCO-NAS-ATL-NAS-TPA-NAS	940	$77,433
2	MIA-KIN-MIA-PLS-NAS-FLL-NAS-MIA	957	$86,770
3	NAS-MIA-GCM-EYW-GCM-MIA-EYW-MIA-NAS	942	$90,119
4	MIA-NAS-PBI-NAS-PLS-MIA-KIN-MIA	953	$82,384

Table 14.5 presents the complete schedule with departure and arrival times incorporating turn-around times derived from the above solution.

Figures 14.2 and 14.3 show the time-space network at each airport and frequency of flights between city pairs respectively.

Table 14.5 Flight schedule and aircraft routing for the case study

Aircraft no.	Flight-leg	Flight no.	Route	Departure time	Arrival time	Block time
1	1	172	NAS-MCO	7:00 AM	8:27 AM	1:27
1	2	127	MCO-NAS	9:10 AM	10:37 AM	1:27
1	3	171	NAS-ATL	11:20 AM	1:59 PM	2:39
1	4	117	ATL-NAS	2:42 PM	5:21 PM	2:39
1	5	173	NAS-TPA	6:04 PM	7:39 PM	1:35
1	6	137	TPA-NAS	8:22 PM	9:57 PM	1:35
2	1	2610	MIA-KIN	7:00 AM	9:03 AM	2:03
2	2	2106	KIN-MIA	9:46 AM	11:49 AM	2:03
2	3	269	MIA-PLS	12:32 PM	2:34 PM	2:02
2	4	297	PLS-NAS	3:17 PM	4:55 PM	1:38
2	5	275	NAS-FLL	5:38 PM	6:38 PM	1:00
2	6	257	FLL-NAS	7:21 PM	8:21 PM	1:00
2	7	276	NAS-MIA	9:04 PM	10:04 PM	1:00
3	1	376	NAS-MIA	8:00 AM	9:00 AM	1:00
3	2	3611	MIA-GCM	9:43 AM	11:35 AM	1:52
3	3	3118	GCM-EYW	12:18 PM	1:50 PM	1:32
3	4	3811	EYW-GCM	2:33 PM	4:05 PM	1:32
3	5	3116	GCM-MIA	4:48 PM	6:40 PM	1:52
3	6	368	MIA-EYW	7:23 PM	7:58 PM	0:35
3	7	386	EYW-MIA	8:41 PM	9:16 PM	0:35
3	8	367	MIA-NAS	9:59 PM	10:59 PM	1:00
4	1	467	MIA-NAS	8:00 AM	9:00 AM	1:00
4	2	474	NAS-PBI	9:43 AM	10:46 AM	1:03
4	3	447	PBI-NAS	11:29 AM	12:32 PM	1:03
4	4	479	NAS-PLS	1:15 PM	2:53 PM	1:38
4	5	496	PLS-MIA	3:36 PM	5:38 PM	2:02
4	6	4610	MIA-KIN	6:21 PM	8:24 PM	2:03
4	7	4106	KIN-MIA	9:07 PM	11:10 PM	2:03

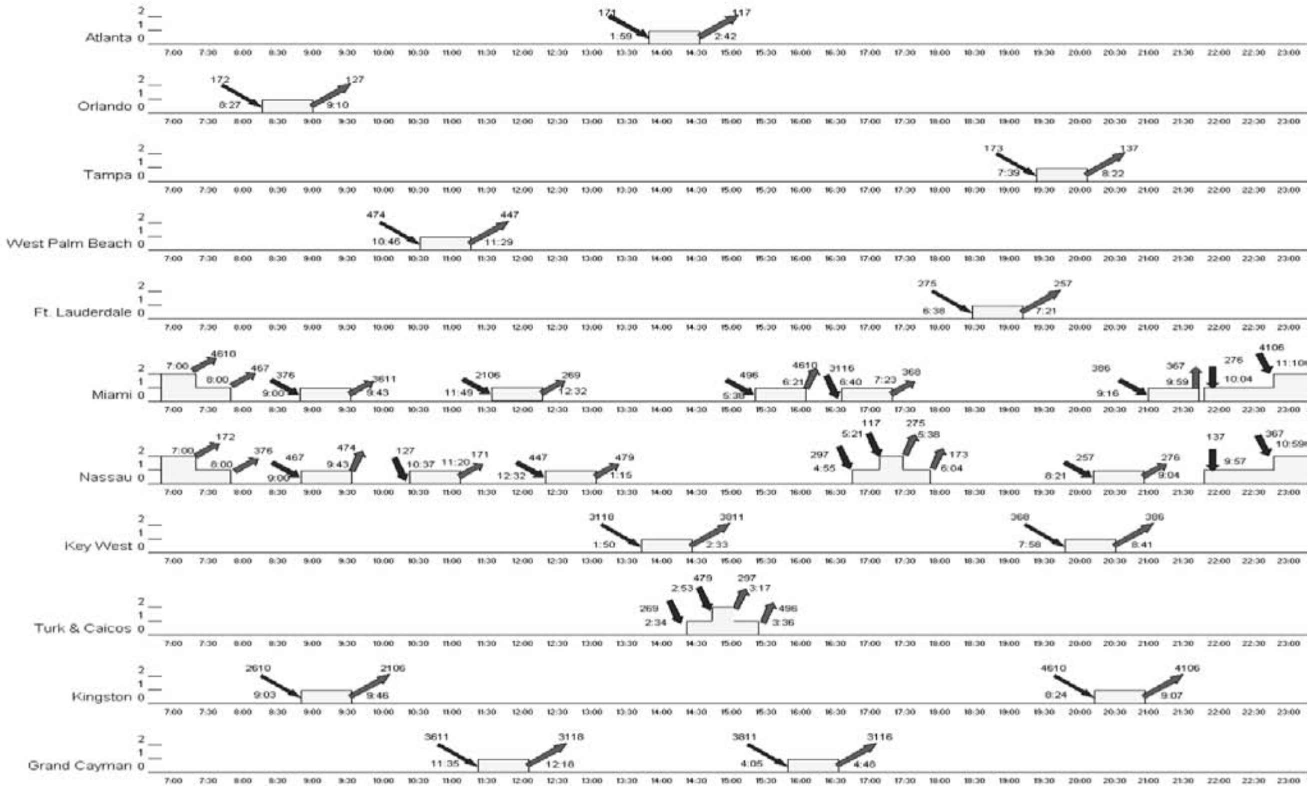

Figure 14.2 Arrival/departure of flights at each airport

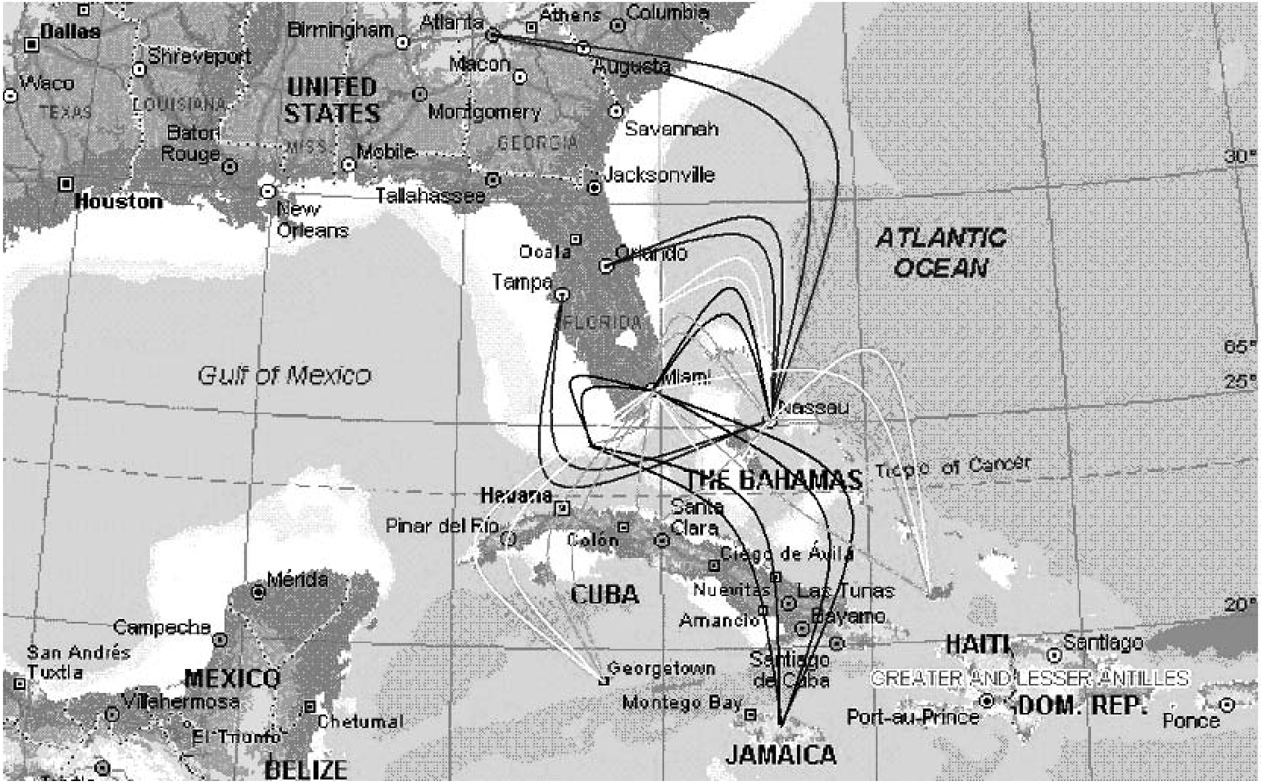

Figure 14.3 Airline's network and aircraft routing

Chapter 15
Manpower Maintenance Planning

Introduction

Manufacturing, transport, financial, health, and distribution systems are frequently in need of an upgrade or improvement. How would the upgraded or improved system perform? What are the unforeseen problems that could occur in the new system? Would the performance of the projected improvement justify the expense? What resources should be purchased and how should they be used, so that the new system would generate the expected benefits?

We can answer the above questions and study the performance of the organization under the new regime, in advance, through *simulation modeling* without actually investing in the new changes.

Simulation Modeling

Simulation modeling is a process by which the basic features of a system are analyzed and simulated by the computer. The simulation model is then used to view the ways in which the new system would operate, typically through animations. It is possible to experiment with simulation models to see what might happen under various conditions the new system might operate.

Simulation study is proving to be an integrated part and an alternative way to mathematical modeling, where the governing parameters are very complex and dynamic. It allows the user to perform *what-if* analysis under different scenarios.

This chapter and Chapters 16–18 present simulation case studies for airlines and airports. The objective of these chapters is not to introduce the concept of simulation but to introduce how they can be utilized in planning for the aviation industry. For readers not familiar with simulation modeling, Law and Kelton (2000) and Kelton et al. (2003) provide comprehensive descriptions of simulation modeling and its application to various industries.

Simulation in Airlines

The AGIFOR's website (www.AGIFORS.org) and Winter Simulation Conferences website (www.wintersim.org) show that a growing number of airlines are adopting simulation study as an important tool for their planning process. Simulation has been applied to manpower planning, fleet assignment, gate assignment, flight scheduling, traffic flow, and so on. Simulation modeling is becoming much more popular as the software used for these models are becoming easier to use, and more powerful. The recent integration of meta-heuristics in this simulation software has enabled them to optimize the parameters within the models. This feature is another important factor for the growing popularity of simulation software.

The case study in this chapter relates to manpower planning for maintenance at Continental Airlines (Bazargan et al. 2003). For maintenance capacity planning, simulation provides the capability of changing many variables simultaneously.

Manpower Planning for Continental Airlines

This case study describes the development of an aircraft line-maintenance simulation model for Continental Airlines to be used at their hub in Newark airport (EWR). The simulation model is developed to support the management of the line-maintenance department in solving various capacity-planning issues related to manpower requirement and scheduling.

Line maintenance (commonly referred to as short routine maintenance) includes the regular short-haul inspection of aircraft between their arrival and departure.

Line maintenance is driven by the flight schedule. Once the flight schedule is finalized, a maintenance schedule is assigned to each maintenance station. The maintenance schedule takes into consideration the fleet/equipment type flying to that station, type of maintenance programs to be carried out, the capabilities of the specific station, task standards for each of these maintenance programs, aircraft ground time availability, and other resources such as tooling, hangar, weather, and events that would conflict with one another.

Aircraft maintenance is a major cost component for the airlines. The following table presents maintenance cost as a percentage of their total operating cost for select US airlines.

Table 15.1 Percentage of maintenance expense in total operating expense for select US airlines

Airline	Maintenance expense/total operating expense
American	8.49%
Continental	6.52%
Delta	5.02%
Frontier	6.66%
JetBlue	6.76%
Southwest	9.80%
United	8.30%
US Airways	7.90%

Source: OAG Form41 iNET

A major challenge for maintenance stations is the availability of mechanics at different times of the day. The stochastic nature of aircraft failure and the time it takes to rectify the failures make manpower planning a challenge for maintenance stations. Labor on average represents 13% of maintenance cost. The following table represents the percentage of labor cost in total maintenance cost:

Table 15.2 Percentage of labor expense in total maintenance expense for select US airlines

Airline	Labor maintenance expense/total maintenance expense
American	14.09%
Continental	13.31%
Delta	11.47%
Frontier	17.36%
JetBlue	12.48%
Southwest	8.74%
United	8.66%
US Airways	19.43%

Source: Form41 iNET

Mathematical modeling techniques have been used for maintenance planning, and are sometimes integrated with other scheduling models. Dijkstra et al. (1991), Clarke et al. (1996), Hane et al. (1995), Rushmeier and Kontogiorgis (1997), Barnhart et al. (1998), Talluri (1998), Sachon, Pate-Cornell (2000), Sarac (2006), Sandu (2007) and Kozanidis (2009), are some of the researchers who have developed mathematical modeling for aircraft-maintenance planning. In most of these mathematical models, maintenance requirements are included as constraints in the problem formulation than as the primary goal of the study.

Over the past few years it has become apparent that better decision-support tools, such as simulation modeling, are needed in the maintenance department. Duffuaa and Andijani (1999) consider that the application of computer simulation to maintenance functions provides a better and more viable alternative to mathematical modeling and analysis. This is due to the difficulty of the mathematical models in capturing the complexities of maintenance operations, uncertainty of parameters in arrivals, sequencing, job contents, and availability of resources.

Simulation studies applied to airline maintenance operations and planning dates back to 1961 (see, for example, Defosse and Bindler 1961, or Stanhel 1961) who applied Monte Carlo simulation models to study engine rates and costs. Other simulation approaches include manpower planning (Kamiko 1978, Bazargan 2003, Bazargan 2005), engine maintenance and overhaul (Getland et al. 1997, Duffuaa and Andijani 1999, Painter et al. 2006). Agifors.org and wintersim.org provide more application areas of simulation to maintenance planning and operations. These studies indicate that the application of computer simulation to maintenance functions provides a better and more viable alternative to mathematical modeling and analysis.

Line Maintenance Department

This simulation study aims at duplicating the maintenance operations for Continental Airlines at their major maintenance station at Newark (EWR). AutoMod Simulation Software (Banks 2000) has been used as the developmental platform for the study. The focus of this study is to analyze and recommend efficient manpower-staffing models.

Continental Airlines, based in Houston, Texas, is the seventh largest airline in the United States in term of revenue passengers carried during 2008. Continental Airlines serves over 260 domestic and international destinations from its Newark, Houston Cleveland, and Guam hubs with a total of 2,663 daily departures.

Equipment/Fleet Type

The aircraft equipment/fleet types operating through Newark, at the time of this study, are presented in Figure 15.1. The abbreviated three-letter number coding system represents the respective aircraft type under each size/range classification (e.g., the 733 under narrow-body is a Boeing 737-300, while the 735 represents a Boeing 737-500, etc.).

Maintenance Schedules: Line maintenance includes the regular short-haul inspections of aircraft between the arrival and the consecutive departure from the airport. An aircraft flying into a station can be classified as a *through*, *day hold* or *remains overnight* flight.

Through Flight: In a through flight, the aircraft would be in transit through the station with minimal ground time. Every through flight goes through a departure check while it is on the ground. The total number of narrow-body, mid-body (domestic), mid-body (International), and wide-body aircraft flying into Newark is presented in Table 15.3. Figure 15.2 shows the workload of through flights on a typical day at the time of the study.

Day Hold: In day hold, the aircraft is scheduled for one of the routine checks, held during the daytime, before its subsequent departure.

Remains Overnight (RON): In remains overnight, the aircraft remains overnight for one of the routine checks before its subsequent departure.

Maintenance Programs

The maintenance program that an aircraft goes through is as follows:

Service Check (SVC): A walk-around service and systems check applicable to all fleets, generally done on an overnight basis. Wide-body aircraft get this check

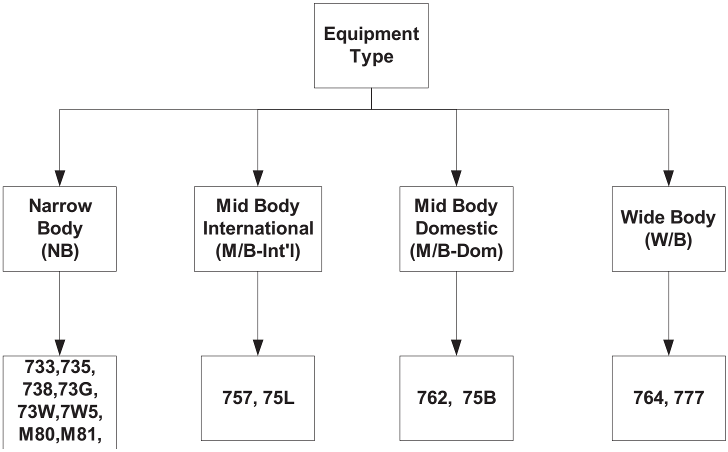

Figure 15.1 Equipment type

Table 15.3 Number of through flights in a day

Equipment type	Number of through flights in a day
Narrow body	145
Mid body – domestic	15
Mid body – international	16
Wide body	7

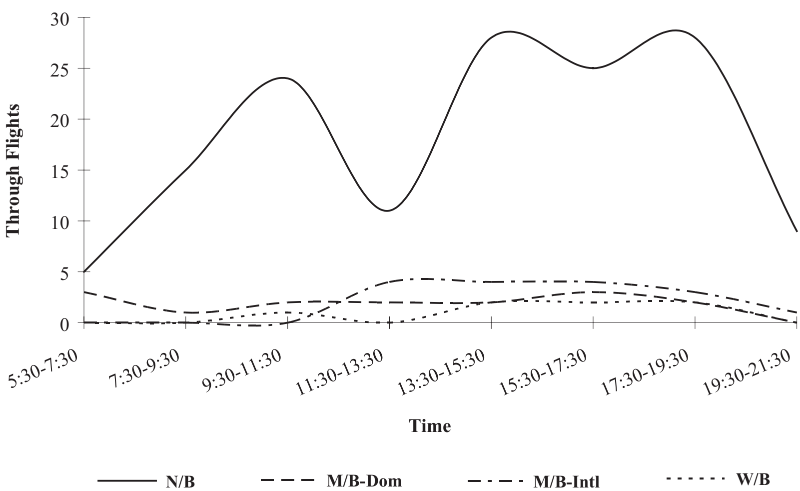

Figure 15.2 Through flights on a typical day

done on *day hold* as well as on remains overnight. If an aircraft remains overnight at a station with sufficient ground time, a service check will be performed, regardless of the number of days it has been since its last service check. If a higher-level check has instead been performed, it supersedes or signs-off the service check.

Level 3 Service Check (SC3): Level 3 service check is a more in-depth service check applicable to all fleet types. This check is also done on an overnight basis. It generally takes between 8 to 10 hours to complete this check for a narrow-body aircraft and 12 or more hours for a wide-body aircraft. A level 3 service check is a higher level of check than a service check, so a service check is not performed if a SC3 check is due.

Line Package Visit (LPV): A scheduled check applicable to all narrow-body aircraft, generally done on an overnight basis. LPV requires 75 man-hours, and generally just one LPV is scheduled at the Newark maintenance station in a night. LPV are handled by the night-shift technicians.

Table 15.4 presents the daily demand for various checks for the different fleet type classifications.

Table 15.4 Total number of checks scheduled on each equipment type daily

Equipment type	Total number of checks		
	SVC	SC3	LPV
Narrow body	35	7	1
Mid body – domestic	4	1	0
Mid body – international	5	1	0
Wide body	9	1	0

Standard Maintenance Timings

Table 15.5 gives the standard man-hours (M/H), ground time (in hours), and technician requirements for each maintenance program for *day holds* and *remains overnights* for all fleet types at Newark.

Tables 15.6 and 15.7 show the standard man-hours (M/H), ground time (hours), variability (+/-) and technician requirements for *through flights* for all fleet types at Newark.

Shift Schedule

There are three working shifts in 24-hour, *day*, *swing* (afternoon) and *night shifts*. Each shift is divided into sub-shifts. Table 15.8 projects the shift and sub-shifts schedules at Newark.

Table 15.5 Man-hours, ground-time, and technician requirements for day holds and remains overnights (RON)

Fleet type	Ground time	Number of technicians
N/B	<0.75 hrs	2
	≥0.75 hrs	1
M/B-Dom	<0.75 hrs	2
	≥0.75 hrs	1
M/B-Int'l	<1.5 hrs	3
	≥1.5 hrs	2
W/B	<1.5 hrs	3
	≥1.5 hrs	2

Table 15.6 Service-check (SVC) man-hours, ground–time, and technician requirements for through flights

Fleet type	SVC			
	M/H	Ground-time (hrs)	+/- (hrs)	Number of technicians
N/B	6	6	0.25	1
M/B-Dom	8	8	0.25	1
M/B-Intl	10	5	0.25	2
W/B	25	6.25	0.25	4

Table 15.7 Level 3 Service-check (SC3) man-hours, ground–time, and technician requirements for through flights

Fleet type	SC3			
	M/H	Ground-time (hrs)	+/- (hrs)	Number of technicians
N/B	16	8	0.25	2
M/B-Dom	18	9	0.25	2
M/B-Intl	30	7.5	0.25	4
W/B	75	18.75	0.5	4

Table 15.8 Shift and sub-shift schedules at Newark

Shifts	Sub-shifts	Start time	End time
Day	1	05:30	14:00
	2	06:00	14:30
	3	06:00	16:30
	4	11:00	21:30
Swing	1	13:00	21:30
	2	13:30	22:00
	3	14:00	22:30
	4	14:30	23:00
Night	1	20:30	07:00
	2	21:30	08:00

Manpower Challenge

The challenge faced by the maintenance department was determining the number of technicians required and their shift schedules based on the flight schedule and the maintenance programs to be carried out.

Continental Airlines was using basic quantitative models to compute the shift schedules, but these models were incapable of capturing the peaks and troughs in the arrivals and departures of flights.

A simulation approach therefore seemed promising in capturing the complexity of operations at the maintenance department.

Assumptions of the Simulation Model

The proposed simulation model incorporated the following assumptions:

- The daily flight schedule at Newark was used for the arrival process.
- There are three technician pools – day, swing ,and night shifts, each divided into several sub-shifts.
- The model extracts the technicians from a requisite pool whenever there is a requirement.
- A technician already assigned to a job cannot be utilized for another job until he/she finishes the job which he/she began.
- A technician becomes available to work on a new job immediately after finishing a previous job.

- Every technician is qualified to work on any job. There is no distinction between the technicians who works on *through flights* and *routine checks* (i.e., day holds and remains overnights).

Process Logic

The flowchart in Figure 15.3 presents a sample logic behind the development of the simulation model for through flights.

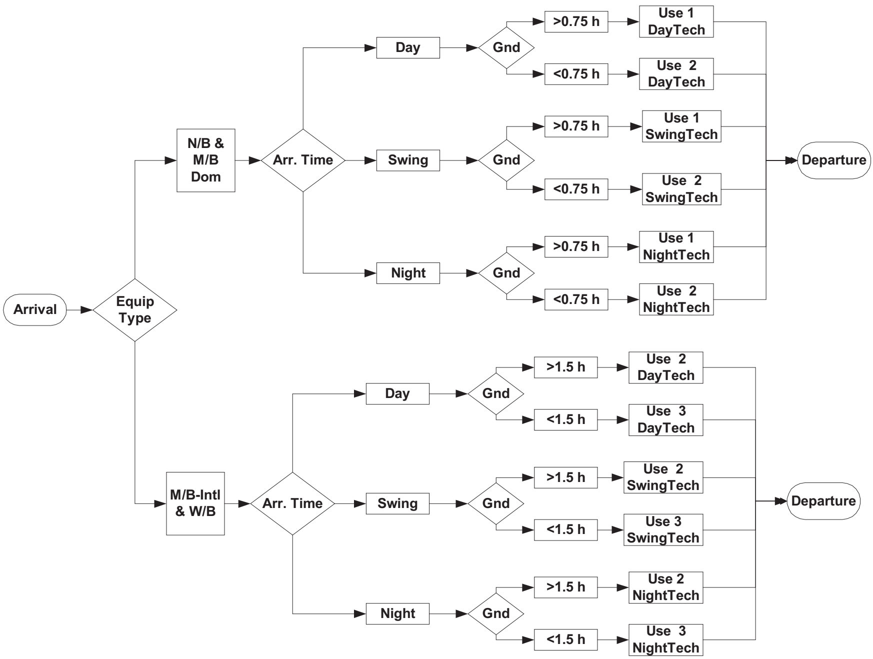

Figure 15.3 Maintenance cycle for through flights (narrow body, mid-body domestic, mid-body international and wide-body aircraft)

Analysis (Base Scenario)

In the base scenario, the focus was to developing a simulation model representing the existing maintenance practices. The validity of the results of the simulation model was confirmed through meetings with the airline personnel and feedback received from the maintenance department. The following are the results of the existing practices at the time of the study, which is referred to as *base scenario* in this case study. AutoStat analysis tool (Banks 2000) was used to derive the various performance measures for the system. The model simulates an entire day of operations. Multiple replications were made for each scenario to increase the reliability of the output.

The airline was interested in identifying three performance measures for maintenance technicians, namely number of aircraft serviced, utilization, and unfinished jobs. The following section describes these performance measures under the base and proposed scenarios.

Total Technician Requirement

Figure 15.4 presents the output of the simulation model for the total technician requirement during each sub-shift of the base scenario. As the figure suggests, there is more demand for technicians during the night shift than during the day and swing shifts.

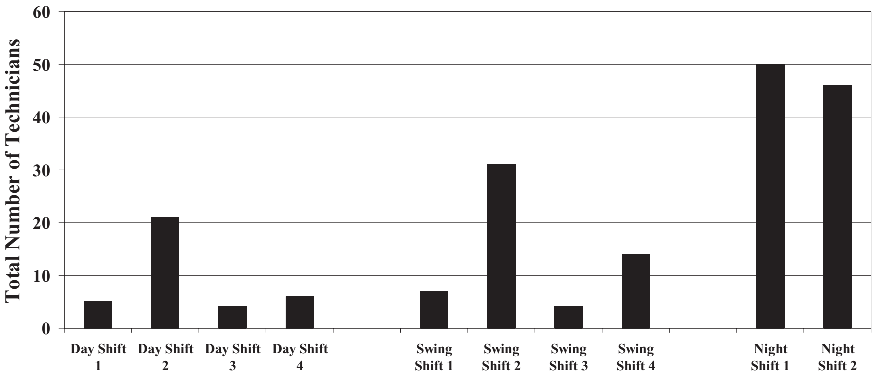

Figure 15.4 Total technician requirements for each sub-shift in a day

Total Number of Aircraft Serviced by each Technician

Table 15.9 summarizes the average workload in terms of number of aircraft serviced by a technician in each shift.

The number of aircraft serviced by day and swing shift technicians increases with the major workload of *through flights* during the day and swing shifts, as *through flights* require less time to service. However, the major workload during the night shift consists of *routine checks* that require comparatively more ground time to complete, thus decreasing the total number of aircraft serviced by night-shift technicians.

Utilization of Technicians

The *utilization* of each technician is calculated by adding the total amount of time a technician works on each job divided by the total shift time (calculated as a percentage). A technician working near his/her maximum capacity represents a

Table 15.9 Average number of aircraft serviced by each technician in each shift

Shift	Average workload for a technician
Day	2.5 aircraft
Swing	3.5 aircraft
Night	1.7 aircraft

bottleneck, and a technician with a low percentage of utilization is considered *underutilized*. Figure 15.5 summarizes the average percentage utilization of technicians in each shift. The day-shift technician utilization is comparatively less than the other shifts. This can be attributed to the nature of the workload for *through flights*, experienced by day-shift technicians. The policy requiring a technician to be available to greet an aircraft upon arrival generates underutilized technicians. In reality, these utilization percentages are higher as the technicians can also be utilized elsewhere as needed to work on other *unscheduled jobs*. The introduction of *part-time technicians* could improve the utilization of day-shift technicians.

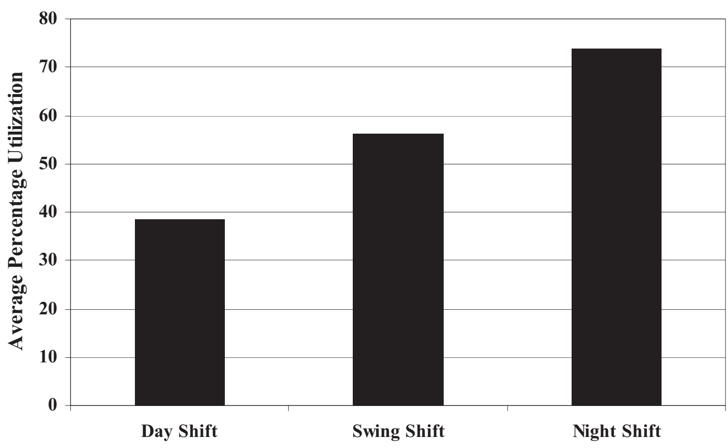

Figure 15.5 Average percentage utilization of technicians in a day

Number of Technicians with Unfinished Jobs

A technician will only take up a job if it arrives between his/her shift start and end-times. If a technician is still busy on a job after the shift end time, the job is transferred to a technician in the next shift. The management considers that a lower number of jobs transferred to the next shift will improve the spread of workload across all shifts. Thus, a required performance measure in evaluating the efficiency of the existing shift schedule was to determine the number of unfinished jobs in each shift.

The total number of technicians with unfinished jobs after their shift end-times for each shift is shown in Table 15.10. The number of technicians with unfinished jobs for all other shifts is zero. As it can be observed, the later swing shift and especially the night shifts need to be better scheduled for a more uniform spread of workload.

Table 15.10 Number of technicians with unfinished jobs at the end of each shift

Shift	Number of technicians with unfinished jobs
Swing shift 4	4
Night shift 1	8
Night shift 2	34

Sensitivity Analysis

Various analyses and changes were made to the model in order to answer questions raised by the airline on how the system would perform under different scenarios. These scenarios included changes to daily flight schedules, the number of technicians, the start/end of sub-shifts, and so on. Reports detailing the impact of such changes to the operation of line maintenance were submitted to the airline. In this section, a proposed schedule is presented that improves the performance measures.

An interesting feature of recent simulation software is optimization. Through this feature, the model makes changes to a set of parameters within specified boundaries in an effort to optimize some objective function. The optimization algorithm of the AutoMod simulation software automates the process of changing the necessary parameters. It uses meta-heuristics (Banks 2000) to determine the optimum set of parameters.

In this study we adopted the optimal scenario, to determine the start/end of sub-shifts. The optimal scenario corresponds to the situation in which the system uses its resources to achieve the highest point of efficiency. Here, the management was interested to see how the number of unfinished jobs (aircraft) carried from one shift to another could be reduced.

Optimal Shift Schedule

Our study showed that the day, swing and night-shift schedules (starting/ending times) have a major impact on the spread of workload and carrying unfinished jobs to another shift. The software was allowed to make changes to these schedules in an effort to minimize the unfinished jobs. Utilizing the optimization feature of the software generated the results in Table 15.11, which presents the best start/end time for each shift/sub-shift based on the optimization module of the software.

Table 15.11 Optimal shift schedule

Shifts	Sub-shifts	Start time	End time
Day	Shift 1	05:30	14:00
	Shift 2	06:00	14:30
	Shift 3	08:00	16:30
	Shift 4	11:00	19:30
Swing	Shift 1	13:00	21:30
	Shift 2	13:30	22:00
	Shift 3	14:00	22:30
	Shift 4	15:00	23:30
Night	Shift 1	22:00	06:30
	Shift 2	24:00	08:30

Figure 15.6 presents a comparison of the total number of technicians with unfinished jobs at their shift end-times for the base and the optimal scenarios.

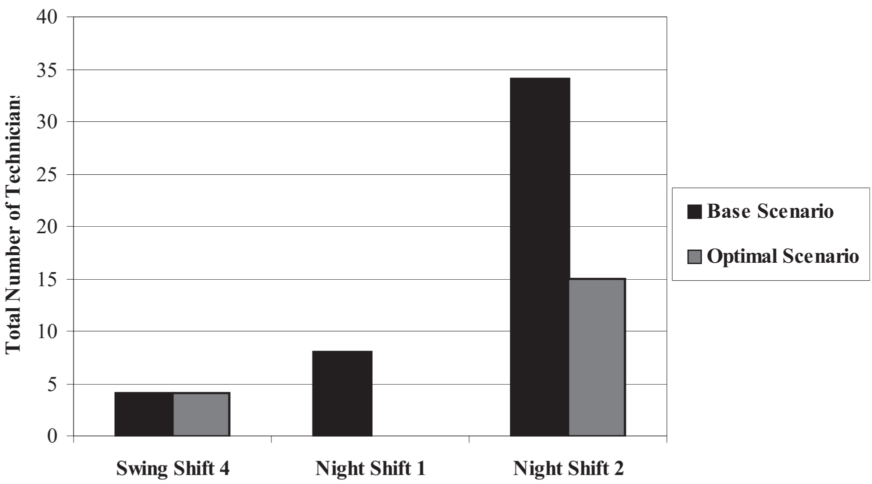

Figure 15.6 Total number of technicians with unfinished jobs in any shift

As the figure shows, the optimal scenario offers a better spread of workload across the shifts by reducing the workload passed on to the next shift.

Conclusions

The simulation model captures the daily operations of the line maintenance facility at Newark. Various system parameters were evaluated, and their validity confirmed by comparison with the airline's existing figures. Some of the benefits of this simulation study include:

- Effective estimation of technician requirement on a sub-shift basis. The model results closely matched actual numbers.
- Simulation analysis generated performance measures, like *technician utilization* and *work overflow*, which could not be estimated earlier.
- Low utilization of technicians, which brings forth the idea of using part-time technicians especially during the day shifts.
- Optimization studies, which show that changing of the shift schedule can greatly enhance the efficiency of the existing system by spreading the workload more uniformly across shifts.

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Chapter 16
Aircraft Tow-tugs

Introduction

As indicated in chapter 9, fuel cost is a major component of operating cost within the airlines. The increasing cost of crude oil and subsequently jet fuel continues to motivate airlines to seek more efficient ways of operations. One of these strategies relates to adopting and utilizing tow-tugs for aircraft ground-movements. This chapter presents an economic and operational feasibility study of using tow-tugs to move aircrafts at airports. Specifically, this simulation study pertains to using aircraft tow-tugs at AirTran Airways at their hub in Atlanta-Hartsfield Jackson International Airport (ATL). These vehicles were intended to tow aircraft to and from the airport terminals (concourses C and D) to the airline's maintenance hangar, located about 3 miles from the terminals.

Background

AirTran Airways, a leading low-cost carrier operating in the United States, initiated this study to investigate potential cost reductions through saving jet fuel by purchasing and utilizing aircraft tow-tugs. The airline's maintenance hangar is about 3 miles from C and D terminals at their hub in Atlanta International Airport (see Figure 16.3). At the time of this study in 2005, aircraft that remained overnight (RON) at ATL and were scheduled for maintenance taxied from these terminals to the maintenance hangar on their own power. The airline basically has two types of fleet: Boeing 717-200 and Boeing 737-700. Two mechanics are needed to taxi an aircraft within each fleet. The taxiing of the aircraft scheduled for maintenance at the hangar was typically performed between 6 p.m. and 4 a.m. the next day. Considering that some major airlines have been using aircraft tow-tugs, AirTran Airways was interested in identifying potential benefits from purchasing and utilizing them at their ATL hub. In particular:

- Considering the high purchasing cost of the tow-tugs, is there any economic justification for purchasing them? If so, what is the payback period of the investment?
- How many of these tow-tugs, if economically justified, are needed for the airline's taxi operations at ATL?
- What is the utilization rate for each tow-tug?

Aircraft Tow-tugs

The tow-tug considered by AirTran (see Figure 16.1) is designed for narrow-body aircraft and is capable of towing both types of aircraft within the AirTran fleet. The attachment of the tow-tug to the aircraft is through two low-level arms. These arms when engaged secure the aircraft's retractable nose gear from either side and raise it slightly off the ground. The aircraft is then moved using the tow-tug's power. Features typically built into the tow-tug include automatic aircraft pick-up sequence, jack-knifing prevention system, emergency aircraft release, and emergency steering system.

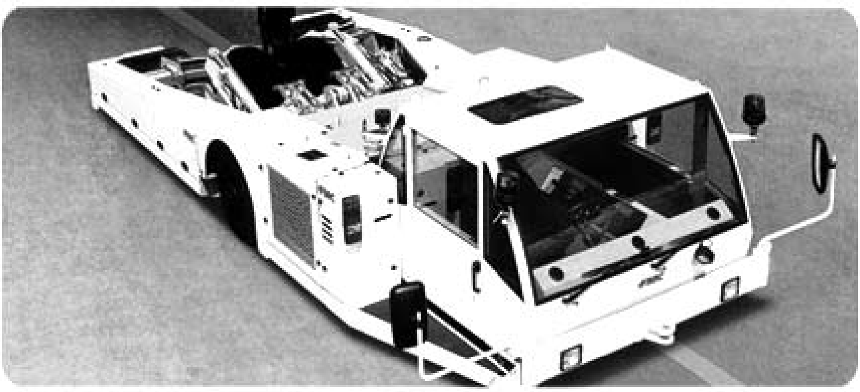

Figure 16.1 Narrow body tow-tug (Expediter 160 – FMC Technologies)

Simulation Model

Owing to the stochastic nature of arrival/departure times, taxi times, maintenance service times, and jet fuel prices, a simulation modeling approach to study the problem seemed to be appropriate. Chapter 15 presented the benefits of simulation modeling and its applications to airline industry.

Several visits to AirTran Atlanta hub were made to observe aircraft taxiing to and from the maintenance hangar. Two simulation models, current and proposed, were developed based on our visits using Arena Simulation Modeling (Rockwell Software).The first model, current scenario, is based on the current operations where aircraft taxi to and from the maintenance hangar on their own power using jet fuel. The second model, proposed scenario, is based on purchasing and utilizing tow-tugs to relocate the aircraft to and from the maintenance hangar. Figure 16.2 presents the basic logic for both models, and a description of each module follows.

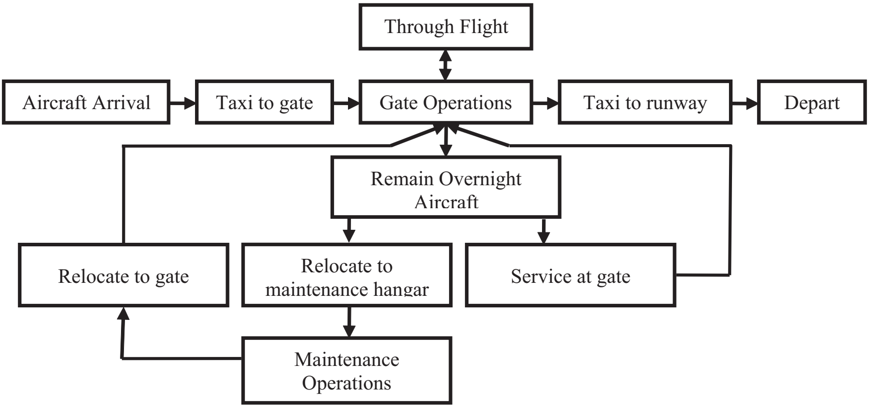

Figure 16.2 Basic logic of the current and proposed models

Simulation Modules

Aircraft Arrival

The airline's weekly flights schedule was used to generate the arrival and departure of aircraft to and from ATL airport. Stochastic variations based on past historical data were also incorporated into the arrival/departure times.

Taxi to Gate

The arriving aircraft are routed to the vacant gates. The stochastic taxi times to gate are determined based on historical data.

Gate Operations

The aircraft arriving at ATL are either through flights or remain overnight (RON). The remain-overnight (RON) aircraft stay at ATL until the next morning. The stochastic unloading, loading, and maintenance times are determined based on historical airline data for both through flight and remain-overnight aircraft.

Remain-Overnight Aircraft

Any aircraft that remain overnight will receive some type of maintenance service. The maintenance hangar has the capacity and space to accommodate six aircraft at any time. Accordingly, six aircraft that are scheduled for heavy maintenance are taxied to the hangar and the rest are serviced at the gates.

Relocate to Maintenance Hangar

This module routes the aircraft that have been identified for heavy maintenance from the gates to the maintenance hangar. The module will request resources (mechanics and tow-tugs if used) for such relocation. The stochastic times for such relocation with or without the tow-tugs are provided in the module based on historical data or tow-tug manufacturer's specification. The tow-tug speed without aircraft being attached to it is 12–15 miles/hr and with the aircraft attached is 10–12 miles/hr.

RON Maintenance Services

Each aircraft entering the hangar is assigned a maintenance program according to the historical frequency tables. The stochastic types and duration of each maintenance operation performed at the hangar on each aircraft are generated from the probability distributions within this module.

Relocate to Gates

When the maintenance program on the aircraft is finished at the maintenance hangar, it is ready to return to a gate at the terminal. The module checks for resources (mechanics, available gates, tow-tugs if used) and if all conditions are satisfied the aircraft is returned to a gate (not necessarily the same gate that the aircraft was moved from) according to a predefined probability distribution for the duration of this transfer.

Taxi to Runway and Depart

All the through-flight and remain-overnight flight that have completed their maintenance program enter this module according to their departure times.

Simulation Analyses

The key performance measures for the two simulation models included cost comparison, operations feasibility and tow-tug utilization. The simulation models were run for a week of airline operations. Each model was run with enough replications to reduce the half-width confidence intervals to 5% of the means.

Current Scenario

As explained earlier, this model pertains to aircraft taxiing on their own powers to/from the hangar. Figure 16.3 presents a rough layout of terminals C and D and the maintenance hangar at ATL. The distance from these terminals to the hangar is about 3 miles.

In order to determine the cost under the current scenario, we needed to identify the breakdown of each of the cost components involved. The primary cost components under the current scenario are jet fuel, labor, and overheads.

The simulation software enables users to utilize cost module while running the model. This module requires cost/minute of the aircraft operations during the taxi to and from the hangar. Historical logbook data on jet fuel and labor were analyzed to estimate the average cost of taxiing aircraft per minute as follows:

- Jet fuel consumption: The airline's logbook showed that on average 30 pounds of fuel were burned per minute for an empty aircraft to taxi between the terminals and the hangar. The jet fuel price was $1.90 per gallon at ATL at the time of this study. Considering 1gallon = 6.6 pounds, the fuel cost per minute is $8.64.
- Labor: Two mechanics are needed to taxi the aircraft. Each mechanic was paid $20/hour. Therefore, on average the labor cost (for two mechanics) per minute is $0.67.
- Overheads: The logbooks did not have any data on overhead costs such as wear and tear or other costs of aircraft taxiing on their powers. We could not identify a reliable figure to represent this cost from other sources either.

Therefore the cost per minute of $9.31 for the base scenario comprises fuel cost and labor cost.

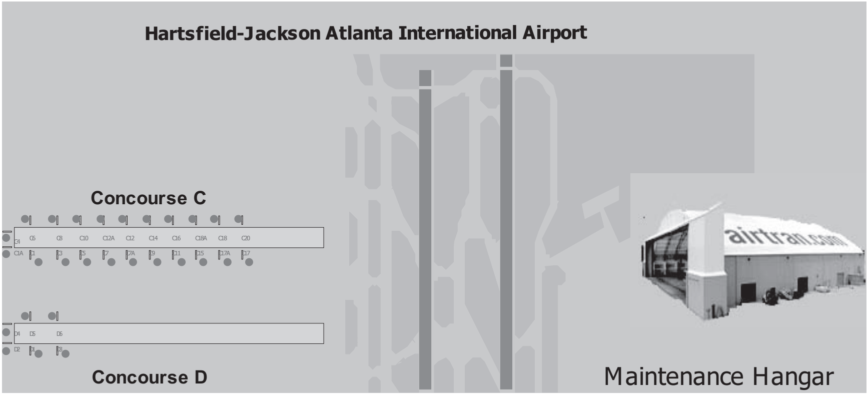

Figure 16.3 Location of gates and the maintenance hangar at ATL

As indicated above, the model was run to simulate one week of aircraft transfer between the terminals and the maintenance hangar. The performance measures based on this simulation model are average weekly costs of $16,805 and average taxi time per trip of 21.5 minutes. Note that this weekly cost is based on $1.90/

gallon of jet fuel. A major interest for this study was to determine the impact of the growing cost of jet fuel. Accordingly, the simulation model was run with different jet-fuel prices. Figure 16.4 presents the total weekly operation cost with jet-fuel price fluctuating from $1.50 to $2.50/gallon.

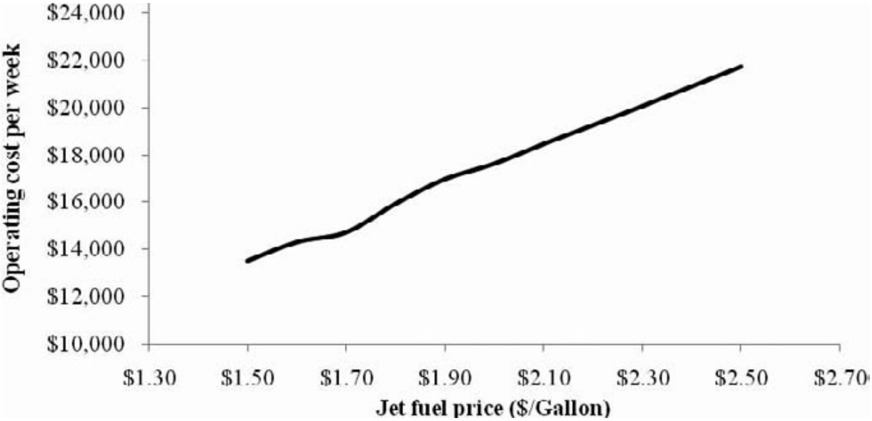

Figure 16.4 Average weekly cost for aircraft taxis without tow-tugs

Proposed Scenario

Another simulation model was developed to incorporate the utilization of aircraft tow-tugs. In this scenario, the aircraft is towed using the tow-tug's power. Two mechanics are needed to operate the tow-tug and safely transfer the aircraft to the maintenance hangar. The variable costs of using tow-tug per minute are as follows. The following cost figures are based on the airlines' estimates:

- Fuel: The tow-tug runs on diesel and fuel cost is estimated to be $0.50/ minute.
- Labor: Similar to the current scenario, two mechanics are needed to operate the tow-tug. Each mechanic will be paid $20/hour. Therefore the labor cost per minute is $0.67
- Overheads: Overheads are estimated to be $2.90 per minute. This relatively high cost was suggested by the airline to protect against the unanticipated operating cost of the tow-tug.

Therefore, the total operating cost of the tow-tug is $4.06/ minute. It should be noted that it takes longer to move the aircraft using the tow-tug than using the aircraft's own power. Based on the tow-tug manufacturer's recommendation and other airlines' experiences, the estimated time to attach/detach the aircraft to a tow-tug is 4–6 minutes.

The model was run with one operational tow-tug, incorporating the attach/detach times and slower tow speeds. Performance measures of the proposed model are an average weekly cost of $11,058, average taxi time with aircraft attached per trip of 42 minutes, average tow-tug taxi time without aircraft attached per trip of 23 minutes, average tow-tug utilization of 30% and average waiting time for the tow-tug of 30 minutes.

The reason for the low utilization of the tow-tug is that the aircraft are moved between the terminals and the maintenance hangar between 6 p.m. and 4 a.m. only. The tow-tug is idle at other times. Also on average it takes about 40 minutes longer to use the tow-tug to move the aircraft than to taxi the aircraft on its own power for a round trip. Most of this extra time is spent attaching/detaching the tow-tug to the aircraft. The average waiting time represents the time in minutes that an aircraft needs to wait until the tow-tug arrives for its transfer. The tow-tug is typically parked outside the maintenance hangar while it is idle.

Similar to the base scenario, it was of interest to evaluate the impact of changes in the weekly operating cost of the tow-tug. Accordingly, we ran the model with tow-tug operating cost fluctuating from $3.50 to $4.50 per minute. The following figure presents the total weekly operating cost as the operating cost per minute varies.

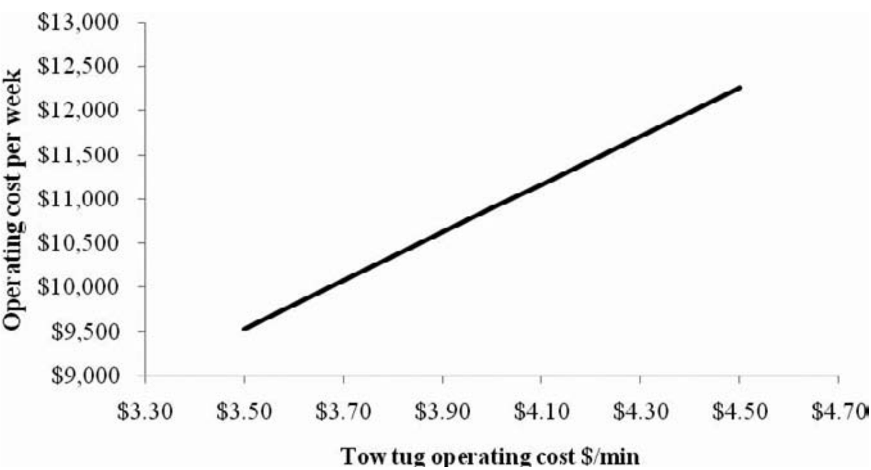

Figure 16.5 Average weekly operating cost using the tow-tug

It should be noted that the aircraft tow-tug manufacturer considers the $2.90/min overhead cost suggested by the airline to be excessive. The tow-tug manufacturer instead recommends an overhead cost of $1.00/min.

Investment Analysis

To get an estimate for annual savings, the costs above were multiplied by 52. Thus the annual cost without tow-tug is $873,860 and the annual cost with tow-tug is $575,016, representing an annual saving of $298,844.

The purchasing price of the proposed aircraft tow-tug vehicle is $250,000. We can now determine the payback period of the tow-tug as below. The payback period provides an indication of the time (in years) that it takes to recover the initial investment.

$$\text{Payback period} = \frac{\text{Initial investment}}{\text{Net annual cash flow}}$$

$$\text{Payback period} = \frac{\$250{,}000}{\$298{,}844} = 0.84$$

It is interesting to see that the tow-tug will be paid off in 10 months. The net present value (NPV) to purchase and operate the tow-tug for 10 years, based on the above annual cash flows and 15% discount rate (suggested by the airline) is as follows:

Table 16.1 NPV for purchasing and operating the tow-tug for a period of 10 years

Year	Present value of future cash flow	Year	Present value of future cash flow
0	-$250,000	6	$129,198
1	$259,864	7	$112,346
2	$225,969	8	$97,692
3	$196,494	9	$84,950
4	$170,865	10	$73,869
5	$148,578	NPV	$1,249,829

As the table implies the net present value (NPV) of purchasing and operating the tow-tug for 10 years is more than $1,200,000. Analysis using payback period and NPV suggests that the purchase and operation of the proposed tow-tug is highly beneficial and financially rewarding, even with low utilization of the tow-tug. This justification is mainly due to the high cost of jet fuel.

It is of interest to identify the jet-fuel prices that would provide the same operating cost under both scenarios (break-even point for jet-fuel prices). So the weekly current scenario model with different cost/min values was run with the objective of achieving total cost to be around $11,058/week, which is the total cost under the proposed scenario. The cost/minute for the aircraft to taxi on its own power was found to be around $4.30. Considering that $0.67 is attributed to labor, the price of jet fuel consumed per minute should be $3.63 ($4.30 – $.067). This cost relates to 30 pounds of jet fuel. Accordingly, the cost per gallon of jet fuel should be less than $0.80 to justify the current scenario over the proposed scenario.

Multiple tow-tugs

As explained earlier, the aircraft are primarily moved between the terminals and the maintenance hangar between 6 p.m. and 4 a.m. the next day. Owing to limited space availability at the hangar, only six aircraft can be accommodated at any time. These two restrictions probably explain for the low 30% utilization of the single tow-tug.

All the performance measures and the analyses for operating a single tow-tug were shared with the airline. The airline was interested in further investigating the impact of purchasing multiple tow-tugs and their economic justifications.

The simulation model was run with multiple tow-tugs. Figure 16.6 presents the average utilization of each tow-tug when multiple tow-tugs are used. As it was described earlier, the average utilization for a single tow-tug is 30%. This utilization drops to about 5% when five tow-tugs are used.

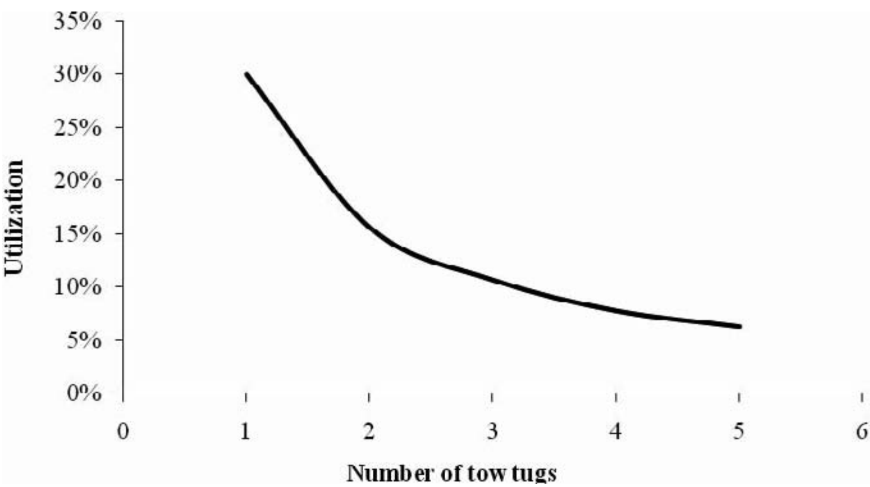

Figure 16.6 Average utilization with multiple tow tugs

The following figure represents the weekly total cost of operating multiple tow-tugs.

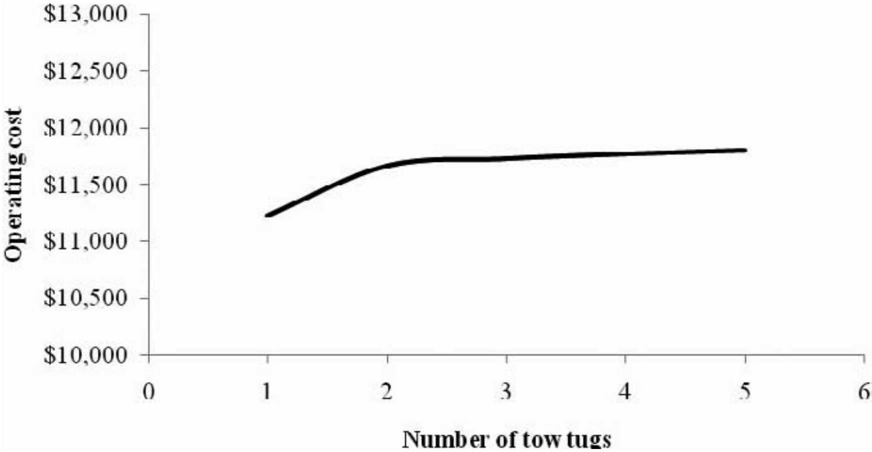

Figure 16.7 Total weekly operating cost in a multi tug operation

It should be noted that even though the operating costs of multi tow-tugs are comparable to a single one, their high purchasing prices and initial investment need to be addressed for any long-term financial analyses. The following table presents the payback period and NPV, when including the initial investment as the number of tow-tugs increases.

Table 16.2 Payback period and NPV for multiple tow-tugs

Number of tow-tugs	Payback period (years)	NPV
1	0.84	$1,249,829
2	1.87	$839,409
3	2.84	$573,148
4	3.82	$314,320
5	4.80	$56,185

The only advantage of having multiple tow-tugs is reduced waiting times at the gates for the aircraft to be moved to the maintenance hangar. Currently the wait time for a tow-tug is around 30 minutes, based on the airline's flight arrivals to ATL. Considering the only feasible parking space for the tow-tugs is outside the maintenance hangar (see Figure 16.1), this waiting time can be reduced to 23 minutes, which is the taxi time of the tow-tug from the hangar to the gates.

Therefore on average the waiting time is reduced by 7 minutes when using multiple tow-tugs. It should be noted that the aircraft needing the tow-tugs are to remain overnight at ATL. Therefore the minor saving in wait time by purchasing extra tow-tugs could not be economically justified.

Recommendation and Conclusion

The simulation analyses on single and multi tow-tugs identified potential financial incentives for utilizing them. Considering the low utilization for tow-tugs and their high initial investment, purchase of a single tow-tug was recommended to AirTran Airways. Multiple tow-tug operations are justified if:

- there are more than six available spaces for aircraft at the maintenance hangar;
- more aircraft remain over night at the airport;
- parking space for tow-tugs is available closer to the terminals;
- the tow-tugs can be used for more operations such as taking an empty aircraft to a parking space or relocating empty aircraft from gates.

Apart from economic incentives, utilizing the tow-tugs is more environmentally friendly than aircraft moving on their own powers for taxiing. On average an aircraft burns jet fuel at a rate of 4.55 gallons/minute when taxiing on its own powers. In comparison, a tow-tug burns diesel at a rate of .25 gallons/minute when towing an aircraft.

Following the study, AirTran purchased one tow-tug in 2006. Figure 16.8 shows the tow-tug in action.

Figure 16.8 A tow tug towing AirTran's 737-700 aircraft

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Chapter 17
Runway Capacity Planning

Introduction

Airports play a key role in the commercial aviation system by allowing airlines and their customers to converge. However, since the early 1970s, the peaking of traffic at airports has been a problem of increasing concern to airport operators around the world. Though the systems put in place by airports today are extensive and highly developed, the busiest airports still face the problems of congestion and delay. Facilities at most airports are not adequate enough to accommodate demand at all times and in all conditions of weather and visibility. The resulting delays lead to inefficiency and increased expenses to airlines, inconvenience and opportunity costs for passengers, and increased workload for the FAA air traffic control system. In fact, a lack of airport capacity has been forecast by the FAA to be one of the most serious constraints to the growth of commercial and private aviation (Wells 2000). Table 17.1 shows the percentage of on-time arrivals at major airports in the United States during 2008.

Table 17.1 Percentage of on-time arrivals at major airports in the US during 2008

Airport identifier	Airport	On-time arrival
ATL	Hartsfield-Jackson Atlanta International Airport	75.52%
BOS	Boston's Logan International Airport	73.36%
BWI	Baltimore Washington International Airport	80.31%
DEN	Denver International Airport	78.34%
DFW	Dallas Fort Worth International Airport	76.16%
LAS	McCarran International Airport	77.76%
LAX	Los Angeles International Airport	76.89%
LGA	LaGuardia Airport	62.80%
MCO	Orlando International Airport	77.81%
MDW	Chicago Midway Airport	80.68%

Source: Federal Aviation Administration.

According to the US Department of Transportation, in 2007 more than 166,000 commercial flights were delayed with a total of 100 million delayed minutes at a total cost of more than $40 billion to the airline industry and passengers. Non-weather related delays and specifically airport-related delays account for more than 54% of all delay costs reported. Figures 17.1 and 17.2 present the number of delayed flights and total minutes of delay respectively for both weather and non-weather-related delays from 2004 to 2008 as reported by the Department of Transportation (US Department of Transportation). These figures suggest an upward trend in both weather and non-weather delays. Airport-related delays represent a major contributor towards non-weather delays. As the figures suggest, the non-weather-related delays are growing at a faster and more alarming rate than weather-related delays. The average cost of delay per minute for commercial airlines in 2007 was $342.4 and is calculated in Table 17.2.

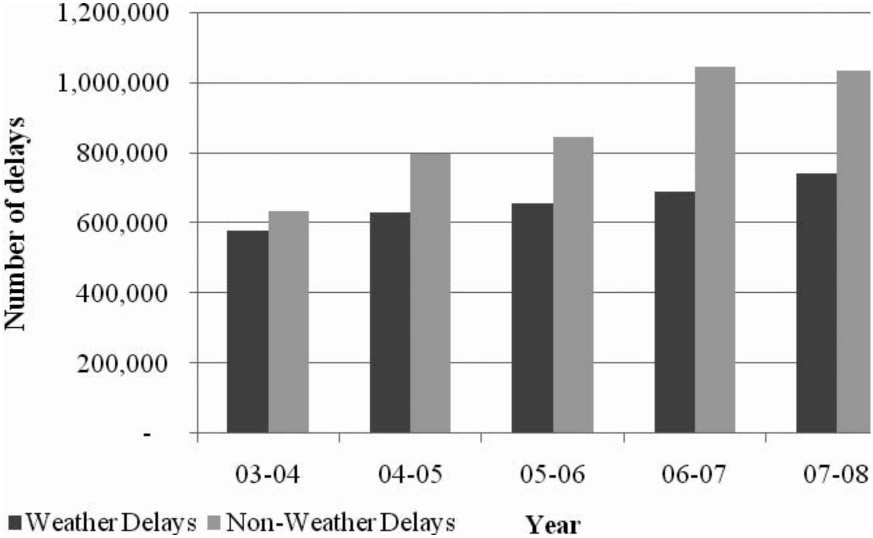

Figure 17.1 Number of weather and non-weather related delays from 2003–2008

One main reason for the lack of capacity and delay is that airport development projects are enormously capital-intensive and probably some of the largest infrastructure development projects that are undertaken. For example, the construction of a new runway at Lambert St. Louis International Airport involved costs of $1.1 billion, acquisition of more than 1,500 acres of land, reconfiguration of seven major roads and displacement of many homes, a school, and some airport-support operations (Cohen and Coughlin 2003). Hence it is a challenging task for airports to keep pace with the rapidly growing demand for air transport

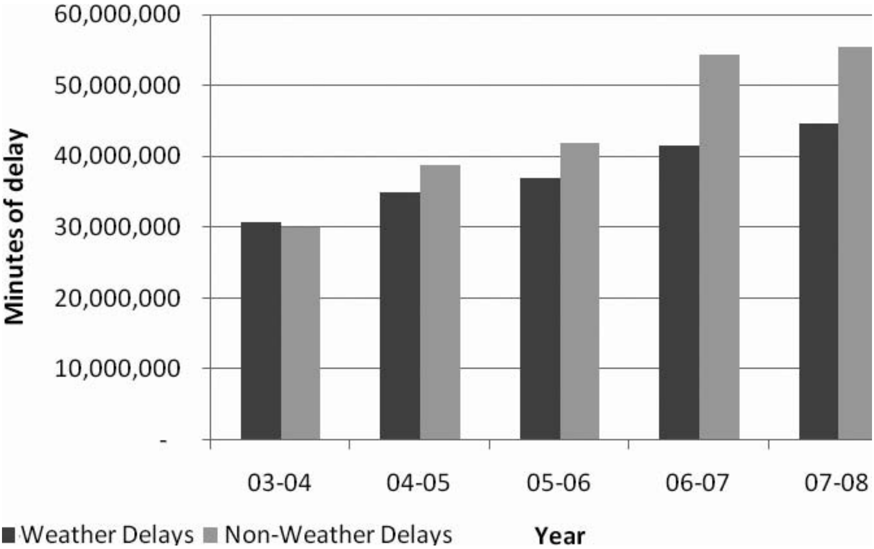

Figure 17.2 Total weather and non-weather related delay in minutes from 2003–2008

Table 17.2 Cost of delay per minute for commercial airlines during 2007

Cost item	Cost/minute
Airline operating costs	$160.7
Value of passengers time	$101
Spill over costs to economy	$80.7
Total	$342.4

Source: US Bureau of Transportation.

(Dempsey 2000). This fact also accentuates the importance of thorough analysis of the various options and their outcomes in the planning stage.

Therefore demand-capacity analysis, a vital component of the airport-planning process, is crucial in defining the physical requirement of airport facilities to meet future demand.

Airport facilities broadly include the airfield (runway, taxiway, gates), the terminal building, and airport access/parking facilities (Mumayiz 1999). Approaches to improving these facilities, thereby expanding airport capacity, may be categorized as:

- techniques to increase runway operation rate and hence augment airside capacity or mitigate aircraft delay;

- techniques to move the aircraft from the runway to the passenger-loading gates and back again as quickly as possible to shorten the taxi-in and taxi-out components of delay;
- techniques to aid in the transit of passengers through the terminal building and the flow of vehicles on airport circulation and access roads (Wells 2000).

A prerequisite to an airport-planning process is an evaluation of the existing operational environment. The next step would be to estimate the effect of proposed developments on the airport's performance. This is then compared with the performance of the existing system to justify the proposed developments. This chapter deals with runway capacity planning and the following sections describe, assess, and evaluate various runway layouts.

Airport Layouts in General

Most airport layouts and runway layouts are customized to represent the most useful configuration, given the airport environment. The airport's environment is characterized by (Wells 2000):

- Airfield characteristics: Basic determinants of the airfield's ability to accommodate different types of aircraft and the handling rate. These include the physical layout of the runways, taxiways, aprons, and so on.
- Airspace characteristics: The situational relationship of the airfield to other airports and to natural and manmade obstacles and the navigable airspace hence developed.
- Air traffic control: ATC rules and procedures.
- Meteorological conditions: Visual Meteorological Conditions (VMC), atmospheric conditions, which allow pilots to land and take-off visually and Instrument Meteorological Conditions (IMC), atmospheric conditions, which do not allow visual reference and require ATC rules and procedures for safe conduct of operations.
- Demand characteristics: The number of aircraft seeking service, their performance characteristics and their usage of the airport.

As a result, airport operations, including runway dependencies, airspace procedures and limitations, and other characteristics, are usually unique to every airport. A more generic description of runway configurations and their corresponding dependencies has been laid out by the FAA. These configurations include the following:

- single runway;
- close parallels (distance between runway centerlines less than 2,500 feet);

- intermediate parallels (distance between runway centerlines 2,500 – 4,300 feet);
- far parallels (distance between runway centerlines greater than 4,300 feet);
- dual lane (two pairs of close parallel runways separated by more than 4,300 feet).

Under instrument flight conditions, simultaneous independent approaches are permissible on far parallels. Intermediate parallels can employ simultaneous dependent approaches, requiring a diagonal separation between approaching aircraft. Close parallels are treated as a single runway and simultaneous operations are not permitted (Burnham, Hallock, and Greene 2001). Airport layouts may correspond with one of the above configurations or may be a combination of two or more of them.

Runway System Capacity

The Airports Council International (ACI) and International Air Transport Association (IATA) guidelines for airport capacity/demand management (1996) defines the most significant aspect of an airport's capacity, Runway System Capacity, as the hourly rate of aircraft operations which may be reasonably expected to be accommodated by a single runway or a combination of runways under given local conditions.

The Runway System Capacity is primarily dependent on the runway occupancy times of, and separation standards applied to, successive aircraft in the traffic mix. Other key items affecting runway capacity include availability of exit taxiways, especially that of high-speed exits that help minimize runway occupancy times of arriving aircraft; aircraft type/performance; traffic mix; Air Traffic Control (ATC) and wake vortex constraints on approach separation; weather conditions [Visual Meteorological Conditions (VMC)/Instrument Meteorological Conditions (IMC)]; spacing between parallel runways; intersecting point of intersecting runways; and whether the mode of operation is segregated or mixed.

To better explain the capacity measures introduced here, we may begin with the concepts of practical capacity λ_p. Practical capacity λ_p is defined as the number of operations that can be accommodated in a given time period, considering all constraints incumbent on the airport, and with no more than a given amount of delay (Wells 2000). On a typical delay curve, this may be depicted as in Figure 17.3 (Raguraman 1999). The key here is that capacity is determined at a given level of delay. This capacity level does not necessarily reflect the maximum throughput capacity of the runway configuration.

As an illustration of practical capacity, we may consider the following example. Let us assume that capacity at 10 minutes of delay is 100 movements per hour and that at 20 minutes of delay is 125 movements per hour. On a typical delay curve,

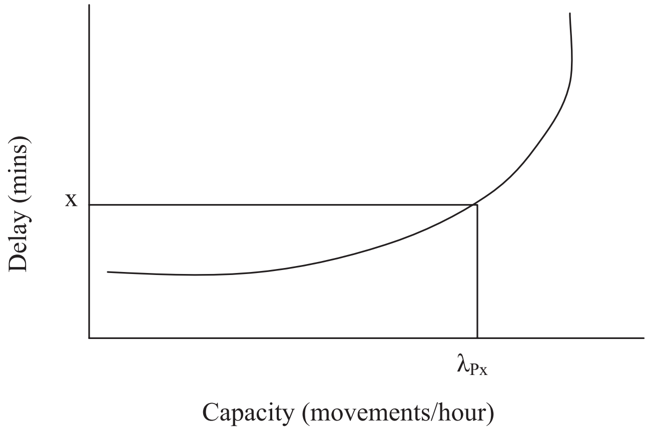

Figure 17.3 Practical capacity λ_P

this could be represented as in Figure 17.4. From the figure, it may be observed that, at 20 minutes of delay, the airport has almost reached its maximum capacity. However, capacity at 10 minutes of delay does not represent the maximum throughput capacity of the airport.

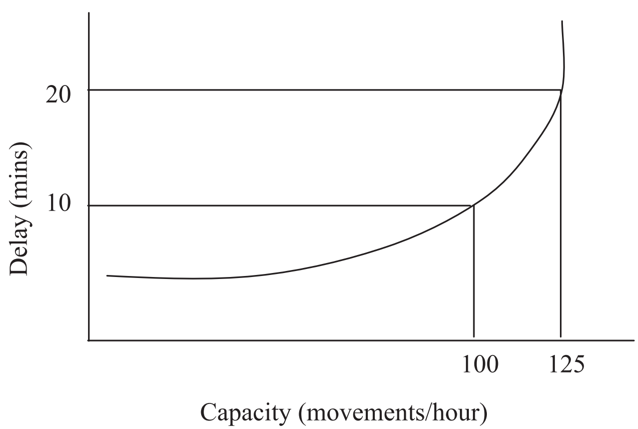

Figure 17.4 Example of practical capacity

Expanding on the concept of practical capacity, if we were to disregard delay, the airport's capacity would only increase until a certain maximum level. In the above example, this would be about 125 movements per hour. Every movement above this level in the same hour would contribute more to delay than to the airport's capacity. This level may be regarded as the point of negative returns, beyond which every additional movement would only contribute to the overall delay without improving capacity; this concept is called the maximum throughput capacity or saturation capacity λ_S. It can be measured as the number of operations that can be accomplished in a given period of time disregarding any delay that aircraft might experience and assuming that the aircraft will always be present, waiting to land or take-off (Wells 2000, Ashford and Wright 1992). This concept is depicted as in Figure 17.5. Put simply, this is the capacity level where the layout gets saturated.

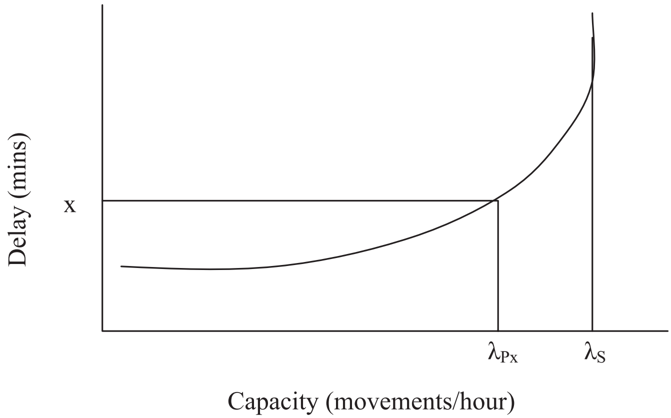

Figure 17.5 Saturation capacity λ_S

Saturation capacity λ_S is the key concept for this study and it is used for three different measures of capacity for each proposed runway configuration. The capacity measures differ in the sense that each one represents a capacity that has a separate set of constraints associated with it. Each of these is discussed below:

λ_{S1} Fully Constrained Capacity

Fully constrained capacity λ_{S1} takes into account all constraints that exist in an airport environment. These include both layout/ground factors as well as airspace factors. Ground constraints include the location of runway exits and taxiway and apron capacity. Airspace constraints arise from factors such as increased controller workloads owing to the absence of sufficient procedural and technological support. This measure of capacity is similar to what is described by Reynolds-Feighan and Button (1999) as ultimate capacity.

λ_{S2} Semi-Constrained Capacity

The second measure of capacity (λ_{S2}), which may also be called semi-constrained capacity, assumes that technological and procedural improvements are in place. These improvements aid in maintaining separation standards more precisely, thereby increasing runway throughput. However, the airport layout constraints discussed above are still considered in determining this measure of capacity.

λ_{SU}: Unconstrained Capacity

Finally, unconstrained capacity λ_{SU}, assumes away all constraints except those posed by safety requirements. These would broadly include separation standards established in order to allow for wake turbulence and runway occupancy rules. The concept of unconstrained capacity has been advanced by IATA and represents the maximum possible capacity of a given runway configuration (Pitfield and Jerrard, 1999).

These three concepts may be represented diagrammatically as in Figure 17.6. Again, note that each of these is essentially a saturation capacity. They fall on different curves because each one represents a different level of constraints on the system and is hence a separate scenario. As the constraints on the system decrease, capacity increases and the curve moves in the positive-x direction.

For example, for a particular layout, the fully constrained saturation capacity λ_{S1}, may be 110 movements per hour. For the same scenario, the semi-constrained capacity λ_{S2}, could be 130 movements per hour and the unconstrained capacity λ_{SU}, could be 160 movements per hour.

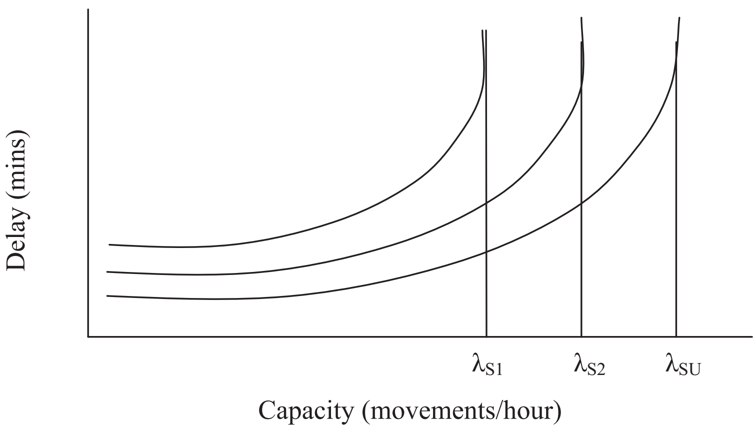

Figure 17.6 Capacity measures λ_{S1}, λ_{S2} and λ_{SU}

Capacity Estimation Models

Analytical and/or simulation models are mainly used to estimate capacities at an airport. Analytical models are mathematical representations of the airport, airspace characteristics and operations, and tend to have low levels of detail. Analytical models are mainly used for policy analysis, strategy development, and cost-benefit evaluation (Odoni et al. 1997). Some of the analytical models include Harris (1972), Odoni et al. (1997), Inniss (2000) and Janic (2009).

Simulation of the airport environment has been increasingly used recently to obtain more realistic estimates of capacity by randomizing the various input parameters. Fishburn and Stouppe (1997) have suggested that simulation modeling and analysis be integrated into the airport planning process rather than being simply used for final evaluations. Some of the simulation models used for capacity estimation include Pitfield and Jerrard (1999), Khoury, Kamat and Ioannou (2006) and Kageyama (2006).

The case study in this chapter (discussed later) employs a 3-D (dimensional) microscopic simulation model that is designed to simulate the traffic flows through the airport and airspace with regards to actual constraints and uncertainties.

Airport Layout Evaluation

To begin with, the three saturation capacity measures λ_i described in the earlier section are determined for each of the layouts.

$\lambda_{S1}/\lambda_{SU}$ Efficiency

$\lambda_{S1}/\lambda_{SU}$ indicates the runway system utilization owing to all constraints incumbent at an airport. This would show where the layout stands, in capacity terms, in light of its maximum potential. Hence, $[(\lambda_{SU}-\lambda_{S1}) / \lambda_{S1}]$ indicates the potential for maximum runway system utilization and is an index of the efficiency in terms of design functionality.

$\lambda_{S1}/\lambda_{S2}$ Sensitivity to Changes

$\lambda_{S1}/\lambda_{S2}$ provides an estimate of the utilization as a result of airspace constraints. Therefore, the sensitivity of the layout to technological and procedural changes that improve the traffic flow in and out of the airport is indicated by $[(\lambda_{S2}-\lambda_{S1}) / \lambda_{S1}]$.

$\lambda_{S2}/\lambda_{SU}$ Overall Utilization of Capacity

$\lambda_{S2}/\lambda_{SU}$ indicates the utilization constrained by the airport layout design factors affecting taxiing, gate usage, and so on, thus throwing light on the layout's functionality or what may be called its design efficiency. Here again, $[(\lambda_{SU}-\lambda_{S2}) / \lambda_{S2}]$ shows the potential for runway system utilization by improving airport design.

Total Airspace and Airport Modeler (TAAM)

TAAM is a leading simulation package for modeling entire air traffic systems offered by Jeppesen, a subsidiary of The Boeing Company. The model is a four-dimensional flight-path simulator and allows greater realism than mesh-based simulations such as SIMMOD (Odoni et al. 1997). A number of factors may be randomized in the simulation to reflect day-to-day fluctuations. A versatile simulation model, TAAM has been used in a wide variety of applications including airport capacity estimation (gate, taxiway, runway capacity), planning airport improvements, extensions, de-icing, noise impact, effect of severe weather, design of terminal area procedures (SIDs/STARs) and terminal area ATC sectors, controller workload assessment, impact of new ATC rules, system wide delays and cost/benefit studies.

Being a large scale simulation of an air traffic system, TAAM requires comprehensive input data files describing the entire Air Traffic system. The level of detail, however, is variable and can be adapted to suit individual project needs. Typical inputs include the airport layout, air traffic schedule, environment description, aircraft flight-plans and air-traffic control rules. These are used to investigate the usage of the airport and airspace, conflict-detection and resolution, and to compute aggregate metrics using TAAM's internal algorithms and user-specified rules (Odoni et al. 1997). These aggregated metrics include system delay and its distribution; costs; fuel, non-fuel, and total; airport movements; operations on taxiways and runways; runway occupancy and airspace-operation metrics such as usage of routes, sectors, fixes and coordination.

TAAM has been verified by many users on many different scenarios. TAAM simulation outputs have been compared with some FAA studies on aspects of new ATM concepts and have shown comparable results. In fact, the four-dimensional movement of aircraft can be simulated in TAAM to get within 3–4% of the actual aircraft profiles. Airport movement rates and other characteristics can be modeled with similar accuracy (Odoni et al. 1997).

Runway Capacity Planning at Philadelphia Airport

The FAA Capacity Benchmark Report (2001) estimated the current capacity benchmark at Philadelphia International Airport (PHL) to be 100–110 flights per hour in good weather (VFR conditions) and 91–96 flights (or fewer) per hour in adverse weather conditions (IFR conditions), which could include poor visibility or low cloud base. Figure 17.7 represents a westerly usage of the runways in VFR conditions. In this figure, the callouts provide the runway names. The arrows show the usage of the runways. An arrow toward a runway represents arrivals to that runway while an arrow away from the runway represents departures from it.

One of the current problems faced at PHL is that of significant delays. For example, in 2000, over 4% of all flights at Philadelphia experienced significant

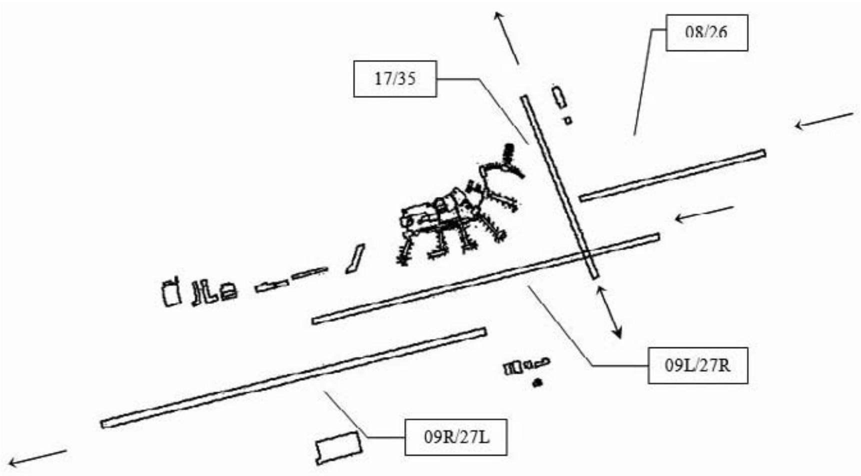

Figure 17.7 Current West-VFR operations at PHL

delay (defined by the FAA as more than 15 minutes of delay). Under adverse weather conditions, capacity is exceeded for about $3^1/_2$ hours of the day resulting in about 14% of the flights experiencing significant delay. Moreover, traffic at PHL is expected to increase by 23% over the next decade, which will further increase delays. The capacity estimates in the FAA report assume that the short runways 17/35 and 8/26 provide for 25% of airport traffic operations. The airport's capacity stands to decrease if this percentage declines (Federal Aviation Administration 2001).

Because of these current capacity problems, a number of enhancement initiatives are being undertaken by the airport authorities. Technological and procedural improvements to be implemented include:

- Automatic Dependent Surveillance-Broadcast/Cockpit Display of Traffic Information to help pilots maintain desired separations more precisely;
- Flight Management System/Area Navigation (FMS/RNAV) Routes to enable a more consistent flow of aircraft to the runway;
- Land and Hold Short Operations (LAHSO) allowing independent arrivals for specific aircraft types on intersecting runways;
- Precision Runway Monitor (PRM), a sophisticated radar system that allows simultaneous instrument approaches to parallel runways as close as 3,000 feet apart (Federal Aviation Administration 2001).

According to the Capacity Benchmark Report, these changes will improve Philadelphia's capacity in good weather by 17% (to 117–127 flights per hour) over the next 10 years, while capacity under adverse weather is expected to increase by 11% (to 101–106 flights per hour).

Besides these, major expansions involving the construction of new and/or expansion of existing runways and taxiways, improved and/or new terminal area and cargo handling facilities are being planned. These expansion plans may be categorized under two broad concepts: the parallel concept and the diagonal concept. The parallel concept is an extension of the current layout, and the diagonal concept involves a complete change of the layout including new runway orientations, new terminal area design, new apron and taxiway designs.

Under each of these concepts, two proposed layouts were chosen for the purpose of this case study. Therefore, in total, five layouts will be examined in this chapter – the baseline or the airport layout, as it exists, two parallel concept layouts and two diagonal concept layouts.

Parallel Concept: Full-Build Parallel Layout with Crosswind Runway (parallel-1)

As Figure 17.8 depicts, parallel-1 concept involves:

- shifting runway 09L/27R to the south and west to provide more taxiways closer to the apron area just above the runway;
- extending runways 17/35 and 08/26 to enable turboprops and jets other than wide-bodies to use these runways;
- constructing a new runway, 09R/27L, south of the airfield to be used as a departure runway;
- extending the existing southerly runway, 09R/27L, which would then be called 09C/27C. This would be the primary arrival runway.

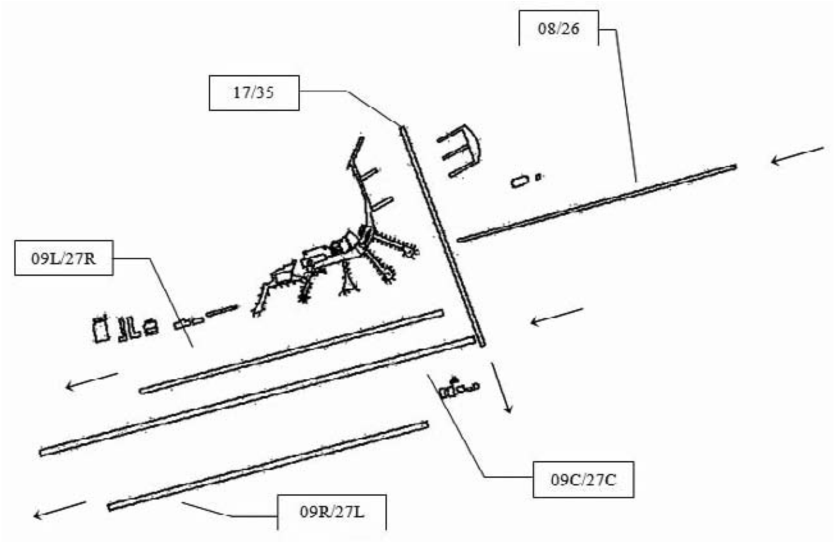

Figure 17.8 Parallel-1 West VFR operations

Parallel Concept: Baseline Layout with 4th Parallel Runway (parallel-2)

As Figure 17.9 depicts, this configuration is essentially the same as the parallel-1 except that:

- The crosswind runway, 17/35, would be converted to a taxiway in order to provide for easier taxiing to and from the northern aprons. Other advantages from avoiding the use of this runway would include the removal of the dependencies associated with it.
- 27R/09L would be as in the baseline scenario and not shifted south and west as in parallel-1.
- Runway 08/26 would not be built to the full length as in parallel-1 and would hence be unavailable for use by jets.

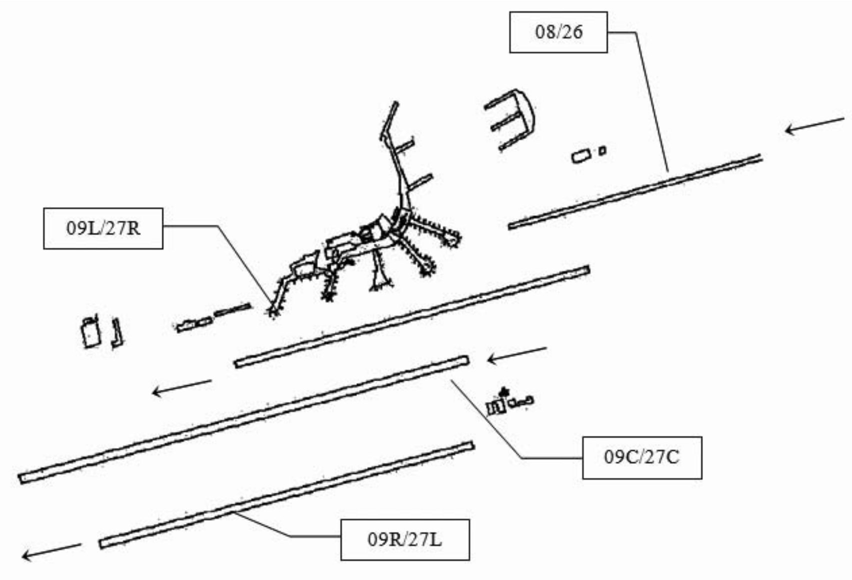

Figure 17.9 Parallel-2: West VFR operations

Diagonal Concept: Full-Build Diagonal Layout with 4 Runways (diagonal-1)

As Figure 17.10 represents, diagonal-1 concept involves:

- two new pairs of close parallel runways separated by more than 4,300 feet;
- the new runways that would be oriented 30 degrees clockwise from 09C/27C;

- the terminal area in this concept which would also be redesigned to a more symmetric one allowing more structured taxi patterns;
- the two inner runways, 11R/29L and 12L/30R, being used as departure runways;
- 11L/29R and 12R/30L, the two outer runways, being used as arrival runways.

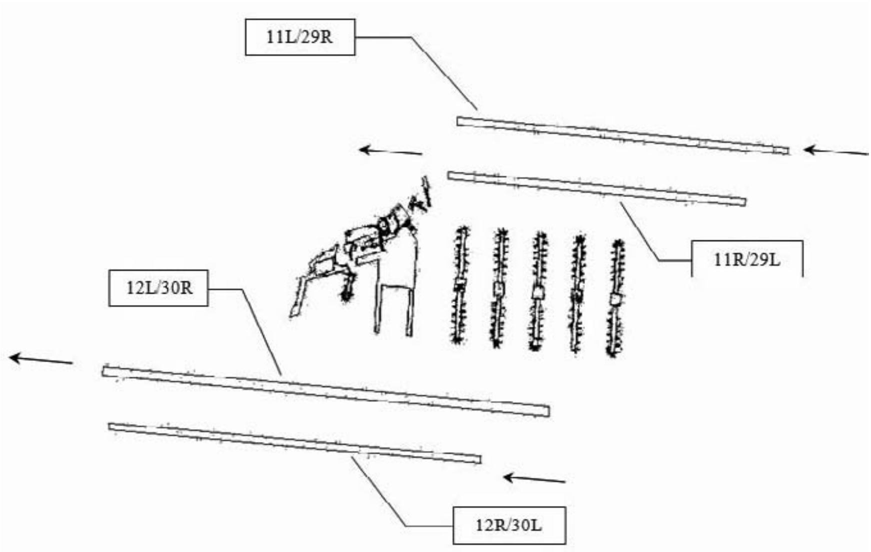

Figure 17.10 Diagonal-1: West VFR operations

Diagonal Concept: Full-Build Diagonal Layout with 3 Runways (diagonal-2)

As Figure 17.11 depicts:

- This configuration is the same as the diagonal-1 with the exception of the northernmost runway;
- Runway usage is similar to that of diagonal-1 with runway 11R/29L being used as a dual-use runway. Dual usage of a runway means the runway is used for arrivals as well as for departures. Departures are normally interleaved between arrivals.

Inputs

The inputs, common to all the scenarios evaluated, were the routes, airports, waypoints, and the traffic schedule. The routes, airports, and waypoints are files in TAAM format that represent those in the current National Airspace System (NAS).

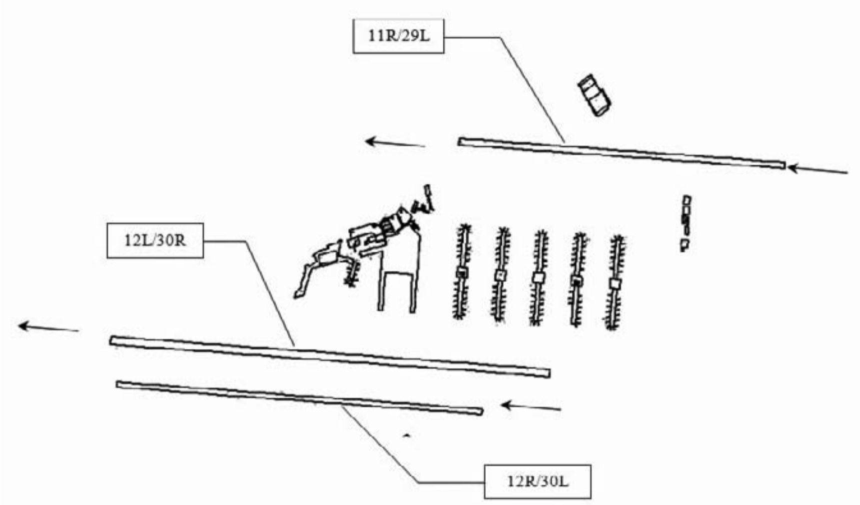

Figure 17.11 Diagonal-2: West VFR operations

To satisfy the assumption of an ever-present traffic flow the traffic schedule was restricted to a one-hour time frame with a total of 364 flights – equal arrivals and departures. The following represent the basis on which the schedule was generated:

- The traffic mix representing the forecast for the year 2020 for PHL was used.
- The arrivals, departures, and different types of aircraft were evenly distributed through the one-hour time period.
- The year 2020 was chosen, as this is the expected date of completion of the full-build layouts in either concept.

Inputs that were unique to each scenario included the airport layout, and rules governing the airport usage such as Air Traffic Control (ATC) and sequencing rules and taxiway, gate and runway-usage rules. Instrument Departure Procedure (DP)/Standard Terminal Arrival (STAR) were input to guide aircraft to and from the departure and arrival runways.

Simulation Results

Table 17.3 summarizes the saturation capacities under varying constraint levels for each of the scenarios evaluated.

The ratios computed from Table 17.3 are presented in Table 17.4.

Table 17.3 Saturation capacities under varying constraint levels for each of the scenarios

	λ_{S1}			λ_{S2}			λ_{SU}		
	Arrs	**Deps**	**All**	**Arrs**	**Deps**	**All**	**Arrs**	**Deps**	**All**
Baseline	56	61	117	69	61	130	69	67	136
Diagonal-1	77	86	163	84	88	172	84	90	174
Diagonal-2	75	62	137	78	65	143	78	69	147
Parallel-1	76	69	145	82	73	155	81	95	176
Parallel-2	77	69	146	80	70	150	80	82	162

Table 17.4 Ratios comparing the different layouts

	λ_{S1} vs. λ_{SU}		λ_{S1} vs. λ_{S2}		λ_{S2} vs. λ_{SU}	
	$\lambda_{S1}/\lambda_{SU}$	$[(\lambda_{SU}-\lambda_{S1})/\lambda_{S1}]$	$\lambda_{S1}/\lambda_{S2}$	$[(\lambda_{S2}-\lambda_{S1})/\lambda_{S1}]$	$\lambda_{S2}/\lambda_{SU}$	$[(\lambda_{SU}-\lambda_{S2})/\lambda_{S2}]$
Baseline	86%	16.2%	90%	11.1%	95.6%	4.6%
Diagonal-1	93.7%	6.7%	94.8%	5.5%	98.9%	1.2%
Diagonal-2	93.2%	7.3%	95.8%	4.4%	97.3%	2.8%
Parallel-1	82.4%	21.4%	93.5%	6.9%	88.1%	13.5%
Parallel-2	90.1%	11.0%	97.3%	2.7%	92.6%	8%

Comparisons and Conclusion

When the diagonal concepts are compared (diagonal-1 vs. diagonal-2), both layouts are largely similar with respect to the parameters evaluated. But diagonal-1 is marginally better than diagonal-2 with respect to runway-system capacity-utilization and efficiency in terms of taxiing and gate usage.

Between the parallel concepts (parallel-1 vs. parallel-2), parallel-2 is better than parallel-1 with respect to runway-system capacity-utilization and efficiency in terms of taxiing and gate usage. Probable reasons as observed from the simulation include the absence of the crosswind runway in parallel-2 and hence the elimination of related dependencies, and the use of the crosswind runway as taxiway, which provides for more efficient taxiing.

When the baseline is compared with the two proposed concepts (diagonal vs. parallel), the diagonal concept layouts were found to be better than either the baseline or the parallel concept layouts. Diagonal concept layouts were better in terms of runway system capacity utilization and efficiency in terms of taxiing and gate usage. This may be due to a more structured and symmetric taxiway and terminal design in the diagonal concept, which facilitates more structured flow of traffic on the ground. Besides, the fact that no runway crossing is required for departures ensures a continuous feed to the departure runways, which is not influenced by the arrival flow

Also, the baseline is better than either parallel concept layout with respect to design factors affecting taxiing and gate usage. This could be as a result of the constraints posed by the number of runways that departures have to cross in either parallel concept layout. For example, departures on 27L have to cross the departure runway 27R, as well as the arrival runway 27. In the event of continuous arrival and departure flows on these runways, the feed to 27L is greatly constrained. The solution to this would involve holding the departures on 27R and arrivals on 27 periodically in order to let aircraft cross these runways. However, this would negatively affect the overall runway system throughput.

Finally, parallel-1 and baseline are more sensitive to technological and procedural improvements. This is primarily caused by the use of the crosswind runway 17/35 in both these configurations. Using this runway imposes dependencies on arrivals and departures, which are eliminated in the other configurations.

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Chapter 18
Small Aircraft Transportation System (SATS)

Introduction

Similar to Chapter 17, this case study presents simulation study of an airport. The main objective of this case is to study the impact of additional aviation traffic flow on airport infrastructure. The following represents some recent and new trends that will affect the transportation industry in general and airlines in particular.

- Small Aircraft Transportation System (SATS) – SATS was originally proposed by the National Aeronautics and Space Administration (NASA). SATS represented an innovative program intended to provide travelers with a safe and affordable traveling alternative to current transportation systems. SATS will be discussed in more details later in this chapter.
- Fractional Ownership Programs – because of growing security issues with the airlines and airports, besides the presence of problems and delays caused by these security concerns, some companies have moved to Fractional Ownership Programs. In this program, companies with many business travelers jointly purchase and maintain small jet(s). Based on their contribution to this program (fraction), these companies will use the aircraft for their business travels. According to the FAA's definition, the fractional ownership program is possible when an individual or corporation purchases at least $^{1}/_{16}$ share of an airplane. The aircraft is then placed in a 'pool' to share with other owners of aircraft. The pooled aircraft are managed by a company that provides aviation management services with the necessary expertise for the owners.
- Air Taxis – Because of the growth in the demand of small jet aircraft, we have witnessed an increase in the number of their manufacturers. These companies utilizing advances in aircraft manufacturing, avionics, and falling component prices have been able to offer small jets (4–8 seaters) at very reasonable prices. A large number of entrepreneurs have placed orders for these aircraft to start up air-taxis in various parts of the nation. These start-up companies use regional airports and provide full service to their passengers. Some of these air-taxis promise a one-hour advance call for the service. Once the passenger(s) calls, the air taxi sends a car to pick-up the customer(s) from their home or work place. The car then drives the passenger(s) to the nearest regional airport. The waiting jet will fly the

passenger(s) to their destination (in most of the cases, another quiet regional airport). At the destination, the process repeats again, with a waiting car taking passenger(s) to their homes or businesses. All this at the price of a first-class airline ticket! It is anticipated that with increased competition, increased demand, and falling aircraft prices, this service will be offered at the current airline's economy fares.

Considering these trends, an important question is: can the existing airports and airspace accommodate such increased flow of aircraft? Will there be congestions and delays? The study presented in this chapter, primarily sponsored by the Florida Department of Transportation (FDOT), uses simulation to address the introduction of SATS and its integration with the current traffic for the Tallahassee Regional Airport for the years 2002–2025.

Small Aircraft Transportation System (SATS)

An efficient and reliable transportation system is the backbone of every successful economy (Ashford 1992, Wells 2000, Dempsey 2000). The demand for transportation continues to grow, while current highways and hub-and-spoke systems become more congested. Increasing congestion and delays continue in the current infrastructure, while national investments to reduce these issues are reaching a point of diminishing effectiveness. If these concerns are not addressed, delays in the hub–and-spoke system will limit economic activity to the few well-connected regions and communities. With 98% of the US population living within a 30-minute drive of over 5,000 public-use landing facilities, this infrastructure is an untapped national resource of mobility.

Introduction and commitment to the hub-and-spoke system of routing has focused the development of airports to major cities, increasing air traffic congestion to these specific regions (Reynolds-Feighan et al. 1999, Pitfield et al. 1999). Conversely, many rural airports and their communities have been suffering a lack of essential air service owing to the fact that it has not been financially viable for air carriers to serve these airports. As a result, many major city airports are heavily congested, operating at or above capacity and many rural city airports are increasingly underutilized. This trend has been apparent for some time and is becoming increasingly more significant.

The Small Aircraft Transportation System (SATS) was introduced as a solution to improve this imbalance by decreasing the congestion at major city airports, and improving rural airport utilization. With such vision, NASA, the US Department of Transportation (DOT), the Federal Aviation Administration (FAA), industry stakeholders, and academia joined forces to pursue the SATS viability. Its goal was to utilize next-generation technology, to improve travel between remote communities and urban transportation centers, and by using general aviation airports. Principled on a new generation of fully automated and affordable small aircraft, SATS would operate in a fully distributed system of small airports serving thousands of suburban, rural, and remote communities.

The small aircraft transportation vision was intended as a safe travel alternative, freeing people and products from transportation-system delays by creating access to more communities (for more information see http://sats.erau.edu).

The following represent some of the anticipated benefits of SATS:

- reduction of intercity travel cost on the order of half in many markets, while increasing the number of communities served by air transportation by more than ten-fold in the longer term;
- distribution of transportation capabilities;
- an alternative to delays imposed by grid-lock, hub-lock, and urban sprawl;
- the potential to ease some of the environmental impacts of the ever-expanding transportation consumption in the nation;
- an increase in the radius of action of daily life by ten-fold, the first increase of such magnitude since the cars replaced horses for intercity travel.

It is not known whether the current infrastructure of small rural airports is capable of accommodating SATS concept without encountering significant operational difficulties. The primary objective of this study is to analyze the operations of the integrated system, identify and evaluate the potential congestion points of airports, runways and terminals, and develop solutions to these problems.

Project Focus

The state of Florida was identified as one of the pioneer states to implement the SATS concept. Seven rural and regional airports including the Tallahassee regional airport in Florida were identified as potential and suitable airports for the SATS program. The Florida Department of Transportation (FDOT) showed their interest and sponsored a study to examine the integration of SATS with existing traffic to its regional airport at Tallahassee, the state capital. The study was not focused on the technological or economic feasibility of SATS, but on the operation side of it.[1] More specifically, the study should focus on the following for 2002–2025:

- How would the new integrated system perform? What are the unforeseen bottlenecks/problems that could occur?
- What is the impact of SATS on congestions at the airport?
- What facilities, if any, should be expanded or created in order to streamline the integration of SATS?
- How adversely will the existing air traffic be affected due to sudden and unanticipated SATS growth and expansion rates?

1 This study was conducted in 2000, and accordingly all data relate to that time frame.

In order to conduct this study, our first attempt was to estimate the future flow (growth) of existing traffic as well as SATS.

Future Traffic Flow Forecast for KTLH

Three independent aviation forecasts have been prepared for Tallahassee regional airport (KTLH). These forecasts do not consider SATS airplanes. These three forecasts are made by the FAA, an independent consulting group, and KTLH Management respectively. Figure 18.1 shows the growth of annual operations (without SATS) from 2000–2025 based on these three forecasts. These operations show the number of landings and departures per year. They include commercial, general, and military operations. Further research, meetings, and interviews prompted us to believe that the forecast made by the consulting group is more realistic.

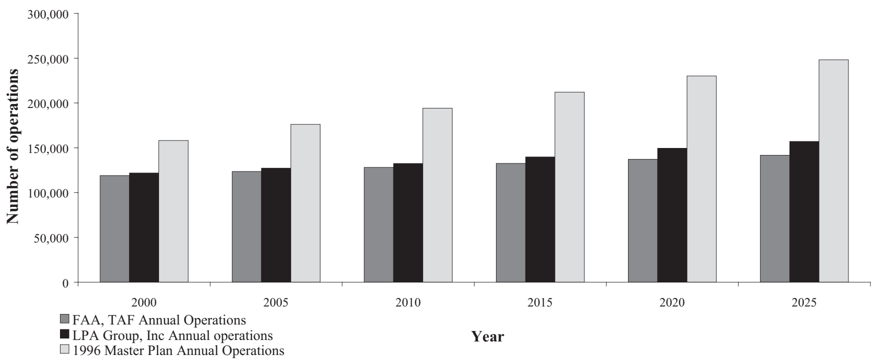

Figure 18.1 Forecasts for number of operations (landings and take-offs) at KTLH

SATS Traffic Flow Forecast

Currently, the scheduled implementation date of the first set of SATS flights is set for the year 2005. Based on NASA's analysis and forecasts, it is estimated that the total SATS operations at KTLH will be around 3,000 in year 2005. It is also anticipated that it will grow to maturity to 80,000 SATS operations in year 2025.

All new products and services typically follow a life-cycle graph (see Figure 18.2). With any new technology, in the beginning the demand is relatively low and grows at a slow rate; later on, the demand picks up and grows at a faster rate, until it reaches maturity where the demand either levels off or declines (Tidd et al. 2001). The SATS program was envisioned to follow this same cycle over the twenty-year projection 2005–2025. A life-cycle curve was fitted to the beginning

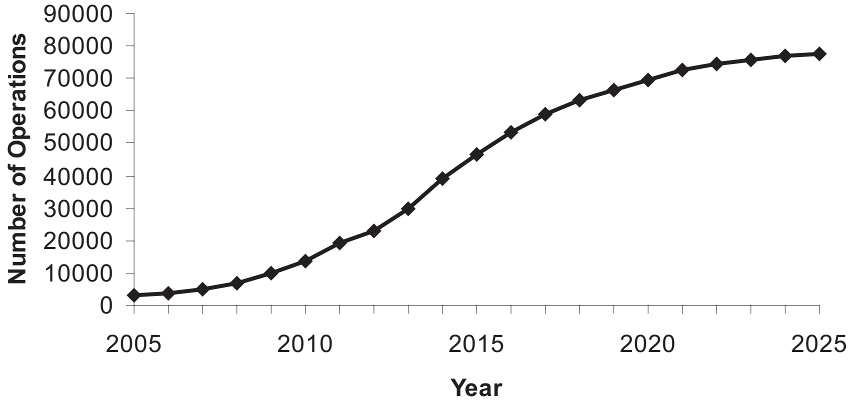

Figure 18.2 Life-cycle forecast for SATS demand at KTLH

(2005) and ending (2025) years of this program at KTLH. Figure 18.2 shows the forecast SATS operations over this 20-year time horizon.

Figure 18.3 presents the SATS operations, existing traffic and total future operation forecasts for KTLH from 2005–2025 (on a five-year basis).

The simulation modeling TAAM (Total Airspace and Airport Modeler) as was explained in Chapter 17, was adopted for this study.

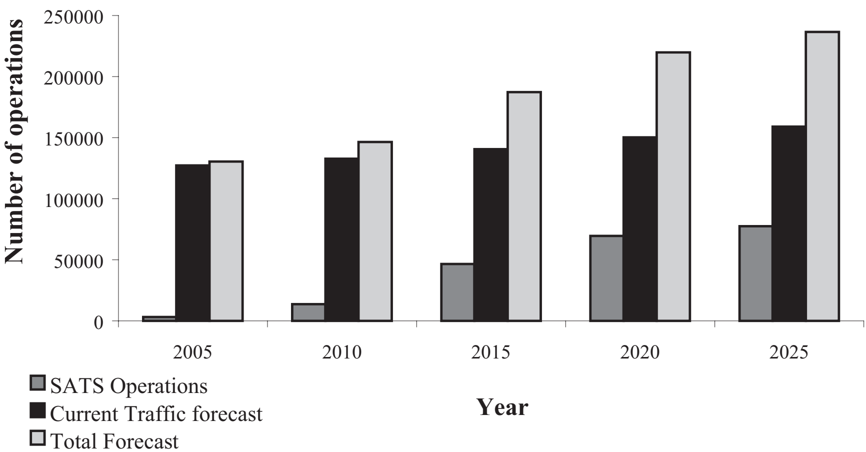

Figure 18.3 Forecast for SATS, existing and total operations for KTLH using Total Airspace Airport Modeler (TAAM)

Tallahassee Regional Airport (KTLH)

The Tallahassee market contains a total population of more than 1.4 million people. This includes Tallahassee, eleven neighboring Florida counties and twelve southern Georgia counties. The Tallahassee Regional Airport (KTLH) has two runways; runway 27 and runway 36. Figure 18.4 presents the layout of the runways and terminal for this airport.

Preliminary activities consisted of gathering all initial data and information about Tallahassee Regional Airport (KTLH) from the airport officials. A baseline layout was produced from drawings provided by KTLH. Gate allocation information was also gathered to develop accurate aircraft terminal parking rules. Information from the KTLH tower was used to design reasonable rule assumptions about accurate runway usage. These assumptions included gate and apron rules.

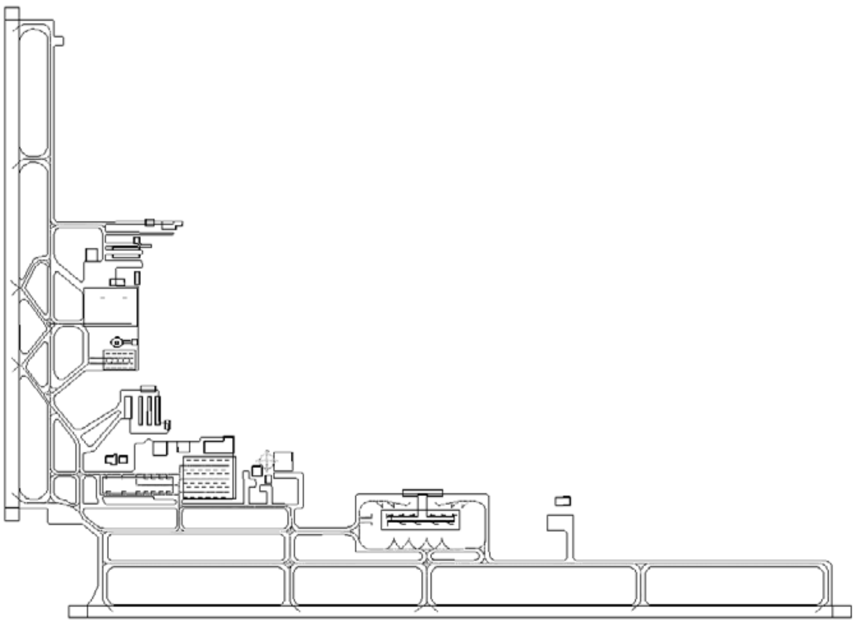

Figure 18.4 KTLH runway, taxiway, and terminal layout

Performance Measures

Similar to many other simulation software, TAAM also generates large amounts of output and reports. We were specifically interested in the following performance measures (Fishburn and Stouppe 1997):

- System Delays – arrival, departure, airspace, and total flight delays;
- Dissection of Delay on a per-aircraft basis;
- Peak Movement Rates – Peaks in arrival, departure and total flights;
- Runway Utilization Percentages by aircraft type and market segment.

Baseline Scenario

To capture the logic and to verify the accuracy of the model, initially a simulation model for a typical day in the year of 2002 (the year of this study) was developed. This model did not include any SATS operations. The intention was to check the validity of the model with the actual figures for KTLH. On a typical day in year 2002, there are around 300 daily operations. These operations include commercial, general, and military activities. Figure 18.5 presents the TAAM output report for the spread of these 300 operations over different times of the day. As the figure suggests the peak operation time at KTLH occurs between 15:00 and 16:00 with a total of 33 operations.

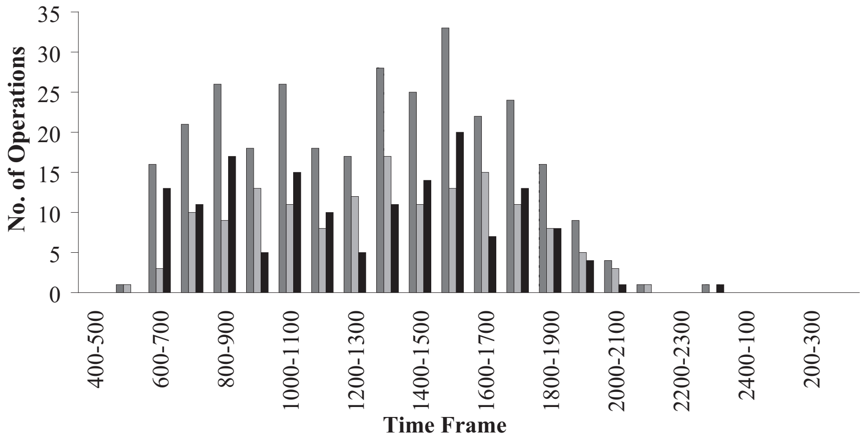

Figure 18.5 Daily arriving, departing, and total flight operation for baseline scenario

Figure 18.6 presents the TAAM report for the delay distribution over the time of the day. This figure shows the total delay in minutes that the arrival and departure flights experience. Note that these delays also consider and include airspace congestion around KTLH. This covers an area within a 20-nautical mile radius of the airfield. According to this figure the peak delay happens between 15:00 and 16:00. The total delay time for all arrivals and departures during this peak hour is 25 minutes. Returning to Figure 14.5 we have 33 operations during this peak hour. This means that during the peak hour, the flights experience on average a delay of less than one minute (25 minute/33 operations).

Figure 18.7 presents the dissection of delays at KTLH. This figure shows how many aircraft experience delays and by how much. According to this figure 250 out of 300 daily operations do not experience any delays at all. Fifty operations experience 0–3 minutes of delay and fewer than 10 operations experience 3–6 minute delays.

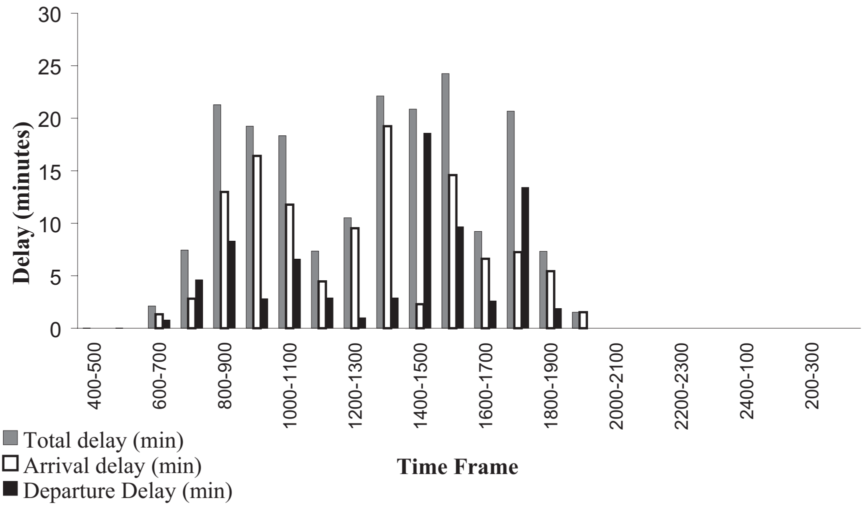

Figure 18.6 Delay distribution for baseline scenario at KTLH

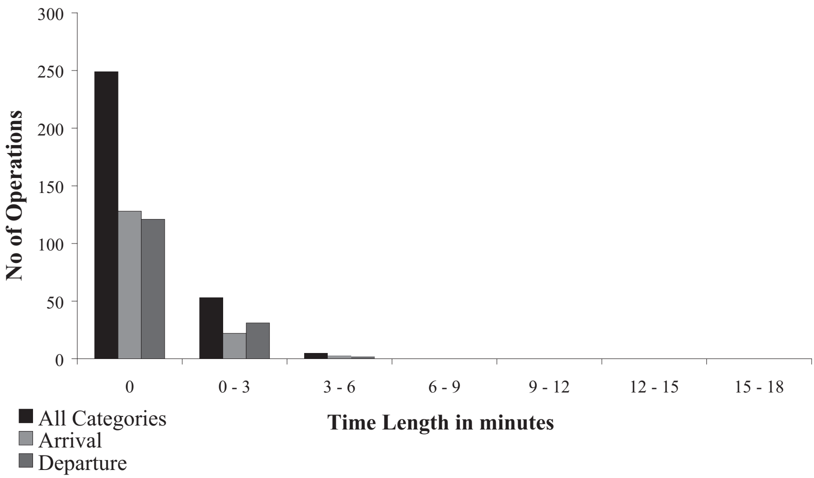

Figure 18.7 Dissection of delays at KTLH

Figure 18.8 presents the TAAM report for the number of arriving and departing flights using each of the two runways at the KTLH.

These results and figures were compared and verified with the actual data. As the figures suggest, KTLH is a quiet airport with no significant flight delays. The FAA defines a significant delay to be those arriving or departure flights that experience more than 15 minutes delay.

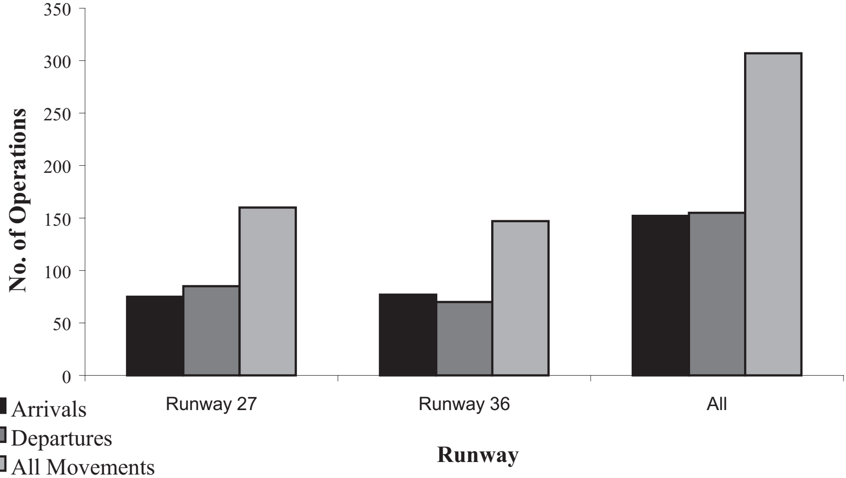

Figure 18.8 Runway usage at KTLH

Simulation Analysis for 2002–2025

Since the TAAM simulation model provided valid results for the baseline scenario, the simulation model was applied to airport operations in the future years. Forecast flow for the SATS and non-SATS traffic at KTLH as described earlier in this chapter and the same parameters such as air-traffic-control rules for arriving and departing flights were used in the future operations simulation. The only change to the model for the future years was the increased flow. Guidelines provided by the KTLH were used to disperse the increased daily flow over different times of the day. Similar performance measures were used to compare the results between the baseline scenario and future operations. The following presents the simulation reports for future operations.

Figure 18.9 presents the peak airport operations from 2002–2025. As was described earlier, the peak number for operations in 2002 is 33. This figure will double by 2025 when the current flow is increased and SATS is fully operational.

Figure 18.10 presents the total delay times in minutes during the peak hour for 2002–2025. According to this figure the peak total time delay in a typical day in 2025 is 150 minutes. This time represents a total delay for 63 flight operations (see Figure 18.9). This total delay translates into an average of less than 3 minutes during the peak time.

Figure 18.11 shows the dissection of delays. As the figure suggests there are no significant delays, or they are very minimal (less than two operations in 2025). More than half of the flights do not experience any delays. Again these delays incorporate both ground and airspace congestions.

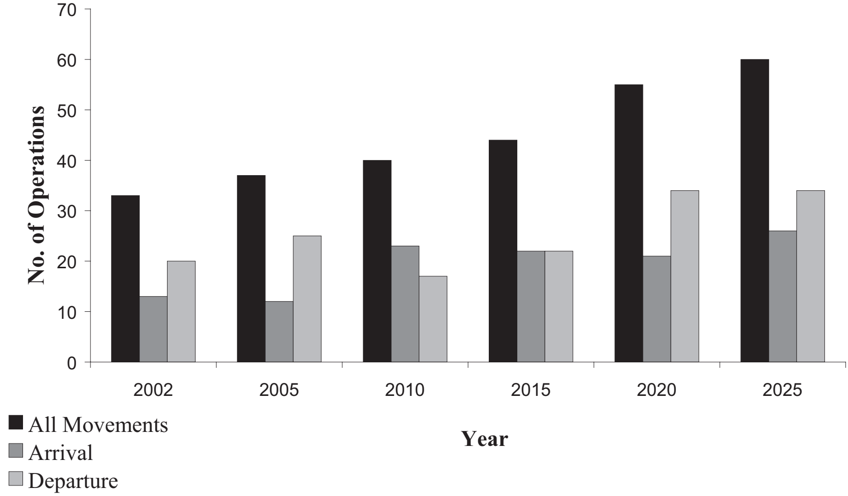

Figure 18.9 Change in peak hourly movements for 2002–2025 study time

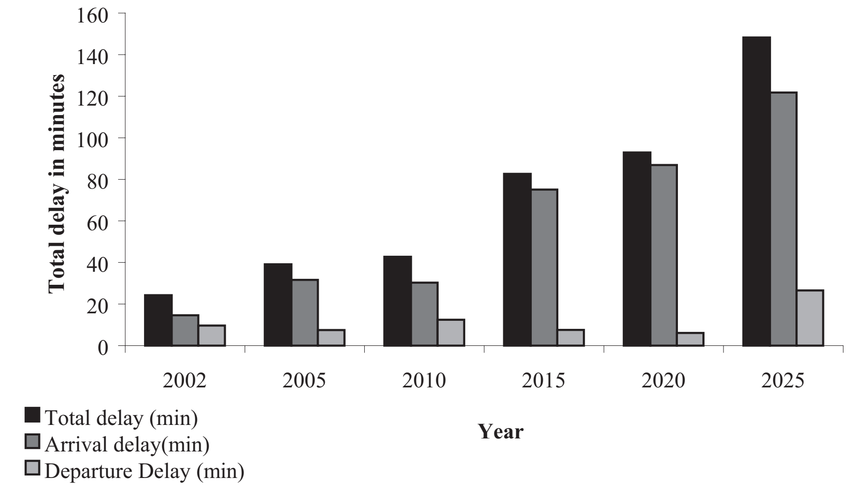

Figure 18.10 Changes in peak delay distribution time for 2002–2025

Finally Figure 18.12 shows the number of times each runway is used for the future operations.

As suggested by the above results of the simulation runs, the current infrastructure at Tallahassee Regional Airport is capable of successfully handling the increased traffic demands placed by the forecast increase in activity, as well as allowing for the smooth integration of the SATS program with existing commercial, general, cargo, and military operations.

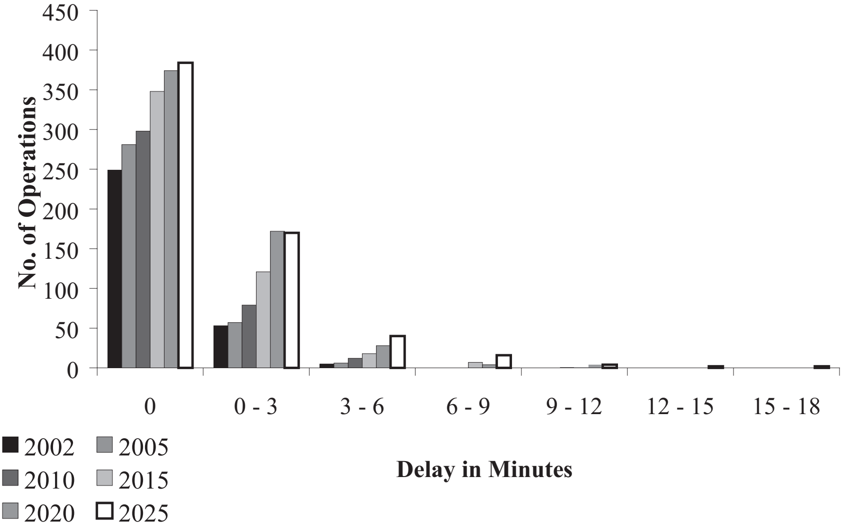

Figure 18.11 Change in dissection of delay 2002–2025

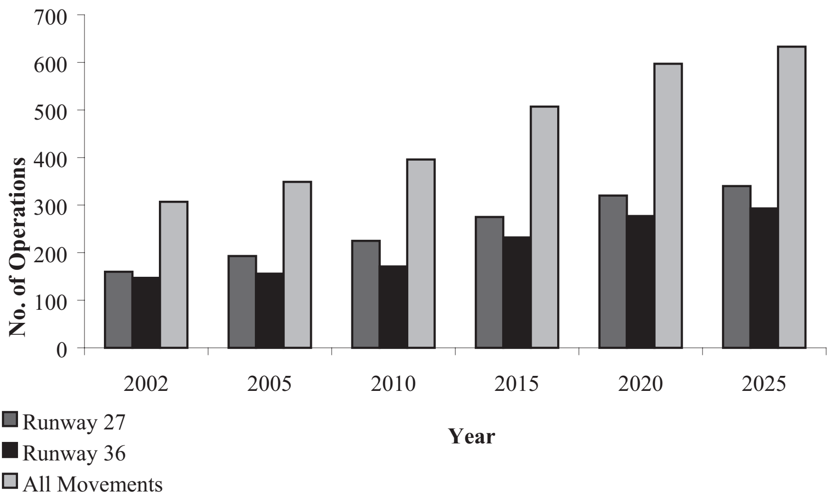

Figure 18.12 Change in runway utilization 2002–2025

Sensitivity Analysis

In an effort to study the KTLH operations under extreme and unanticipated conditions, we increased the flow of SATS flights in 2025 at peak hour by 40%. Even with these increased flows the flights that experienced significant delays (more than 15 minutes) at peak times remained as less than 2% of all flights. This confirms that the existing infrastructure at KTLH is able to handle unanticipated growth in SATS operations without major bottlenecks or significant delays.

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