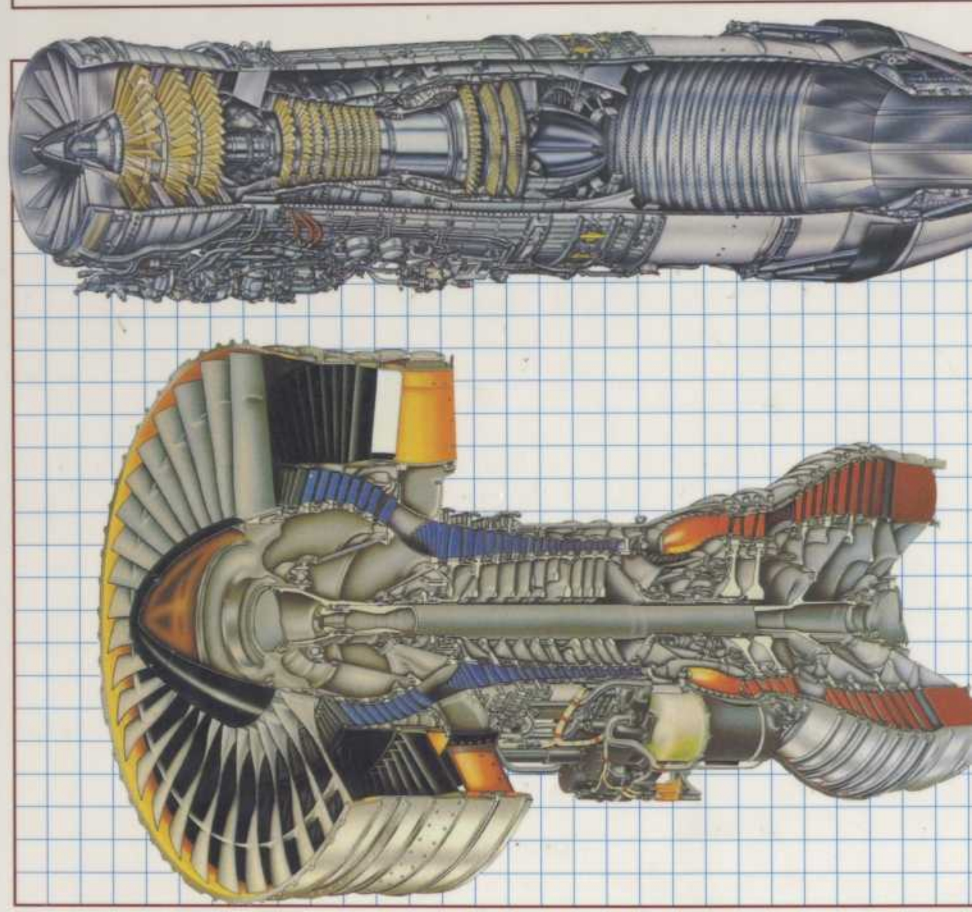
Elements of Gas Turbine Propulsion

Jack D. Mattingly



Foreword by
Hans von Ohain
German Inventor of the Jet Engine

Mattingly
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CHAPTER 1

INTRODUCTION

1-1 PROPULSION

The *Random House College Dictionary* (Ref. 1) defines *propulsion* as "the act of propelling, the state of being propelled, a propelling force or impulse" and defines the verb *propel* as "to drive, or cause to move, forward or onward." From these definitions, we can conclude that the study of propulsion includes the study of the propelling force, the motion caused, and the bodies involved. Propulsion involves an object to be propelled plus one or more additional bodies, called *propellant.*

The study of propulsion is concerned with vehicles such as automobiles, trains, ships, aircraft, and spacecraft. The focus of this textbook is on the propulsion of aircraft and spacecraft. Methods devised to produce a thrust force for the propulsion of a vehicle in flight are based on the principle of jet propulsion (the momentum change of a fluid by the propulsion system). The fluid may be the gas used by the engine itself (e.g., turbojet), it may be a fluid available in the surrounding environment (e.g., air used by a propeller), or it may be stored in the vehicle and carried by it during the flight (e.g., rocket).

Jet propulsion systems can be subdivided into two broad categories: air-breathing and non-air-breathing. Air-breathing propulsion systems include the reciprocating, turbojet, turbofan, ramjet, turboprop, and turboshaft engines. Non-air-breathing engines include rocket motors, nuclear propulsion systems, and electric propulsion systems. We focus on gas turbine propulsion systems (turbojet, turbofan, turboprop, and turboshaft engines) in this textbook.

The material in this textbook is divided into three parts:

- Basic concepts and one-dimensional gas dynamics
- Analysis and performance of air-breathing propulsion systems
- Analysis of gas turbine engine components

This chapter introduces the types of air-breathing and rocket propulsion systems and the basic propulsion performance parameters. Also included is an introduction to aircraft and rocket performance. The material on aircraft performance shows the influence of the gas turbine engine performance on the performance of the aircraft system. This material also permits incorporation of a gas turbine engine design problem such as new engines for an existing aircraft.

Numerous examples are included throughout this book to help students see the application of a concept after it is introduced. For some students, the material on basic concepts and gas dynamics will be a review of material covered in other courses they have already taken. For other students, this may be their first exposure to this material, and it may require more effort to understand.

1-2 UNITS AND DIMENSIONS

Since the engineering world uses both the metric SI and English unit system, both will be used in this textbook. One singular distinction exists between the English system and SI—the unit of force is defined in the former but derived in the latter. Newton's second law of motion relates force to mass, length, and time. It states that the sum of the forces is proportional to the rate of change of the momentum ($\mathbf{M} = m\mathbf{V}$). The constant of proportionality is $1/g_c$.

$$\sum \mathbf{F} = \frac{1}{g_c}\frac{d(m\mathbf{V})}{dt} = \frac{1}{g_c}\frac{d\mathbf{M}}{dt} \tag{1-1}$$

The units for each term in the above equation are listed in Table 1-1 for both SI and English units. In any unit system, only four of the five items in the table can be specified, and the latter is derived from Eq. (1-1).

As a result of selecting $g_c = 1$ and defining the units of mass, length, and time in SI units, the unit of force is derived from Eq. (1-1) as

TABLE 1-1
Units and dimensions

Unit system	Force	g_c	Mass	Length	Time
SI	Derived	1	Kilogram (kg)	Meter (m)	Second (sec)
English	Pound-force (lbf)	Derived	Pound-mass (lbm)	Foot (ft)	Second (sec)

kilogram-meters per square second (kg · m/sec²), which is called the *newton* (N). In English units, the value of g_c is derived from Eq. (1-1) as

$$\boxed{g_c = 32.174 \text{ ft} \cdot \text{lbm}/(\text{lbf} \cdot \text{sec}^2)}$$

Rather than adopt the convention used in many recent textbooks of developing material or use with *only* SI metric units ($g_c = 1$), we will maintain g_c in all our equations. Thus g_c will also show up in the equations for *potential energy* (PE) and *kinetic energy* (KE):

$$\text{PE} = \frac{mgz}{g_c}$$

$$\text{KE} = \frac{mV^2}{2g_c}$$

The total energy per unit mass e is the sum of the specific internal energy u, specific kinetic energy ke, and specific potential energy pe.

$$e \equiv u + \text{ke} + \text{pe} = u + \frac{V^2}{2g_c} + \frac{gz}{g_c}$$

There are a multitude of engineering units for the quantities of interest in propulsion. For example, energy can be expressed in the SI unit of *joule* (1 J = 1 N · m), in British thermal units (Btu's), or in foot-pound force (ft · lbf). One must be able to use the available data in the units provided and convert the units when required. Table 1-2 is a unit conversion table provided to help you in your endeavors.

TABLE 1-2
Unit conversion table

Length	1 m = 3.2808 ft = 39.37 in
	1 km = 0.621 mi
	1 mi = 5280 ft = 1.609 km
	1 nm = 6080 ft = 1.853 km
Area	1 m² = 10.764 ft²
	1 cm² = 0.155 in²
Volume	1 gal = 0.13368 ft³ = 3.785 L
	1 L = 10^{-3} m³ = 61.02 in³
Time	1 hr = 3600 sec = 60 min
Mass	1 kg = 1000 g = 2.2046 lbm = 6.8521 × 10^{-2} slug
	1 slug = 1 lbf · sec²/ft = 32.174 lbm
Density	1 slug/ft³ = 512.38 kg/m³
Force	1 N = 1 kg · m/sec²
	1 lbf = 4.448 N
Energy	1 J = 1 N · m = 1 kg · m²/sec²
	1 Btu = 778.16 ft · lbf = 252 cal = 1055 J
	1 cal = 4.186 J
	1 kJ = 0.947813 Btu = 0.23884 kcal

Power	1 W = 1 J/sec = 1 kg · m²/sec³
	1 hp = 550 ft · lbf/sec = 2545 Btu/hr = 745.7 W
	1 kW = 3412 Btu/hr = 1.341 hp
Pressure (stress)	1 atm = 14.696 lb/in² or psi = 760 torr = 101,325 Pa
	1 atm = 30.0 inHg = 407.2 inH_2O
	1 ksi = 1000 psi
	1 mmHg = 0.01934 psi = 1 torr
	1 Pa = 1 N/m²
	1 inHg = 3376.8 Pa
Energy per unit mass	1 kJ/kg = 0.4299 Btu/lbm
Specific heat	1 kJ/(kg · °C) = 0.23884 Btu/(lbm · °F)
Temperature	1 K = 1.8°R
	K = 273.15 + °C
	°R = 459.69 + °F
Temperature change	1°C = 1.8°F
Specific thrust	1 lbf/(lbm/sec) = 9.8067 N/(kg/sec)
Specific power	1 hp/(lbm/sec) = 1.644 kW/(kg/sec)
Thrust specific fuel consumption (TSFC)	1 lbm/(lbf · hr) = 28.325 mg/(N · sec)
Power specific fuel consumption	1 lbm/(hp · hr) = 168.97 mg/(kW · sec)
Strength/weight ratio (σ/ρ)	1 ksi/(slug/ft³) = 144 ft²/sec² = 13.38 m²/sec²

1-3 OPERATIONAL ENVELOPES AND STANDARD ATMOSPHERE

Each engine type will operate only within a certain range of altitudes and Mach numbers (velocities). Similar limitations in velocity and altitude exist for airframes. It is necessary, therefore, to match airframe and propulsion system capabilities. Figure 1-1 shows the approximate velocity and altitude limits, or *corridor of flight,* within which airlift vehicles can operate. The corridor is bounded by a *lift limit,* a *temperature limit,* and an *aerodynamic force limit.* The lift limit is determined by the maximum level-flight altitude at a given velocity. The temperature limit is set by the structural thermal limits of the material used in construction of the aircraft. At any given altitude, the maximum velocity attained is temperature-limited by aerodynamic heating effects. At lower altitudes, velocity is limited by aerodynamic force loads rather than by temperature.

The operating regions of all aircraft lie within the flight corridor. The operating region of a particular aircraft within the corridor is determined by aircraft design, but it is a very small portion of the overall corridor. Superimposed on the flight corridor in Fig. 1-1 are the operational envelopes of various powered aircraft. The operational limits of each propulsion system are determined by limitations of the components of the propulsion system and are shown in Fig. 1-2.

The analyses presented in this text use the properties of the atmosphere to determine both engine and airframe performance. Since these properties vary with location, season, time of day, etc., we will use the U.S. standard

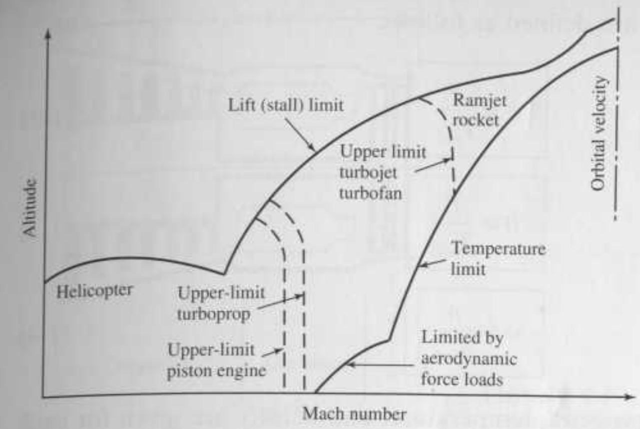


FIGURE 1-1
Flight limits.

atmosphere (Ref. 2) to give a known foundation for our analyses. Appendix A gives the properties of the U.S. standard atmosphere, 1976, in both English and SI units. Values of the pressure P, temperature T, density ρ, and speed of sound a are given in dimensionless ratios of the property at altitude to its value at sea level (SL), (the reference value). The dimensionless ratios of pressure, temperature, and density are given the symbols δ, θ, and σ,

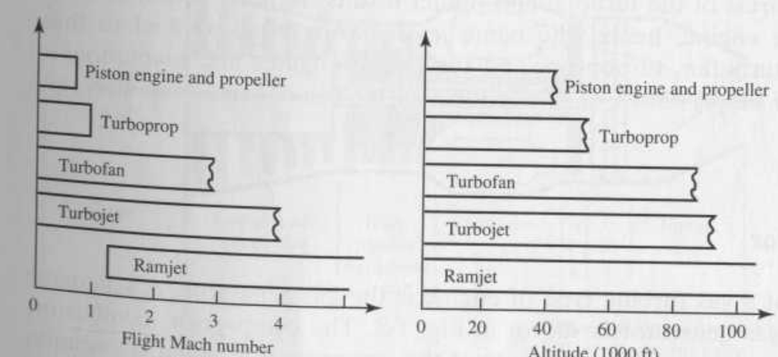


FIGURE 1-2
Engine operational limits.

respectively. These ratios are defined as follows:

$$\delta \equiv \frac{P}{P_{\text{ref}}} \tag{1-2}$$

$$\theta \equiv \frac{T}{T_{\text{ref}}} \tag{1-3}$$

$$\sigma \equiv \frac{\rho}{\rho_{\text{ref}}} \tag{1-4}$$

The reference values of pressure, temperature, and density are given for each unit system at the end of its property table.

For nonstandard conditions such as a hot day, the normal procedure is to use the standard pressure and correct the density, using the perfect gas relationship $\sigma = \delta/\theta$. As an example, we consider a 100°F day at 4-kft altitude. From App. A, we have $\delta = 0.8637$ for the 4-kft altitude. We calculate θ, using the 100°F temperature; $\theta = T/T_{\text{ref}} = (100 + 459.7)/518.7 = 1.079$. Note that absolute temperatures must be used in calculating θ. Then the density ratio is calculated using $\sigma = \delta/\theta = 0.8637/1.079 = 0.8005$.

1-4 AIR-BREATHING ENGINES

The turbojet, turbofan, turboprop, turboshaft, and ramjet engine systems are discussed in this part of Chap. 1. The discussion of these engines is in the context of providing thrust for aircraft. The listed engines are not all the engine types (reciprocating, rockets, combination types, etc.) that are used in providing propulsive thrust to aircraft, nor are they used exclusively on aircraft. The thrust of the turbojet and ramjet results from the action of a fluid jet leaving the engine; hence, the name *jet engine* is often applied to these engines. The turbofan, turboprop, and turboshaft engines are adaptations of the turbojet to supply thrust or power through the use of fans, propellers, and shafts.

Gas Generator

The "heart" of a gas turbine type of engine is the gas generator. A schematic diagram of a gas generator is shown in Fig. 1-3. The compressor, combustor, and turbine are the major components of the gas generator which is common to the turbojet, turbofan, turboprop, and turboshaft engines. The purpose of a gas generator is to supply high-temperature and high-pressure gas.

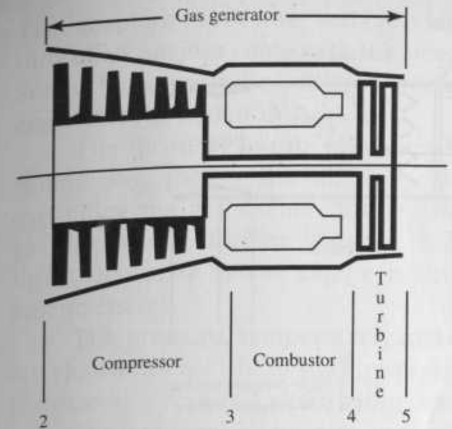


FIGURE 1-3
Schematic diagram of gas generator.

The Turbojet

By adding an inlet and a nozzle to the gas generator, a turbojet engine can be constructed. A schematic diagram of a simple turbojet is shown in Fig. 1-4*a*, and a turbojet with afterburner is shown in Fig. 1-4*b*. In the analysis of a turbojet engine, the major components are treated as sections. Also shown in Figs. 1-4*a* and 1-4*b* are the station numbers for each section.

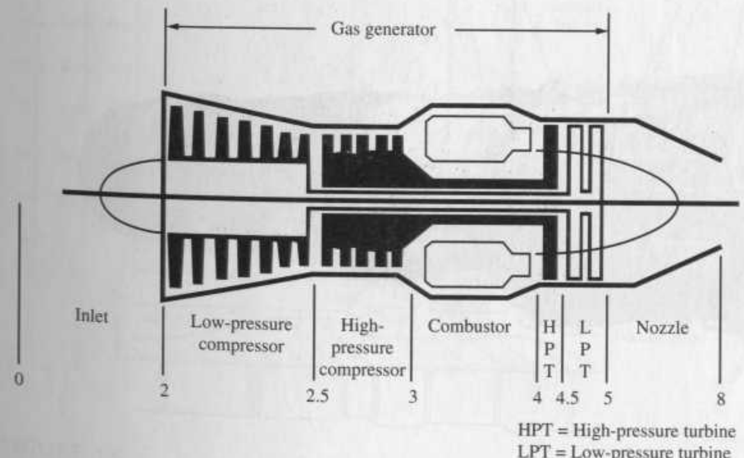


FIGURE 1-4*a*
Schematic diagram of a turbojet (dual axial compressor and turbine).

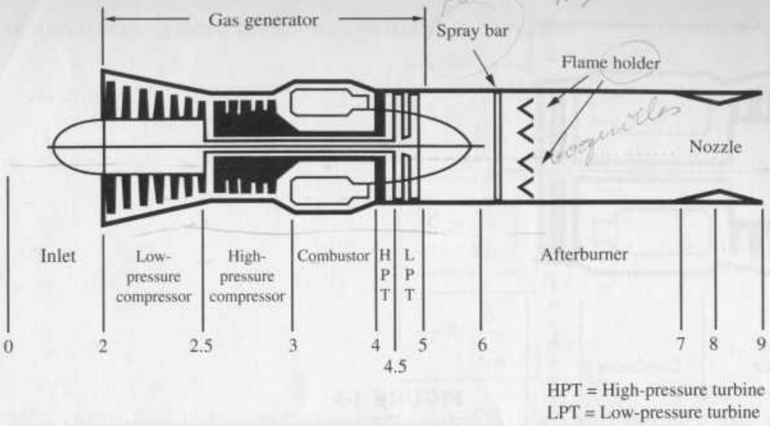


FIGURE 1-4*b*
Schematic diagram of a turbojet with afterburner.

The turbojet was first used as a means of aircraft propulsion by von Ohain (first flight August 27, 1939) and Whittle (first flight May 15, 1941). As development proceeded, the turbojet engine became more efficient and replaced some of the piston engines. A photograph of the J79 turbojet with afterburner used in the F-4 Phantom II and B-58 Hustler is shown in Fig. 1-5.

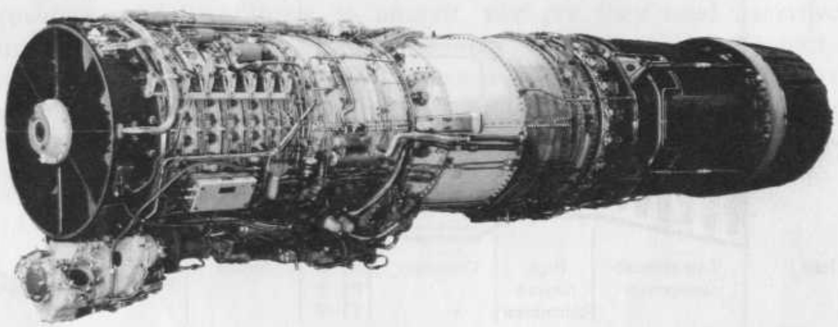


FIGURE 1-5
General Electric J79 turbojet with afterburner. (*Courtesy of General Electric Aircraft Engines.*)

The adaptations of the turbojet in the form of turbofan, turboprop, and turboshaft engines came with the need for more thrust at relatively low speeds. Some characteristics of different turbojet, turbofan, turboprop, and turboshaft engines are included in App. B.

The thrust of a turbojet is developed by compressing air in the inlet and compressor, mixing the air with fuel and burning in the combustor, and expanding the gas stream through the turbine and nozzle. The expansion of gas through the turbine supplies the power to turn the compressor. The net thrust delivered by the engine is the result of converting internal energy to kinetic energy.

The pressure, temperature, and velocity variations through a J79 engine are shown in Fig. 1-6. In the compressor section, the pressure and temperature increase as a result of work being done on the air. The temperature of the gas is further increased by burning in the combustor. In the turbine section, energy is being removed from the gas stream and converted to shaft power to turn the compressor. The energy is removed by an expansion process which results in a decrease of temperature and pressure. In the nozzle, the gas stream is further expanded to produce a high exit kinetic energy. All the sections of the engine must operate in such a way as to efficiently produce the greatest amount of thrust for a minimum of weight.

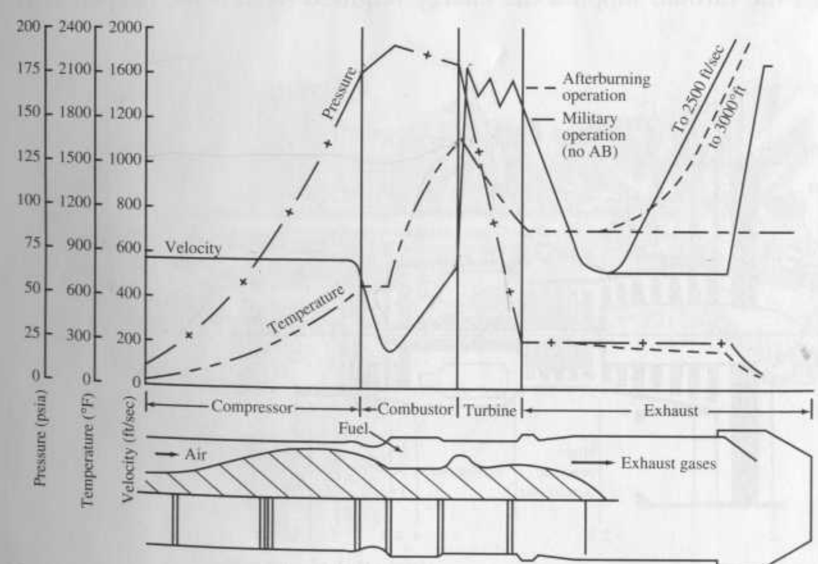


FIGURE 1-6
Property variation through the General Electric J79 afterburning turbojet engine.

The Turbofan

The turbofan engine consists of an inlet, fan, gas generator, and nozzle. A schematic diagram of a turbofan is shown in Fig. 1-7. In the turbofan, a portion of the turbine work is used to supply power to the fan. Generally the turbofan engine is more economical and efficient than the turbojet engine in a limited realm of flight. The *thrust specific fuel consumption* (TSFC, or fuel mass flow rate per unit thrust) is lower for turbofans and indicates a more economical operation. The turbofan also accelerates a larger mass of air to a lower velocity than a turbojet for a higher propulsive efficiency. The frontal area of a turbofan is quite large compared to that of a turbojet, and for this reason more drag and more weight result. The fan diameter is also limited aerodynamically when compressibility effects occur. Several of the current high-bypass-ratio turbofan engines used in subsonic aircraft are shown in Figs. 1-8*a* through 1-8*f*.

Figures 1-9*a* and 1-9*b* show the Pratt & Whitney F100 turbofan and the General Electric F110 turbofan, respectively. These afterburning turbofan engines are used in the F15 Eagle and F16 Falcon supersonic fighter aircraft. In this turbofan, the bypass stream is mixed with the core stream before passing through a common afterburner and exhaust nozzle.

The Turboprop and Turboshaft

A gas generator that drives a propeller is a turboprop engine. The expansion of gas through the turbine supplies the energy required to turn the propeller. A

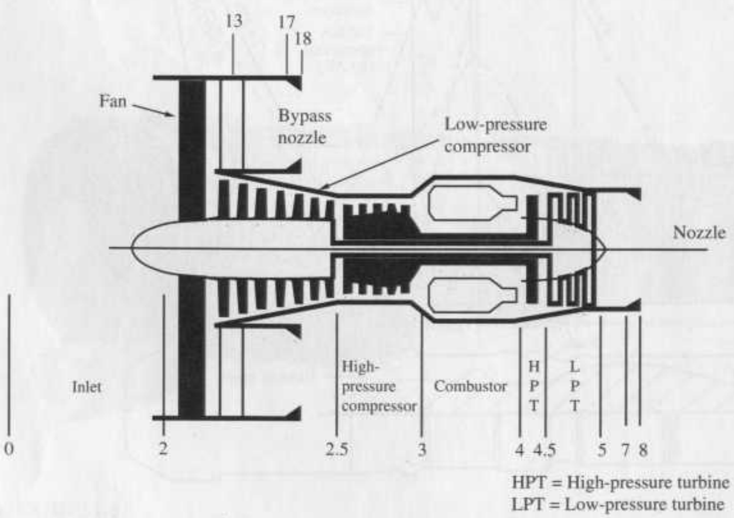


FIGURE 1-7
Schematic diagram of a high-bypass-ratio turbofan.

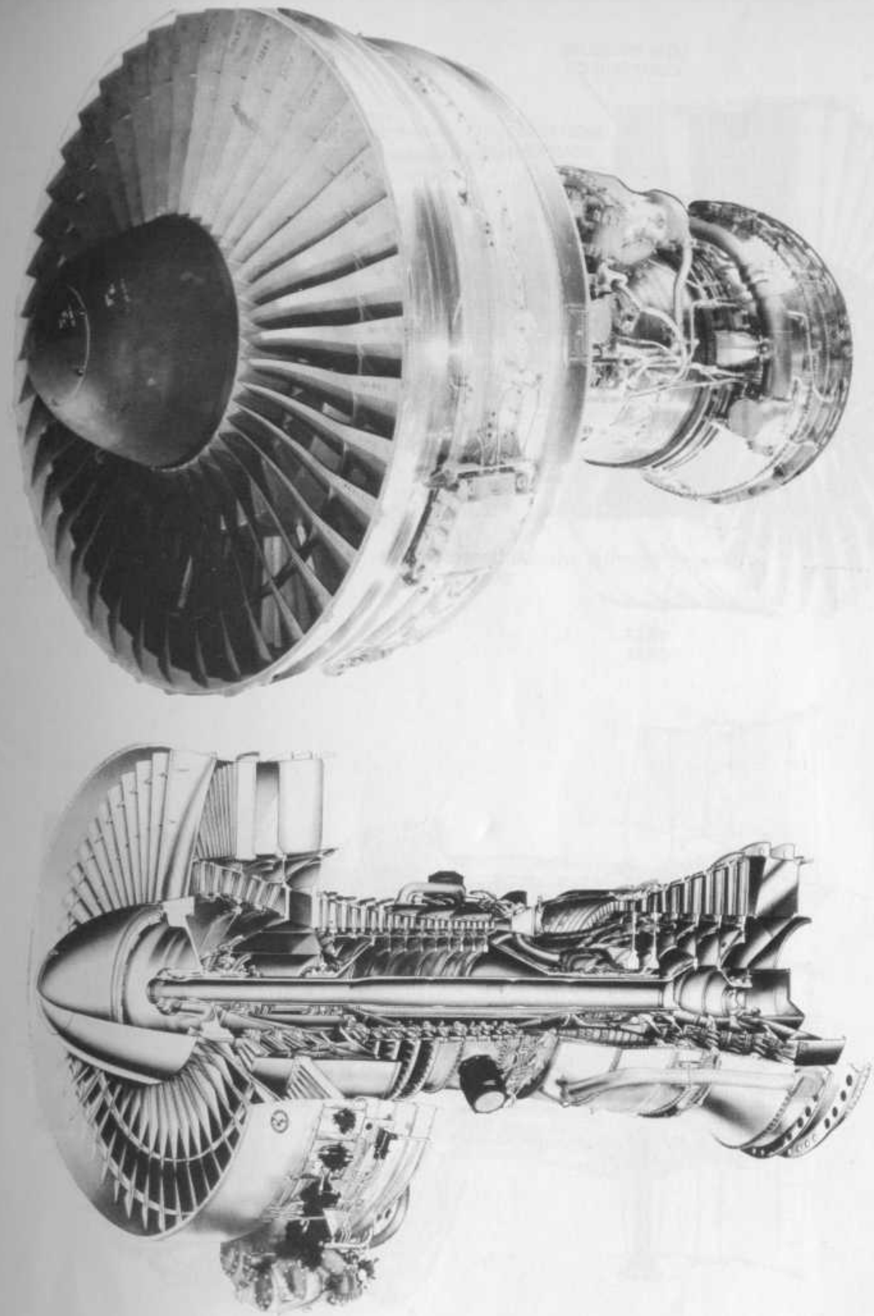
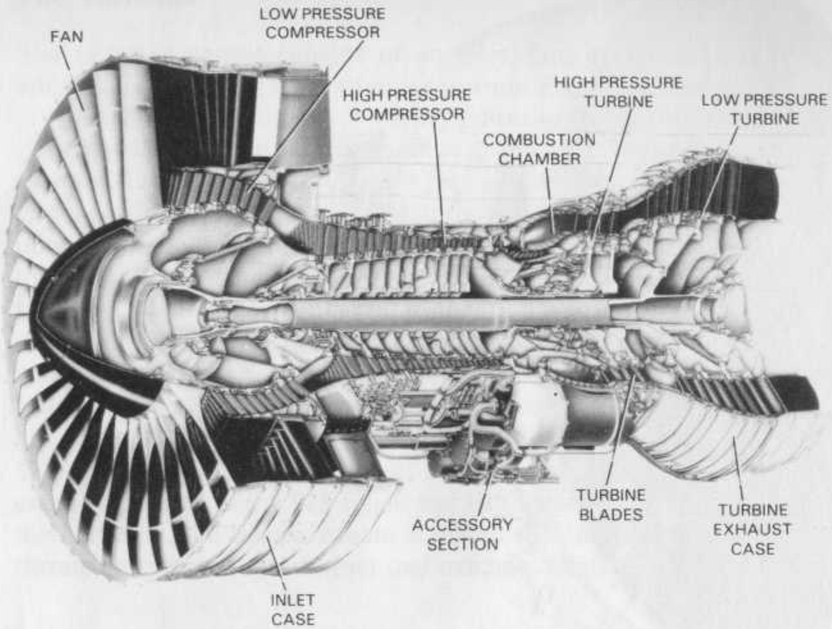


FIGURE 1-8*a*
Pratt & Whitney JT9D turbofan. (*Courtesy of Pratt & Whitney.*)



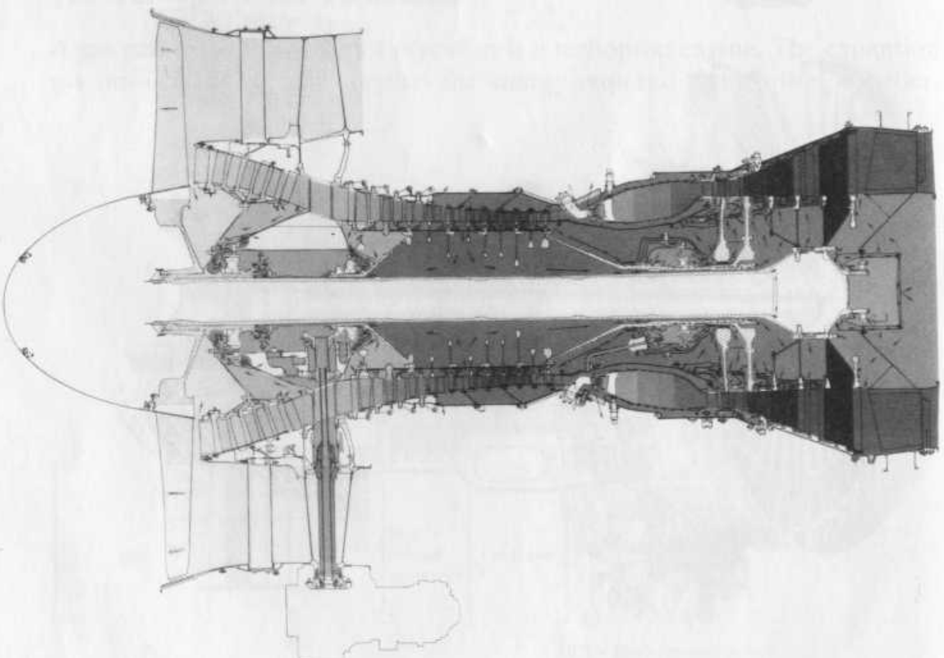


FIGURE 1-8*b*
Pratt & Whitney PW4000 turbofan. (*Courtesy of Pratt & Whitney.*)

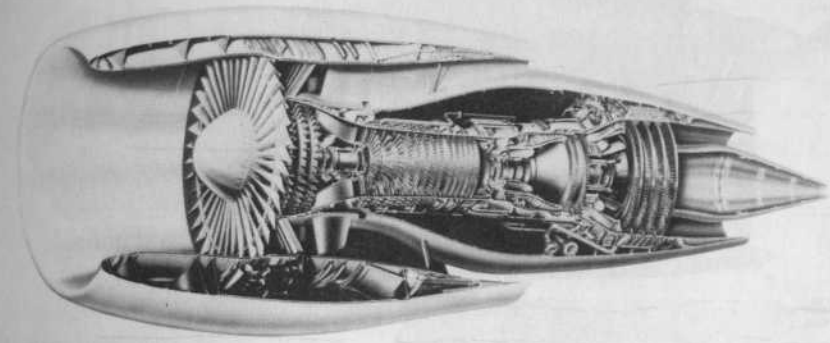


FIGURE 1-8*c*
General Electric CF6 turbofan. (*Courtesy of General Electric Aircraft Engines.*)

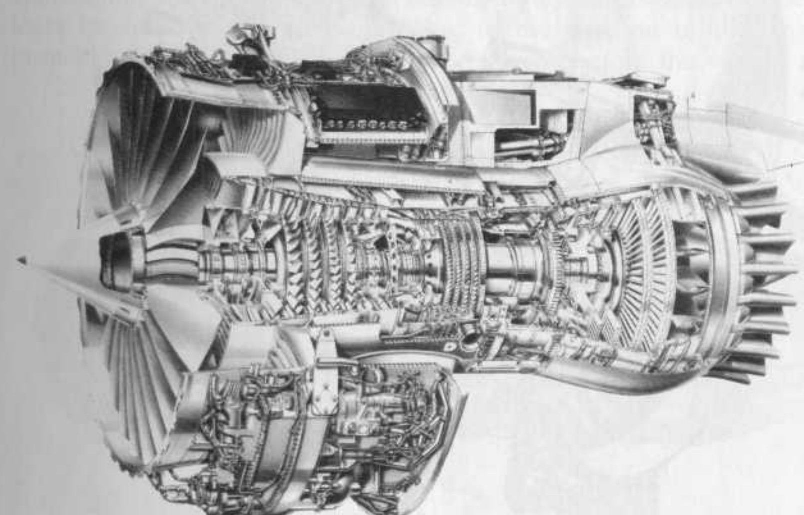


FIGURE 1-8*d*
Rolls-Royce RB-211-524G/H turbofan. (*Courtesy of Rolls-Royce.*)

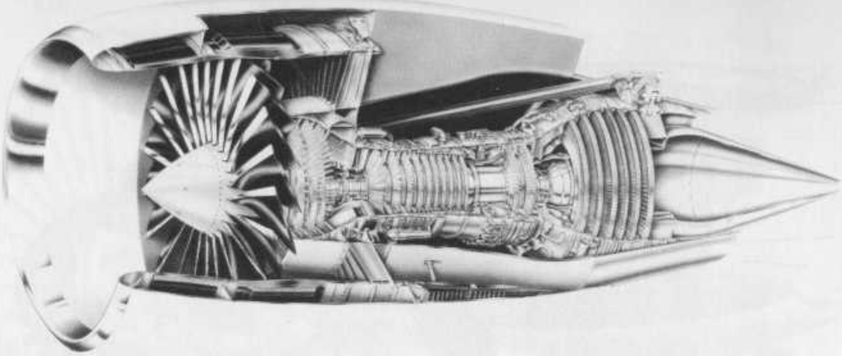


FIGURE 1-8e
General Electric GE90 turbofan. (*Courtesy of General Electric Aircraft Engines.*)

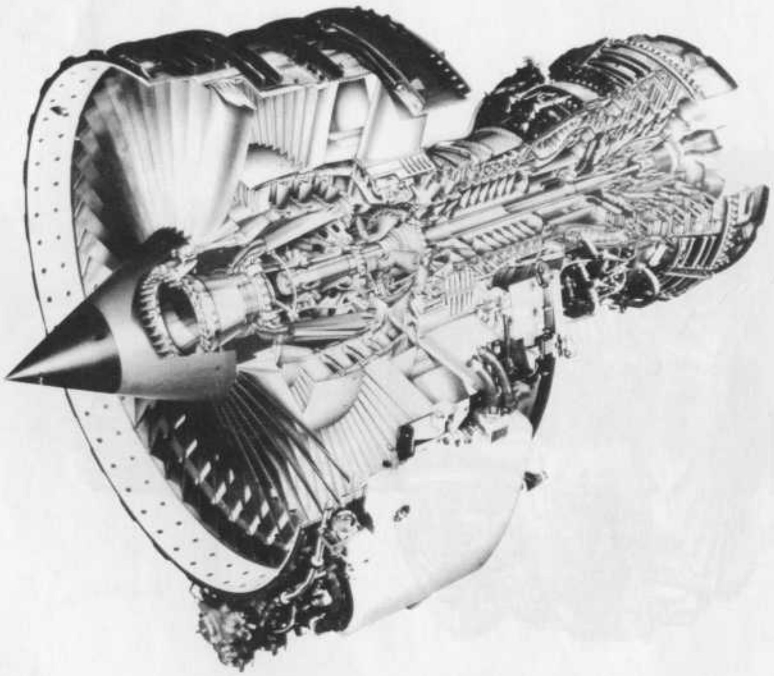


FIGURE 1-8f
SNECMA CFM56 turbofan. (*Courtesy of SNECMA.*)

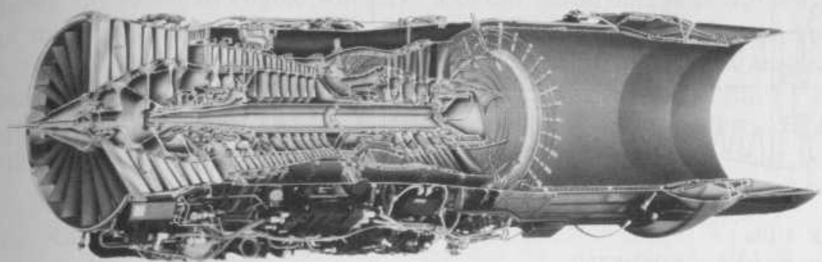


FIGURE 1-9a
Pratt & Whitney F100-PW-229 afterburning turbofan. (*Courtesy of Pratt & Whitney.*)

schematic diagram of the turboprop is shown in Fig. 1-10*a*. The turboshaft engine is similar to the turboprop except that power is supplied to a shaft rather than a propeller. The turboshaft engine is used quite extensively for supplying power for helicopters. The turboprop engine may find application in VTOL (vertical takeoff and landing) transporters. The limitations and advantages of the turboprop are those of the propeller. For low-speed flight and short-field takeoff, the propeller has a performance advantage. At speeds approaching the speed of sound, compressibility effects set in and the propeller loses its aerodynamic efficiency. Due to the rotation of the propeller, the propeller tip will approach the speed of sound before the vehicle approaches

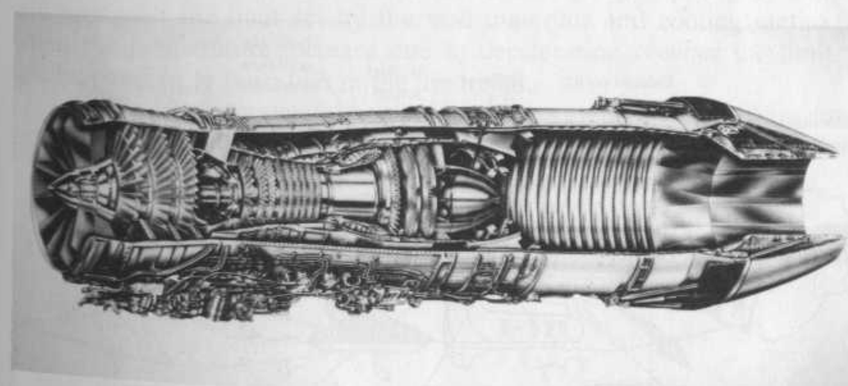


FIGURE 1-9b
General Electric F110-GE-129 afterburning turbofan. (*Courtesy of General Electric Aircraft Engines.*)

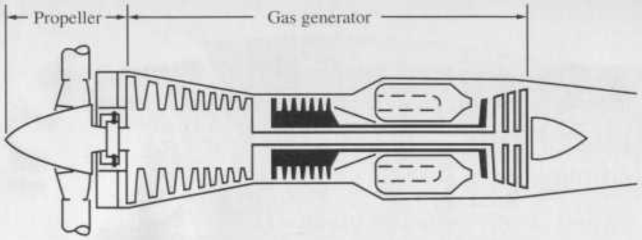


FIGURE 1-10*a*
Schematic diagram of a turboprop.

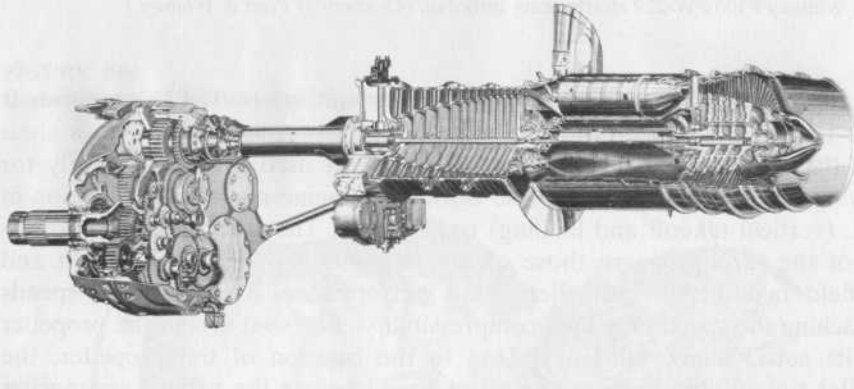


FIGURE 1-10*b*
Allison T56 turboshaft. (*Courtesy of Allison Gas Turbine Division.*)

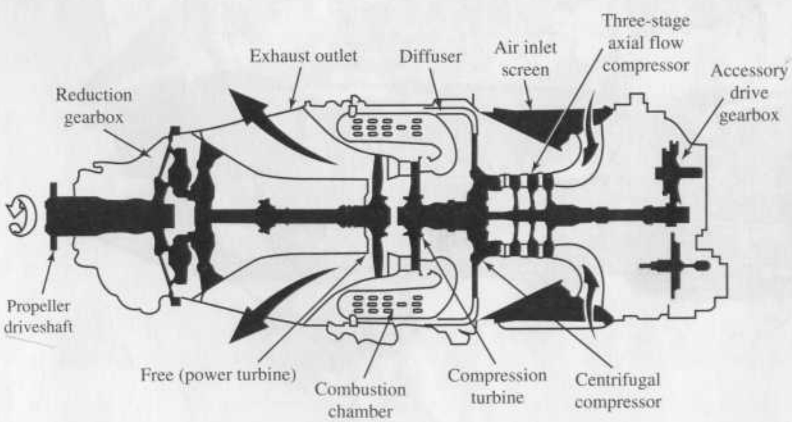


FIGURE 1-10*c*
Canadian Pratt & Whitney PT6 turboshaft. (*Courtesy of Pratt & Whitney of Canada.*)

the speed of sound. This compressibility effect when one approaches the speed of sound limits the design of helicopter rotors and propellers. At high subsonic speeds, the turbofan engine will have a better aerodynamic performance than the turboprop since the turbofan is essentially a *ducted turboprop.* Putting a duct or shroud around a propeller increases its aerodynamic performance. Examples of a turboshaft engine are the Canadian Pratt & Whitney PT6 (Fig. 1-10*c*), used in many small commuter aircraft, and the Allison T56 (Fig. 1-10*b*), used to power the C-130 Hercules and the P-3 Orion.

The Ramjet

The ramjet engine consists of an inlet, a combustion zone, and a nozzle. A schematic diagram of a ramjet is shown in Fig. 1-11. The ramjet does not have the compressor and turbine as the turbojet does. Air enters the inlet where it is compressed and then enters the combustion zone where it is mixed with the fuel and burned. The hot gases are then expelled through the nozzle, developing thrust. The operation of the ramjet depends upon the inlet to decelerate the incoming air to raise the pressure in the combustion zone. The pressure rise makes it possible for the ramjet to operate. The higher the velocity of the incoming air, the greater the pressure rise. It is for this reason that the ramjet operates best at high supersonic velocities. At subsonic velocities, the ramjet is inefficient, and to start the ramjet, air at a relatively higher velocity must enter the inlet.

The combustion process in an ordinary ramjet takes place at low subsonic velocities. At high supersonic flight velocities, a very large pressure rise is developed that is more than sufficient to support operation of the ramjet. Also, if the inlet has to decelerate a supersonic high-velocity airstream to a subsonic velocity, large pressure losses can result. The deceleration process also produces a temperature rise, and at some limiting flight speed, the temperature will approach the limit set by the wall materials and cooling methods. Thus when the temperature increase due to deceleration reaches the limit, it may not be possible to burn fuel in the airstream.

In the past few years, research and development have been done on a ramjet that has the combustion process taking place at supersonic velocities.

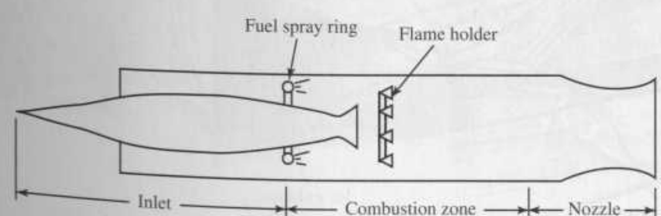


FIGURE 1-11
Schematic diagram of a ramjet.

By using a supersonic combustion process, the temperature rise and pressure loss due to deceleration in the inlet can be reduced. This ramjet with supersonic combustion is known as the *scramjet* (*s*upersonic combustion *ramjet*). Figure 1-12*a* shows the schematic of a scramjet engine similar to that proposed for the National AeroSpace Plane (NASP) research vehicle, the X-30 shown in Fig. 1-12*b*. Further development of the scramjet for other applications (e.g., the Orient Express) will continue if research and development produces a scramjet engine with sufficient performance gains. Remember that since it takes a relative velocity to start the ramjet or scramjet, another engine system is required to accelerate aircraft like the X-30 to ramjet velocities.

Turbojet/Ramjet Combined-Cycle Engine

Two of the Pratt & Whitney J58 turbojet engines (see Fig. 1-13*a*) are used to power the Lockheed SR71 Blackbird (see Fig. 1-13*b*). This was the fastest aircraft (Mach 3+) when it was retired in 1989. The J58 operates as an afterburning turbojet engine until it reaches high Mach level, at which point the six large tubes (Fig. 1-13*a*) bypass flow to the afterburner. When these tubes are in use, the compressor, burner, and turbine of the turbojet are essentially bypassed and the engine operates as a ramjet with the afterburner acting as the ramjet's burner.

Aircraft Engine Performance Parameters

This section presents several of the air-breathing engine performance parameters that are useful in aircraft propulsion. The first performance parameter is the thrust of the engine which is available for sustained flight (thrust = drag), accelerated flight (thrust > drag), or deceleration (thrust < drag).

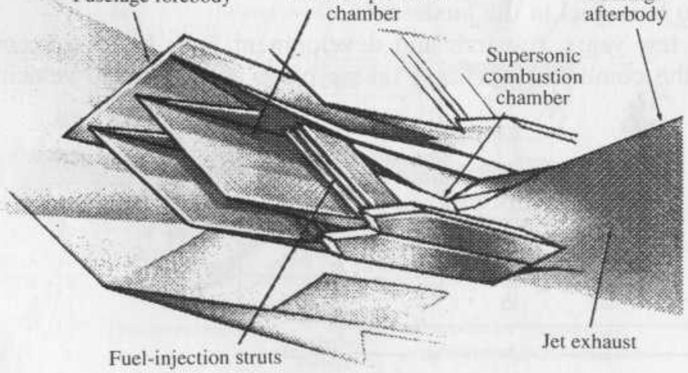


FIGURE 1-12*a*
Schematic diagram of a scramjet.

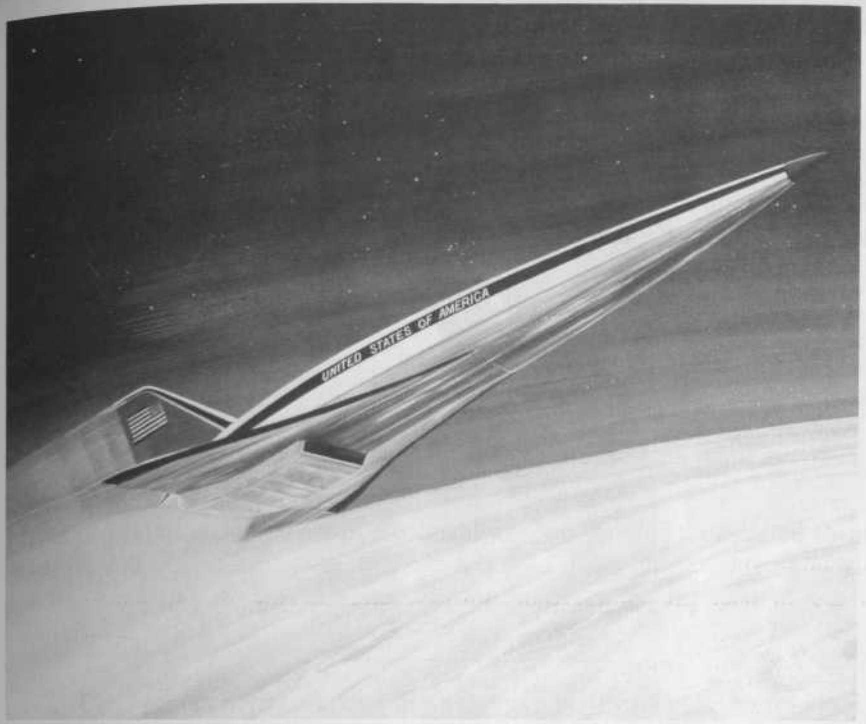


FIGURE 1-12*b*
Conceptual drawing of the X-30. (*Courtesy of Pratt & Whitney.*)

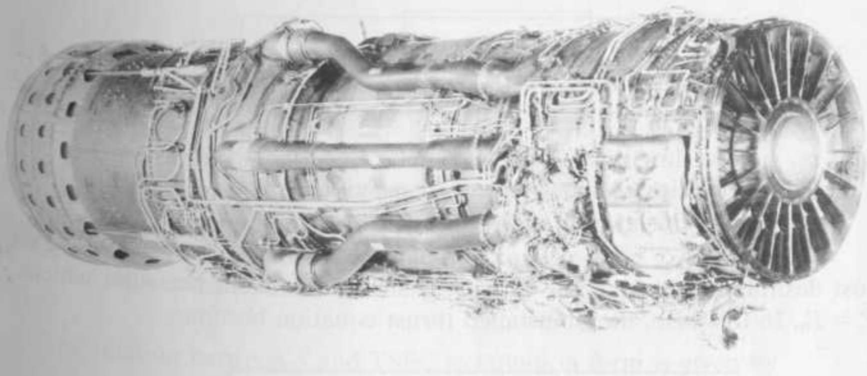


FIGURE 1-13*a*
Pratt & Whitney J58 turbojet. (*Courtesy of Pratt & Whitney.*)

FIGURE 1-13*b*
Lockheed SR71 Blackbird. (*Courtesy of Lockheed.*)

As derived in Chap. 4, the uninstalled thrust F of a jet engine (single inlet and single exhaust) is given by

$$F = \frac{(\dot{m}_0 + \dot{m}_f)V_e + \dot{m}_0 V_0}{g_c} + (P_e - P_0)A_e \tag{1-5}$$

where $\dot{m}_0$, $\dot{m}_f$ = mass flow rates of air and fuel, respectively
V_0, V_e = velocities at inlet and exit, respectively
P_0, P_e = pressures at inlet and exit, respectively

It is most desirable to expand the exhaust gas to the ambient pressure, which gives $P_e = P_0$. In this case, the uninstalled thrust equation becomes

$$F = \frac{(\dot{m}_0 + \dot{m}_f)V_e - \dot{m}_0 V_0}{g_c} \qquad \text{for } P_e = P_0 \tag{1-6}$$

The installed thrust T is equal to the uninstalled thrust F minus the inlet drag D_{inlet} and minus the nozzle drag D_{noz}, or

$$T = F - D_{\text{inlet}} - D_{\text{noz}} \tag{1-7}$$

Dividing the inlet drag D_{inlet} and nozzle drag D_{noz} by the uninstalled thrust F yields the dimensionless inlet loss coefficient ϕ_{inlet} and nozzle loss coefficient ϕ_{noz}, or

$$\phi_{\text{inlet}} = \frac{D_{\text{inlet}}}{F} \qquad \phi_{\text{noz}} = \frac{D_{\text{noz}}}{F} \tag{1-8}$$

Thus the relationship between the installed thrust T and uninstalled thrust F is simply

$$T = F(1 - \phi_{\text{inlet}} - \phi_{\text{noz}}) \tag{1-9}$$

The second performance parameter is the thrust specific fuel consumption (S and TSFC). This is the rate of fuel use by the propulsion system per unit of thrust produced. The uninstalled fuel consumption S and installed fuel consumption TSFC are written in equation form as

$$S = \frac{\dot{m}_f}{F} \tag{1-10}$$

$$\text{TSFC} = \frac{\dot{m}_f}{T} \tag{1-11}$$

where F = uninstalled thrust
S = uninstalled thrust specific fuel consumption
T = installed engine thrust
TSFC = installed thrust specific fuel consumption
$\dot{m}_f$ = mass flow rate of fuel

The relation between S and TSFC in equation form is given by

$$S = \text{TSFC}\,(1 - \phi_{\text{inlet}} - \phi_{\text{noz}}) \tag{1-12}$$

Values of thrust F and fuel consumption S for various jet engines at sea-level static conditions are listed in App. B. The predicted variations of uninstalled engine thrust F and uninstalled thrust specific fuel consumption S with Mach number and altitude for an advanced fighter engine (from Ref. 3) are plotted in Figs. 1-14*a* through 1-14*d*. Note that the thrust F decreases with altitude and the fuel consumption S also decreases with altitude until 36 kft (the start of the isothermal layer of the atmosphere). Also note that the fuel consumption increases with Mach number and that the thrust varies considerably with the Mach number. The predicted partial-throttle performance of the advanced fighter engine is shown at three flight conditions in Fig. 1-14*e*.

The takeoff thrust of the JT9D high-bypass-ratio turbofan engine is given in Fig. 1-15*a* versus Mach number and ambient air temperature for two versions. Note the rapid falloff of thrust with rising Mach number that is characteristic of this engine cycle and the constant thrust at a Mach number for temperatures of 86°F and below (this is often referred to as a *flat rating*). The partial-throttle performance of both engine versions is given in Fig. 1-15*b* for two combinations of altitude and Mach number.

Although the aircraft gas turbine engine is a very complex machine, the basic tools for modeling its performance are developed in the following chapters. These tools are based on the work of Gordon Oates (Ref. 4). They

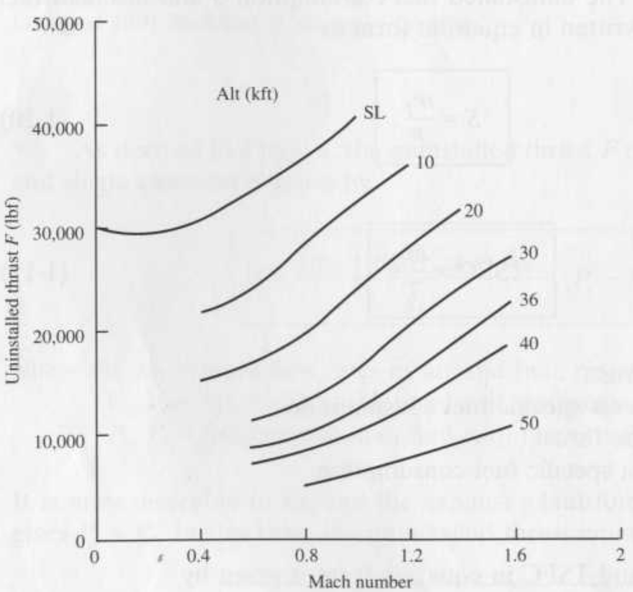


FIGURE 1-14*a*
Uninstalled thrust F of an advanced afterburning fighter engine at maximum power setting, afterburner on. (*Extracted from Ref. 3.*)

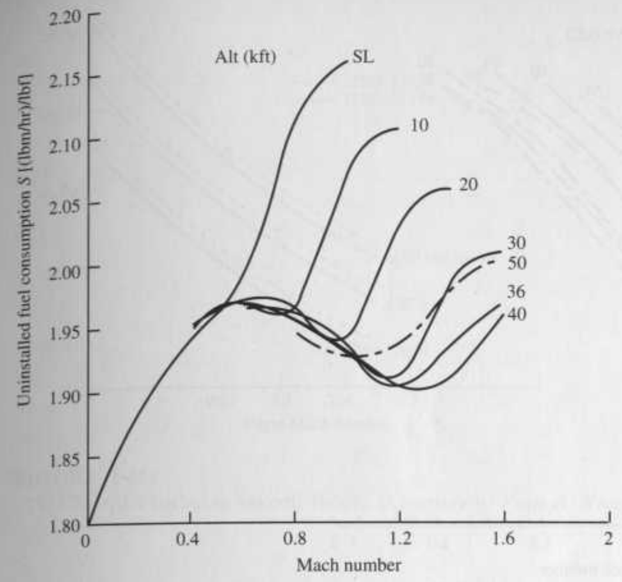


FIGURE 1-14*b*
Uninstalled fuel consumption S of an advanced afterburning fighter engine at maximum power setting, afterburner on. (*Extracted from Ref. 3.*)

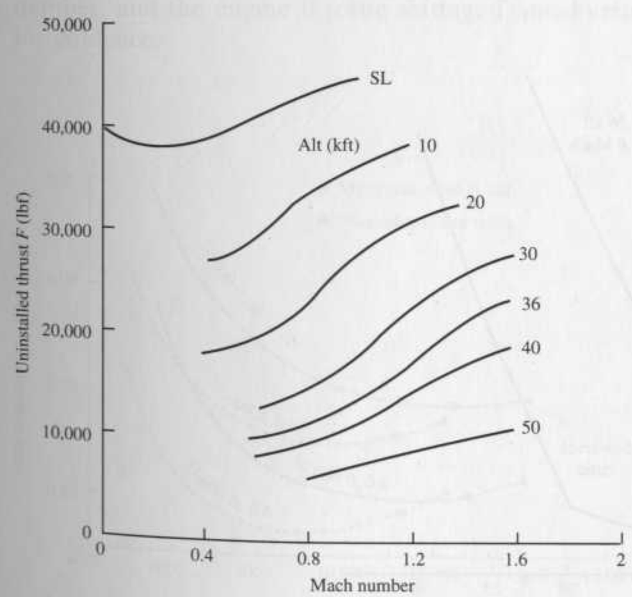


FIGURE 1-14*c*
Uninstalled thrust F of an advanced afterburning fighter engine at military power setting, afterburner off. (*Extracted from Ref. 3.*)

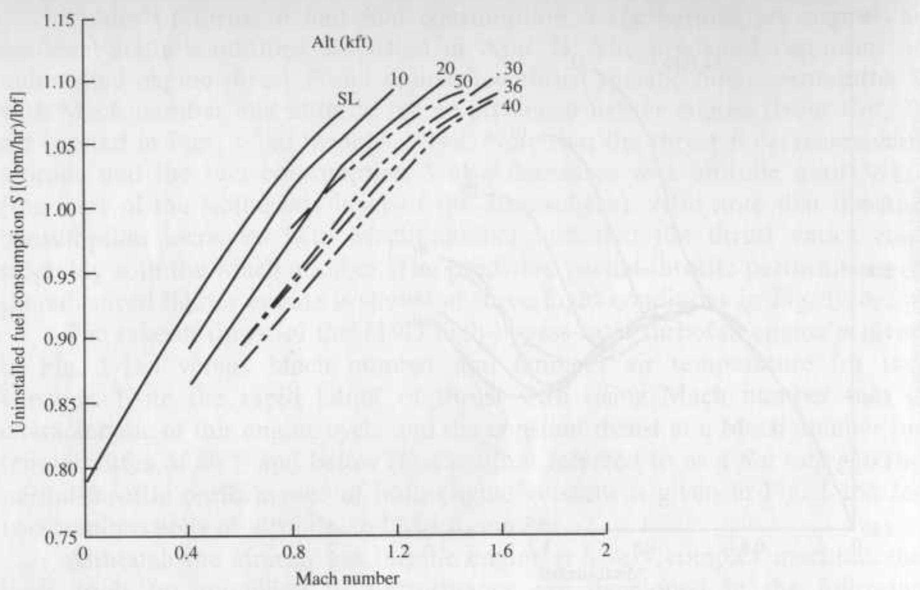


FIGURE 1-14*d*
Uninstalled fuel consumption S of an advanced afterburning fighter engine at military power setting, afterburner off. (*Extracted from Ref. 3.*)

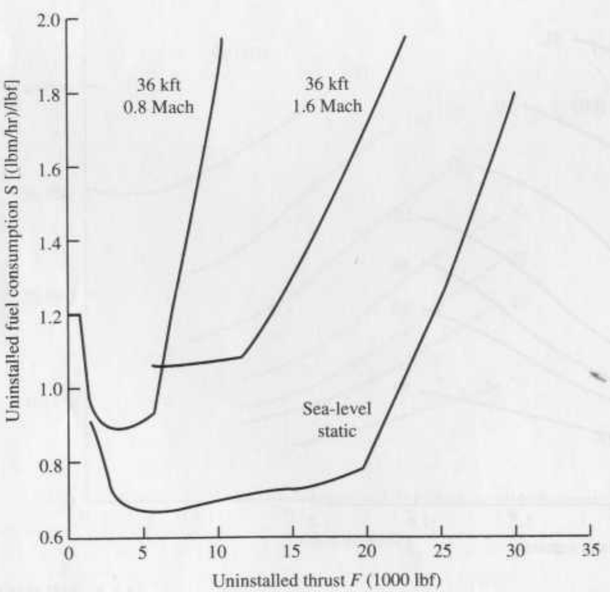


FIGURE 1-14*e*
Partial-throttle performance of an advanced fighter engine. (*Extracted from Ref. 3.*)

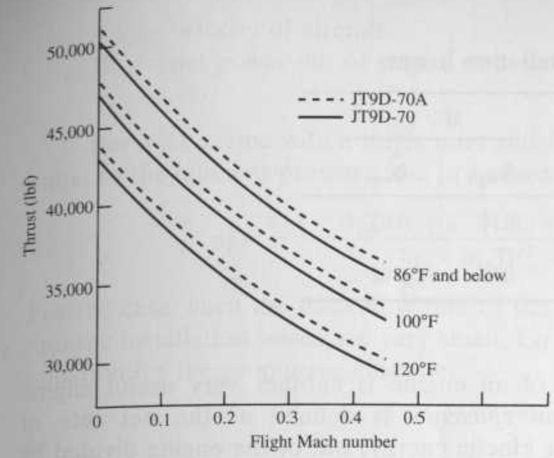


FIGURE 1-15*a*
JT9D-70/-70A turbofan takeoff thrust. (*Courtesy of Pratt & Whitney.*)

permit performance calculations for existing and proposed engines and generate performance curves similar to Figs. 1-14*a* through 1-14*e* and Figs. 1-15*a* and 1-15*b*.

The value of the installation loss coefficient depends on the characteristics of the particular engine/airframe combination, the Mach number, and the engine throttle setting. Typical values are given in Table 1-3 for guidance.

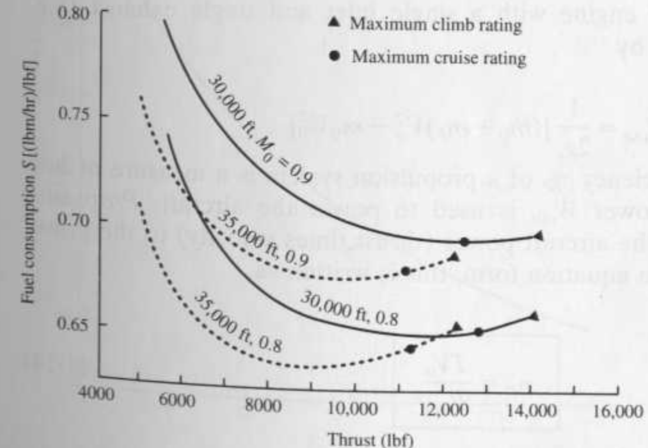


FIGURE 1-15*b*
JT9D-70/-70A turbofan cruise specific fuel consumption. (*Courtesy of Pratt & Whitney.*)

TABLE 1-3
Typical aircraft engine thrust installation losses

Flight condition:	$M<1$		$M>1$	
Aircraft type	ϕ_{inlet}	ϕ_{noz}	ϕ_{inlet}	ϕ_{noz}
Fighter	0.05	0.01	0.05	0.03
Passenger/cargo	0.02	0.01	—	—
Bomber	0.03	0.01	0.04	0.02

The thermal efficiency η_T of an engine is another very useful engine performance parameter. *Thermal efficiency* is defined as the net rate of organized energy (shaft power or kinetic energy) out of the engine divided by the rate of thermal energy available from the fuel in the engine. The fuel's available thermal energy is equal to the mass flow rate of the fuel $\dot{m}_f$ times the fuel heating value h_{PR}. Thermal efficiency can be written in equation form as

$$\boxed{\eta_T = \frac{\dot{W}_{out}}{\dot{Q}_{in}}} \tag{1-13}$$

where η_T = thermal efficiency of engine
$\dot{W}_{out}$ = net power out of engine
$\dot{Q}_{in}$ = rate of thermal energy released ($\dot{m}_f h_{PR}$)

Note: For engines with shaft power output, $\dot{W}_{out}$ is equal to this shaft power. For engines with no shaft power output (e.g., turbojet engine), $\dot{W}_{out}$ is equal to the next rate of change of the kinetic energy of the fluid through the engine. The power out of a jet engine with a single inlet and single exhaust (e.g., turbojet engine) is given by

$$\dot{W}_{out} = \frac{1}{2g_c}[(\dot{m}_0 + \dot{m}_f)V_e^2 - \dot{m}_0 V_0^2]$$

The propulsive efficiency η_P of a propulsion system is a measure of how effectively the engine power $\dot{W}_{out}$ is used to power the aircraft. *Propulsive efficiency* is the ratio of the aircraft power (thrust times velocity) to the power out of the engine $\dot{W}_{out}$. In equation form, this is written as

$$\boxed{\eta_P = \frac{TV_0}{\dot{W}_{out}}} \tag{1-14}$$

where η_P = propulsive efficiency of engine
T = thrust of propulsion system
V_0 = velocity of aircraft
$\dot{W}_{out}$ = net power out of engine

For a jet engine with a single inlet and single exhaust and an exit pressure equal to the ambient pressure, the propulsive efficiency is given by

$$\eta_P = \frac{2(1-\phi_{inlet}-\phi_{noz})[(\dot{m}_0+\dot{m}_f)V_e - \dot{m}_0 V_0]V_0}{(\dot{m}_0+\dot{m}_f)V_e^2 - \dot{m}_0 V_0^2} \tag{1-15}$$

For the case when the mass flow rate of the fuel is much less than that of air and the installation losses are very small, Eq. (1-15) simplifies to the following equation for the propulsive efficiency:

$$\boxed{\eta_P = \frac{2}{V_e/V_0 + 1}} \tag{1-16}$$

Equation (1-16) is plotted versus the velocity ratio V_e/V_0 in Fig. 1-16 and shows that high propulsive efficiency requires the exit velocity to be approximately equal to the inlet velocity. Turbojet engines have high values of the velocity ratio V_e/V_0 with corresponding low propulsive efficiency, whereas turbofan engines have low values of the velocity ratio V_e/V_0 with corresponding high propulsive efficiency.

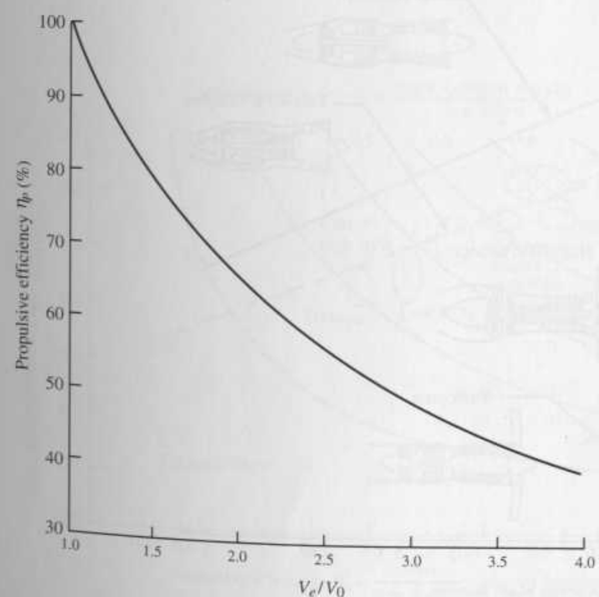


FIGURE 1-16
Propulsive efficiency versus velocity ratio (V_e/V_0).

The thermal and propulsive efficiencies can be combined to give the *overall efficiency* η_O of a propulsion system. Multiplying propulsive efficiency by thermal efficiency, we get the ratio of the aircraft power to the rate of thermal energy released in the engine (the overall efficiency of the propulsion system):

$$\boxed{\eta_O = \eta_P \eta_T} \tag{1-17}$$

$$\boxed{\eta_O = \frac{TV_0}{\dot{Q}_{in}}} \tag{1-18}$$

Several of the above performance parameters are plotted for general types of gas turbine engines in Figs. 1-17*a*, 1-17*b*, and 1-17*c*. These plots can be used to obtain the general trends of these performance parameters with flight velocity for each propulsion system.

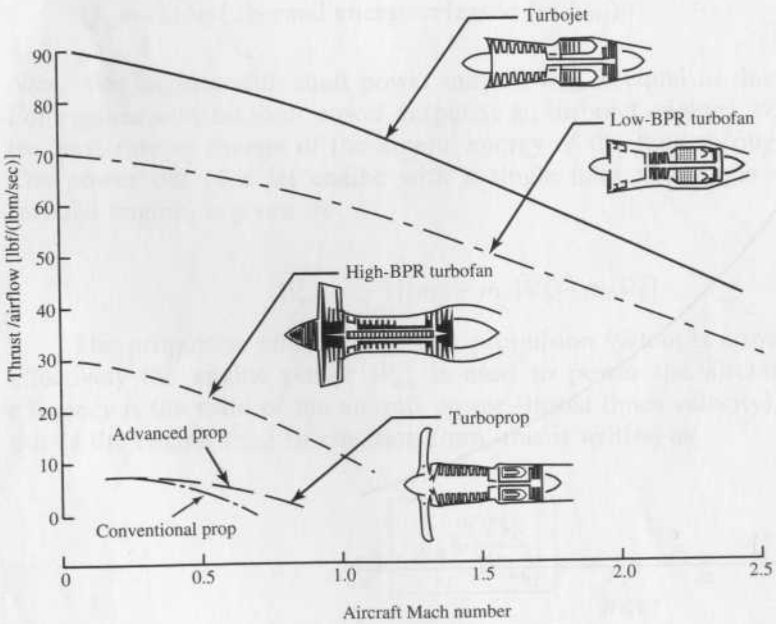


FIGURE 1-17*a*
Specific thrust characteristics of typical aircraft engines. (*Courtesy of Pratt & Whitney.*)

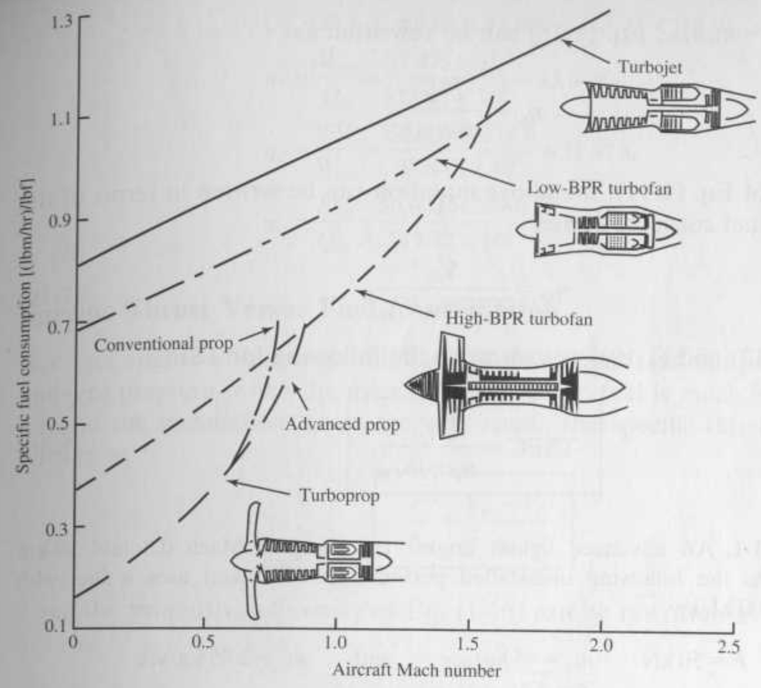


FIGURE 1-17*b*
Thrust specific fuel consumption characteristics of typical aircraft engines. (*Courtesy of Pratt & Whitney.*)

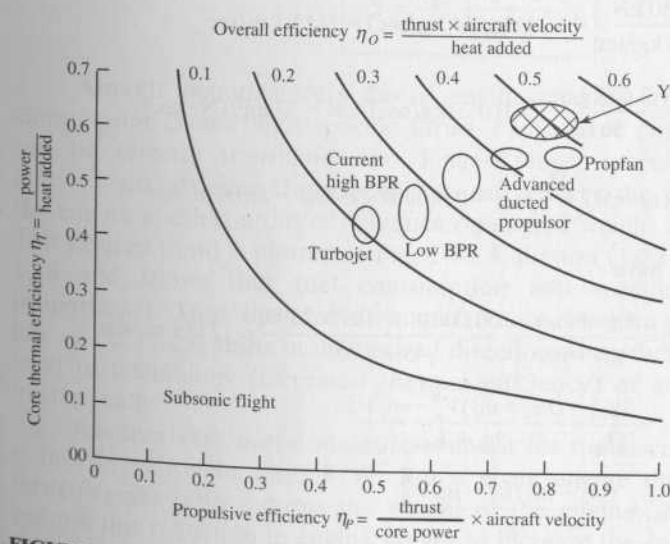


FIGURE 1-17*c*
Efficiency characteristics of typical aircraft engines. (*Courtesy of Pratt & Whitney.*)

Since $\dot{Q}_{in} = \dot{m}_f h_{PR}$, Eq. (1-18) can be rewritten as

$$\eta_O = \frac{TV_0}{\dot{m}_f h_{PR}}$$

With the help of Eq. (1-11), the above equation can be written in terms of the thrust specific fuel consumption as

$$\eta_O = \frac{V_0}{\text{TSFC} \cdot h_{PR}} \tag{1-19}$$

Using Eqs. (1-17) and (1-19), we can write the following for TSFC:

$$\boxed{\text{TSFC} = \frac{V_0}{\eta_P \eta_T h_{PR}}} \tag{1-20}$$

Example 1-1. An advanced fighter engine operating at Mach 0.8 and 10-km altitude has the following uninstalled performance data and uses a fuel with $h_{PR} = 42{,}800$ kJ/kg:

$$F = 50 \text{ kN} \qquad \dot{m}_0 = 45 \text{ kg/sec} \qquad \text{and} \qquad \dot{m}_f = 2.65 \text{ kg/sec}$$

Determine the specific thrust, thrust specific fuel consumption, exit velocity, thermal efficiency, propulsive efficiency, and overall efficiency (assume exit pressure equal to ambient pressure).

Solution.

$$\frac{F}{\dot{m}_0} = \frac{50 \text{ kN}}{45 \text{ kg/sec}} = 1.1111 \text{ kN/(kg/sec)} = 1111.1 \text{ m/sec}$$

$$S = \frac{\dot{m}_f}{F} = \frac{2.65 \text{ kg/sec}}{50 \text{ kN}} = 0.053 \text{(kg/sec)/kN} = 53 \text{ mg/(N} \cdot \text{sec)}$$

$$V_0 = M_0 a_0 = M_0 \left(\frac{M_0}{a_{ref}}\right) a_{ref} = 0.8(0.8802)340.3 = 239.6 \text{ m/sec}$$

From Eq. (1-6) we have

$$V_e = \frac{Fg_c + \dot{m}_0 V_0}{\dot{m}_0 + \dot{m}_f} = \frac{50{,}000 \times 1 + 45 \times 239.6}{45 + 2.65} = 1275.6 \text{ m/sec}$$

$$\eta_T = \frac{\dot{W}_{out}}{\dot{Q}_{in}} = \frac{(\dot{m}_0 + \dot{m}_f)V_e^2 - \dot{m}_0 V_0^2}{2g_c \dot{m}_f h_{PR}}$$

$$\dot{W}_{out} = \frac{(\dot{m}_0 + \dot{m}_f)V_e^2 - \dot{m}_0 V_0^2}{2g_c}$$

$$= \frac{47.65 \times 1275.6^2 - 45 \times 239.6^2}{2 \times 1} = 37.475 \times 10^6 \text{ W}$$

$$\dot{Q}_{in} = \dot{m}_f h_{PR} = 2.65 \times 42{,}800 = 113.42 \times 10^6 \text{ W}$$

$$\eta_T = \frac{\dot{W}_{out}}{\dot{Q}_{in}} = \frac{37.475 \times 10^6}{113.42 \times 10^6} = 33.04\%$$

$$\eta_P = \frac{FV_0}{\dot{W}_{out}} = \frac{50{,}000 \times 239.6}{37.475 \times 10^6} = 31.97\%$$

$$\eta_O = \frac{FV_0}{\dot{Q}_{in}} = \frac{50{,}000 \times 239.6}{113.42 \times 10^6} = 10.56\%$$

Specific Thrust Versus Fuel Consumption

For a jet engine with a single inlet and single exhaust and exit pressure equal to ambient pressure, when the mass flow rate of the fuel is much less than that of air and the installation losses are very small, the specific thrust $F/\dot{m}_0$ can be written as

$$\boxed{\frac{F}{\dot{m}_0} = \frac{V_e - V_0}{g_c}} \tag{1-21}$$

Then the propulsive efficiency of Eq. (1-16) can be rewritten as

$$\eta_P = \frac{2}{Fg_c/(\dot{m}_0 V_0) + 2} \tag{1-22}$$

Substituting Eq. (1-22) into Eq. (1-20) and noting that TSFC = S, we obtain the following very enlightening expression:

$$\boxed{S = \frac{Fg_c/\dot{m}_0 + 2V_0}{2\eta_T h_{PR}}} \tag{1-23}$$

Aircraft manufacturers desire engines having low thrust specific fuel consumption S and high specific thrust $F/\dot{m}_0$. Low engine fuel consumption can be directly translated into longer range, increased payload, and/or reduced aircraft size. High specific thrust reduces the cross-sectional area of the engine and has a direct influence on engine weight and installation losses. This desired trend is plotted in Fig. 1-18. Equation (1-23) is also plotted in Fig. 1-18 and shows that fuel consumption and specific thrust are directly proportional. Thus the aircraft manufacturers have to make a tradeoff. The line of Eq. (1-23) shifts in the desired direction when there is an increase in the level of technology (increased thermal efficiency) or an increase in the fuel heating value.

Another very useful measure of merit for the aircraft gas turbine engine is the thrust/weight ratio F/W. For a given engine thrust F, increasing the thrust/weight ratio reduces the weight of the engine. Aircraft manufacturers can use this reduction in engine weight to increase the capabilites of an aricraft (increased payload, increased fuel, or both) or decrease the size (weight) and cost of a new aircraft under development.

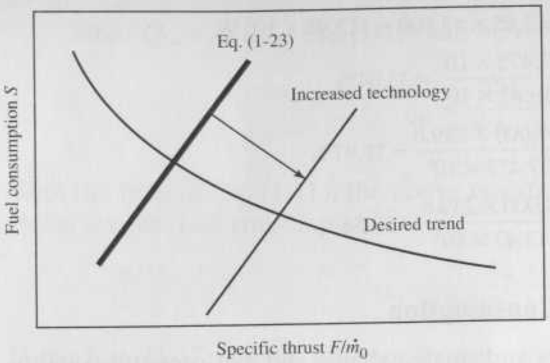


FIGURE 1-18
Relationship between specific thrust and fuel consumption.

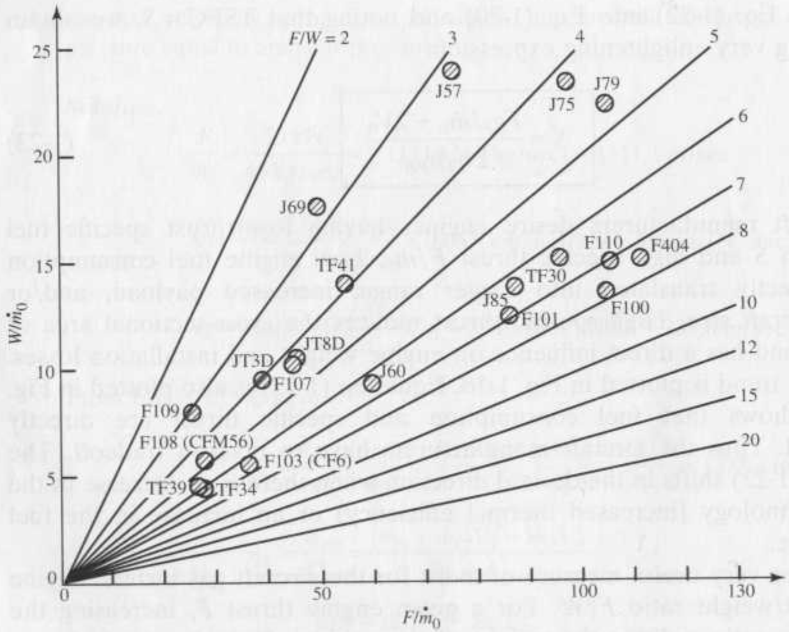


FIGURE 1-19
Engine thrust/weight ratio F/W.

Engine companies expend considerable research and development effort on increasing the thrust/weight ratio of aircraft gas turbine engines. This ratio is equal to the specific thrust $F/\dot{m}_0$ divided by the engine weight per unit of mass flow $W/\dot{m}_0$. For a given engine type, the engine weight per unit mass flow is related to the efficiency of the engine structure, and the specific thrust is related to the engine thermodynamics. The weights per unit mass flow of some existing gas turbine engines are plotted versus specific thrust in Fig. 1-19. Also plotted are lines of constant engine thrust/weight ratio F/W.

Currently, the engine companies, in conjunction with the Department of Defense and NASA, are involved in a large research and development effort to increase the engine thrust/weight ratio F/W and decrease the fuel consumption while maintaining engine durability, maintainability, etc. This program is called the *integrated high-performance turbine engine technology* (IHPTET) *initiative* (see Refs. 5 and 6).

1-5 AIRCRAFT PERFORMANCE

This section on aircraft performance is included so that the reader may get a better understanding of the propulsion requirements of the aircraft (Ref. 7). The coverage is limited to a few significant concepts that directly relate to aircraft engines. It is not intended as a substitute for the many excellent references on this subject (see Refs. 8 through 11).

Performance Equation

Relationships for the performance of an aircraft can be obtained from energy considerations (see Ref. 12). By treating the aircraft (Fig. 1-20) as a moving mass and assuming that the installed propulsive thrust T, aerodynamic drag D, and other resistive forces R act in the same direction as the velocity V, it follows that

$$[T - (D + R)]V = W\frac{dh}{dt} + \frac{W}{g}\frac{d}{dt}\left(\frac{V^2}{2}\right) \tag{1-24}$$

rate of mechanical energy input	storage rate of potential energy	storage rate of kinetic energy

Note that the total resistive force $D + R$ is the sum of the drag of the clean aircraft D and any additional drags R associated with such proturberances as landing gear, external stores, or drag chutes.

By defining the energy height z_e as the sum of the potential and kinetic

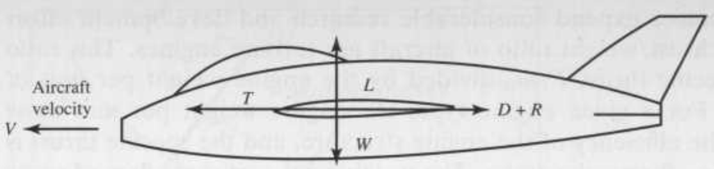


FIGURE 1-20
Forces on aircraft.

energy terms

$$z_e \equiv h + \frac{V^2}{2g} \tag{1-25}$$

Eq. (1-24) can now be written simply as

$$[T - (D + R)]V = W\frac{dz_e}{dt} \tag{1-26}$$

By defining the *weight specific excess power* P_s as

$$P_s \equiv \frac{dz_e}{dt} \tag{1-27}$$

Eq. (1-26) can now be written in its dimensionless form as

$$\frac{T - (D + R)}{W} = \frac{P_s}{V} = \frac{1}{V}\frac{d}{dt}\left(h + \frac{V^2}{2g}\right) \tag{1-28}$$

This is a very powerful equation which gives insight into the dynamics of flight, including both the rate of climb dh/dt and acceleration dV/dt.

Lift and Drag

We use the classical aircraft lift relationship

$$L = nW = C_L q S_w \tag{1-29}$$

where n is the load factor or number of g's perpendicular to V ($n = 1$ for straight and level flight), C_L is the coefficient of lift, S_w is the wing planform area, and q is the dynamic pressure. The dynamic pressure can be expressed in

terms of the density ρ and velocity V or the pressure P and Mach number M as

$$q = \frac{1}{2}\rho\frac{V^2}{g_c} = \frac{1}{2}\sigma\rho_{\text{ref}}\frac{V^2}{g_c} \tag{1-30a}$$

or

$$q = \frac{\gamma}{2}PM_0^2 = \frac{\gamma}{2}\delta P_{\text{ref}}M_0^2 \tag{1-30b}$$

where δ and σ are the dimensionless pressure and density ratios defined by Eqs. (1-2) and (1-4), respectively, and γ is the ratio of specific heats ($\gamma = 1.4$ for air). The reference density ρ_{ref} and reference pressure P_{ref} of air are their sea-level values on a standard day and are listed in App. A.

We also use the classical aircraft drag relationship

$$D = C_D q S_w \tag{1-31}$$

Figure 1-21 is a plot of lift coefficient C_L versus drag coefficient C_D, commonly called the *lift-drag polar*, for a typical subsonic passenger aircraft. The drag coefficient curve can be approximated by a second-order equation in C_L written as

$$C_D = K_1 C_L^2 + K_2 C_L + C_{D0} \tag{1-32}$$

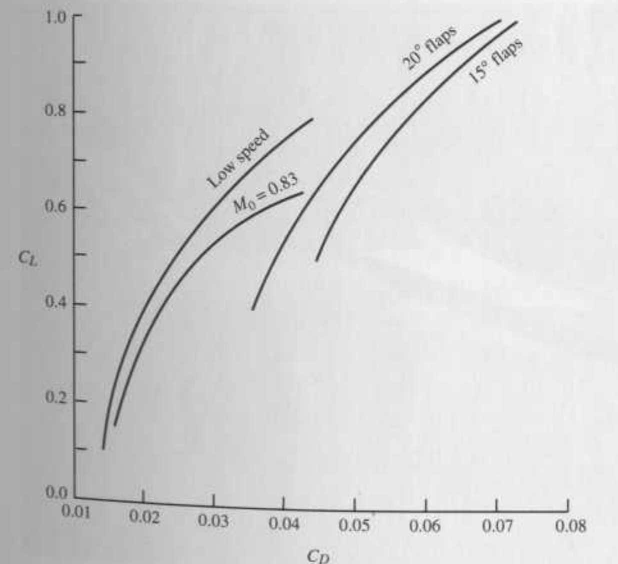


FIGURE 1-21
Typical lift-drag polar.

where the coefficients K_1, K_2, and C_{D0} are typically functions of flight Mach number and wing configuration (flap position, etc.).

The C_{D0} term in Eq. (1-32) is the zero lift drag coefficient which accounts for both frictional and pressure drag in subsonic flight and wave drag in supersonic flight. The K_1 and K_2 terms account for the drag due to lift. Normally K_2 is very small and approximately equal to zero for most fighter aircraft.

Example 1-2. For all the examples given in this section on aircraft performance, two types of aircraft will be considered.

a. An advanced fighter aircraft is approximately modeled after the YF22 Advanced Tactical Fighter shown in Fig. 1-22. For convenience, we will designate our hypothetical fighter aircraft as the HF-1, having the following characteristics:

Maximum gross takeoff weight W_{TO} = 40,000 lbf (177,920 N)
Empty weight = 24,000 lbf (106,752 N)
Maximum fuel plus payload weight = 16,000 lbf (71,168 N)
Permanent payload = 1600 lbf (7117 N, crew plus return armament)
Expended payload = 2000 lbf (8896 N, missiles plus ammunition)
Maximum fuel capacity = 12,400 lbf (55,155 N)
Wing area S_w = 720 ft² (66.9 m²)

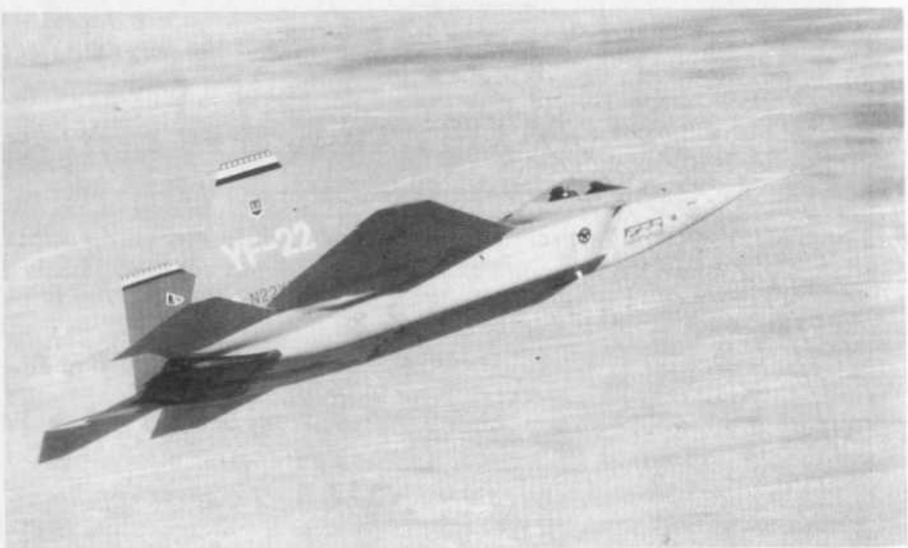


FIGURE 1-22
YF22, Advanced Tactical Fighter. (*Photo courtesy of Boeing Defense & Space Group, Military Airplanes Division.*)

TABLE 1-4
Drag coefficients for hypothetical fighter aircraft (HF-1)

M_0	K_1	K_2	C_{D0}
0.0	0.20	0.0	0.0120
0.8	0.20	0.0	0.0120
1.2	0.20	0.0	0.02267
1.4	0.25	0.0	0.0280
2.0	0.40	0.0	0.0270

Engine: low-bypass-ratio, mixed-flow turbofan with afterburner
Maximum lift coefficient $C_{L\,max}$ = 1.8
Drag coefficients given in Table 1-4

b. An advanced 253-passenger commercial aircraft approximately modeled after the Boeing 767 is shown in Fig. 1-23. For convenience, we will designate our hypothetical passenger aircraft as the HP-1, having the following characteristics:

Maximum gross takeoff weight W_{TO} = 1,645,760 N (370,000 lbf)
Empty weight = 822,880 N (185,500 lbf)

FIGURE 1-23
Boeing 767. (*Photo courtesy of Boeing.*)

TABLE 1-5
Drag coefficients for hypothetical passenger aircraft (HP-1)

M_0	K_1	K_2	C_{D0}
0.00	0.056	−0.004	0.0140
0.40	0.056	−0.004	0.0140
0.75	0.056	−0.008	0.0140
0.83	0.056	−0.008	0.0150

Maximum landing weight = 1,356,640 N (305,000 lbf)
Maximum payload = 420,780 N (94,600 lbf, 253 passengers plus 196,000 N of cargo)
Maximum fuel capacity = 716,706 N (161,130 lbf)
Wing area $S_w = 282.5\ m^2$ (3040 ft^2)
Engine: high-bypass-ratio turbofan
Maximum lift coefficient $C_{L\,max} = 2.0$
Drag coefficients given in Table 1-5

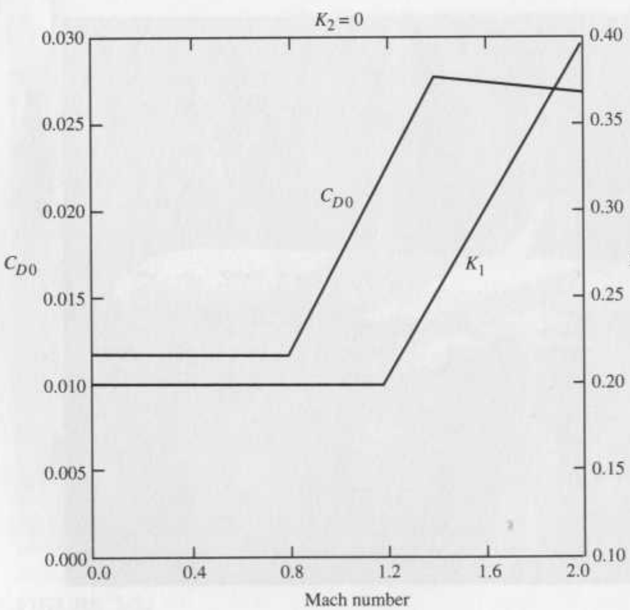


FIGURE 1-24
Values of K_1 and C_{D0} for HF-1 aircraft.

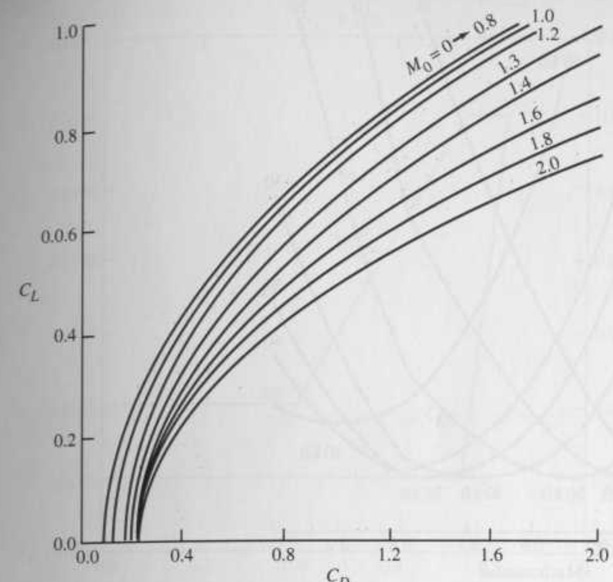


FIGURE 1-25
Lift-drag polar for HF-1 aircraft.

Example 1-3. Determine the drag polar and drag variation for the HF-1 aircraft at 90 percent of maximum gross takeoff weight and the HP-1 aircraft at 95 percent of maximum gross takeoff weight.

a. The variation in C_{D0} and K_1 with Mach number for the HF-1 are plotted in Fig. 1-24 from the data of Table 1-4. Figure 1-25 shows the drag polar at different Mach numbers for the HF-1 aircraft. Using these drag data and the above equations gives the variation in aircraft drag with subsonic Mach number and altitude for level flight ($n = 1$), as shown in Fig. 1-26*a*. Note that the minimum drag is constant for Mach numbers 0 to 0.8 and then increases. This is the same variation as C_{D0}. The variation of drag with load factor n is shown in Fig. 1-26*b* at two altitudes. The drag increases with increasing load factor, and there is a flight Mach number that gives minimum drag for a given altitude and load factor.

b. The variation in C_{D0} and K_2 with Mach number for the HP-1 is plotted in Fig. 1-27 from the data of Table 1-5. Figure 1-28 shows the drag polar at different Mach numbers for the HP-1 aircraft. Using these drag data and the above equations gives the variation in aircraft drag with subsonic Mach number and altitude for level flight ($n = 1$), as shown in Fig. 1-29. Note that the minimum drag is constant for Mach numbers 0 to 0.75 and then increases. This is the same variation as C_{D0}.

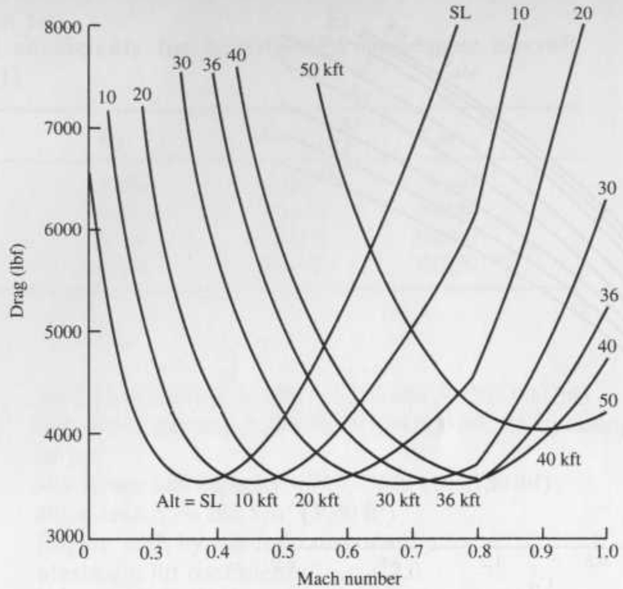


FIGURE 1-26a
Drag for level flight ($n = 1$) for HF-1 aircraft.

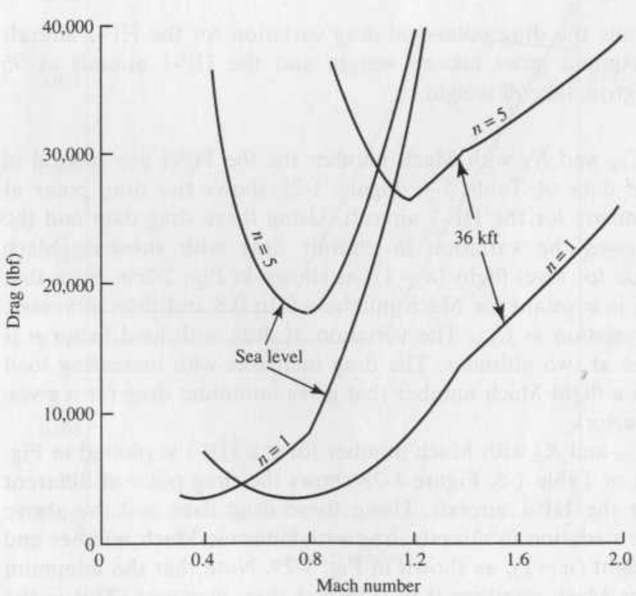


FIGURE 1-26b
Drag of HF-1 aircraft at sea level and 36 kft for $n = 1$ and $n = 5$.

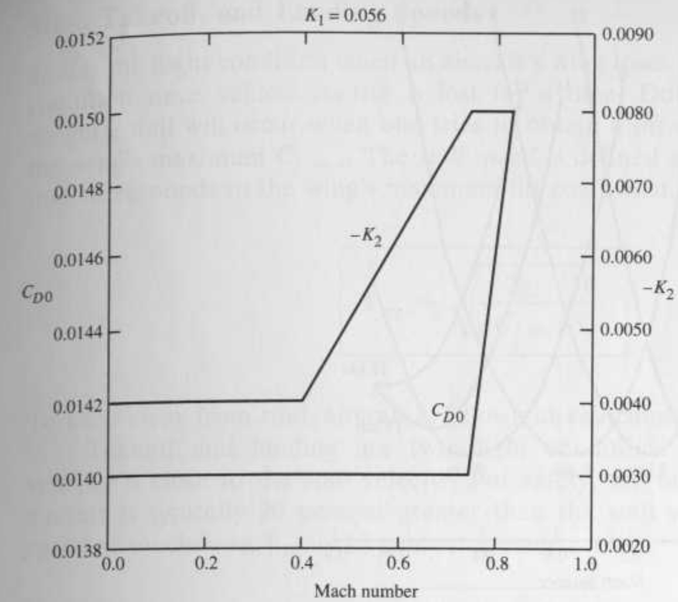


FIGURE 1-27
Values of K_2 and C_{D0} for HP-1 aircraft.

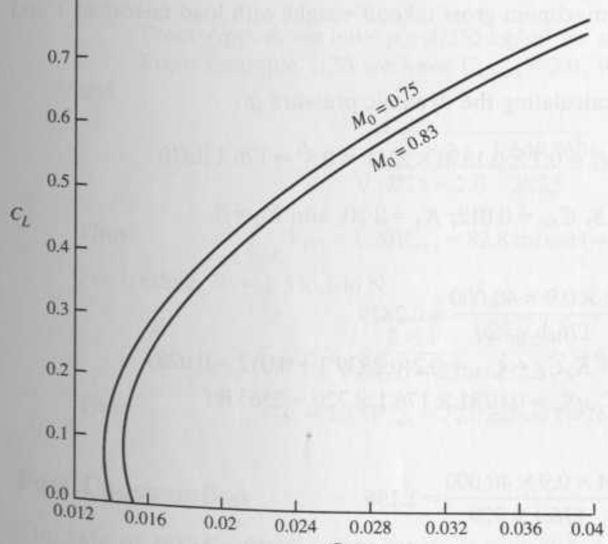


FIGURE 1-28
Lift-drag polar for HP-1 aircraft.

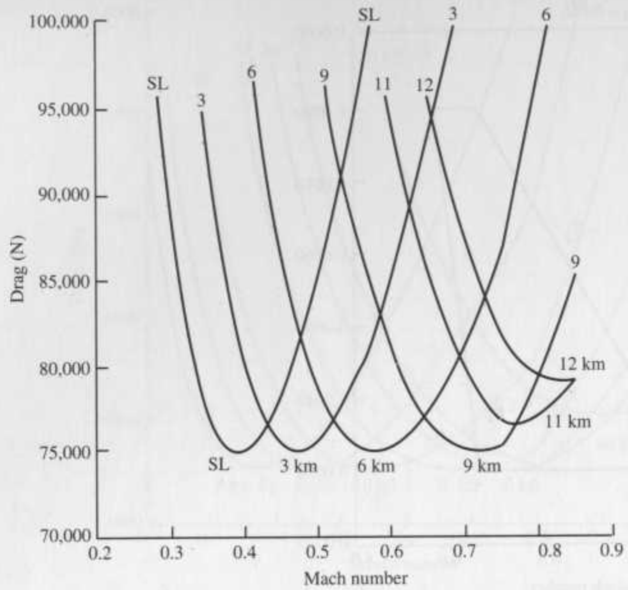


FIGURE 1-29
Drag for level flight ($n = 1$) for HP-1 aircraft.

Example 1-4. Calculate the drag at Mach 0.8 and 40-kft altitude of the HF-1 aircraft at 90 percent of maximum gross takeoff weight with load factors of 1 and 4.

Solution. We begin by calculating the dynamic pressure q.

$$q = \frac{\gamma}{2}\delta P_{\text{ref}} M_0^2 = 0.7 \times 0.1858 \times 2116 \times 0.8^2 = 176.1 \text{ lbf/ft}^2$$

From Fig. 1-24 at $M = 0.8$, $C_{D0} = 0.012$, $K_1 = 0.20$, and $K_2 = 0$.

Case 1: $n = 1$

$$C_L = \frac{nW}{qS_w} = \frac{1 \times 0.9 \times 40{,}000}{176.1 \times 720} = 0.2839$$

$$C_D = K_1 C_L^2 + K_2 C_L + C_{D0} = 0.2(0.2839^2) + 0.012 = 0.0281$$

$$D = C_D q S_w = 0.0281 \times 176.1 \times 720 = 3563 \text{ lbf}$$

Case 2: $n = 4$

$$C_L = \frac{nW}{qS_w} = \frac{4 \times 0.9 \times 40{,}000}{176.1 \times 720} = 1.136$$

$$C_D = K_1 C_L^2 + K_2 C_L + C_{D0} = 0.2(1.136^2) + 0.012 = 0.2701$$

$$D = C_D q S_w = 0.2701 \times 176.1 \times 720 = 34{,}247 \text{ lbf}$$

Note that the drag at $n = 4$ is about 10 times that at $n = 1$.

Stall, Takeoff, and Landing Speeds

Stall is the flight condition when an aircraft's wing loses lift. It is an undesirable condition since vehicle control is lost for a time. During level flight (lift = weight), stall will occur when one tries to obtain a lift coefficient greater than the wing's maximum $C_{L\max}$. The *stall speed* is defined as the level flight speed that corresponds to the wing's maximum lift coefficient, or

$$V_{\text{stall}} = \sqrt{\frac{2g_c}{\rho C_{L\max}} \frac{W}{S_w}} \tag{1-33}$$

To keep away from stall, aircraft are flown at velocities greater than V_{stall}.

Takeoff and landing are two flight conditions in which the aircraft velocity is close to the stall velocity. For safety, the takeoff speed V_{TO} of an aircraft is typically 20 percent greater than the stall speed, and the landing speed at touchdown V_{TD} is 15 percent greater:

$$\begin{aligned} V_{\text{TO}} &= 1.20 V_{\text{stall}} \\ V_{\text{TD}} &= 1.15 V_{\text{stall}} \end{aligned} \tag{1-34}$$

Example 1-5. Determine the takeoff speed of the HP-1 at sea level with maximum gross takeoff weight and the landing speed with maximum landing weight.

From App. A we have $\rho = 1.255 \text{ kg/m}^3$ for sea level.

From Example 1-2*b* we have $C_{L\max} = 2.0$, $W = 1{,}645{,}760$ N, $S_w = 282.5 \text{ m}^2$, and

$$V_{\text{stall}} = \sqrt{\frac{2 \times 1}{1.225 \times 2.0} \frac{1{,}645{,}760}{282.5}} = 69.0 \text{ m/sec}$$

Thus

$$V_{\text{TO}} = 1.20 V_{\text{stall}} = 82.8 \text{ m/sec } (\approx 185 \text{ mi/hr})$$

For landing, $W = 1{,}356{,}640$ N.

$$V_{\text{stall}} = \sqrt{\frac{2 \times 1}{1.225 \times 2.0} \frac{1{,}356{,}640}{282.5}} = 62.6 \text{ m/sec}$$

Thus

$$V_{\text{TD}} = 1.15 V_{\text{stall}} = 72.0 \text{ m/sec } (\approx 161 \text{ mi/hr})$$

Fuel Consumption

The rate of change of the aircraft weight dW/dt is due to the fuel consumed by the engines. The mass rate of fuel consumed is equal to the product of the installed thrust T and the installed thrust specific fuel consumption. For constant acceleration of gravity g_0, we can write

$$\frac{dW}{dt} = -\dot{w}_f = -\dot{m}_f \frac{g_0}{g_c} = -T(\text{TSFC})\left(\frac{g_0}{g_c}\right)$$

This equation can be rewritten in dimensionless form as

$$\boxed{\frac{dW}{W} = -\frac{T}{W}(\text{TSFC})\left(\frac{g_0}{g_c}\right)dt} \tag{1-35}$$

ESTIMATE OF TSFC. Equation (1-35) requires estimates of installed engine thrust T and installed TSFC to calculate the change in aircraft weight. For many flight conditions, the installed engine thrust T equals the aircraft drag D. The value of TSFC depends on the engine cycle, altitude, and Mach number. For preliminary analysis, the following equations (from Ref. 7) can be used to estimate TSFC in units of (lbm/hr)/lbf and θ is the dimensionless temperature ratio T/T_{ref}.

a. High-bypass-ratio turbofan

$$\text{TSFC} = (0.4 + 0.45M_0)\sqrt{\theta} \tag{1-36a}$$

b. Low-bypass-ratio, mixed-flow turbofan

Military and lower power settings:

$$\text{TSFC} = (1.0 + 0.35M_0)\sqrt{\theta} \tag{1-36b}$$

Maximum power setting:

$$\text{TSFC} = (1.8 + 0.30M_0)\sqrt{\theta} \tag{1-36c}$$

c. Turbojet

Military and lower power settings:

$$\text{TSFC} = (1.3 + 0.35M_0)\sqrt{\theta} \tag{1-36d}$$

Maximum power setting:

$$\text{TSFC} = (1.7 + 0.26M_0)\sqrt{\theta} \tag{1-36e}$$

d. Turboprop

$$\text{TSFC} = (0.2 + 0.9M_0)\sqrt{\theta} \tag{1-36f}$$

ENDURANCE. For level unaccelerated flight, thrust equals drag ($T = D$) and lift equals weight ($L = W$). Thus Eq. (1-35) is simply

$$\boxed{\frac{dW}{W} = -\frac{C_D}{C_L}(\text{TSFC})\left(\frac{g_0}{g_c}\right)dt} \tag{1-37}$$

We define the endurance factor EF as

$$\boxed{\text{EF} \equiv \frac{C_L}{C_D(\text{TSFC})}\frac{g_c}{g_0}} \tag{1-38}$$

Then Eq. (1-37) becomes

$$\frac{dW}{W} = -\frac{dt}{\text{EF}} \tag{1-39}$$

Note that the minimum fuel consumption for a time t occurs at the flight condition where the endurance factor is maximum.

For the case when the endurance factor EF is constant or nearly constant, Eq. (1-39) can be integrated from the initial to final conditions and the following expression obtained for the aircraft weight fraction:

$$\boxed{\frac{W_f}{W_i} = \exp\left(-\frac{t}{\text{EF}}\right)} \tag{1-40a}$$

or

$$\boxed{\frac{W_f}{W_i} = \exp\left[-\frac{C_D}{C_L}(\text{TSFC})t\frac{g_0}{g_c}\right]} \tag{1-40b}$$

RANGE. For portions of aircraft flight where distance is important, the differential time dt is related to the differential distance ds by

$$ds = V\,dt \tag{1-41}$$

Substituting into Eq. (1-37) gives

$$\boxed{\frac{dW}{W} = -\frac{C_D}{C_L}\frac{\text{TSFC}}{V}\frac{g_0}{c_g}ds} \tag{1-42}$$

We define the range factor RF as

$$\boxed{\text{RF} \equiv \frac{C_L}{C_D}\frac{V}{\text{TSFC}}\frac{g_c}{g_0}} \tag{1-43}$$

Then Eq. (1-42) can be simply written as

$$\frac{dW}{W} = -\frac{ds}{\text{RF}} \tag{1-44}$$

Note that the minimum fuel consumption for a distance s occurs at the flight condition where the range factor is maximum.

For the flight conditions where the RF is constant or nearly constant, Eq. (1-42) can be integrated from the initial to final conditions and the following expression obtained for the aircraft weight fraction:

$$\boxed{\frac{W_f}{W_i} = \exp\left(-\frac{s}{\text{RF}}\right)} \tag{1-45a}$$

or

$$\boxed{\frac{W_f}{W_i} = \exp\left(-\frac{C_D}{C_L}\frac{\text{TSFC} \times s}{V}\frac{g_0}{g_c}\right)} \tag{1-45b}$$

This is called the *Breguet range equation*. For the range factor to remain constant, C_L/C_D and V/TSFC need to be constant. Above 36-kft altitude, the ambient temperature is constant and a constant velocity V will correspond to constant Mach and constant TSFC for a fixed throttle setting. If C_L is constant, C_L/C_D will remain constant. Since the aircraft weight W decreases during the flight, the altitude must increase to reduce the density of the ambient air and produce the required lift ($L = W$) while maintaining C_L and velocity constant. This flight profile is called a *cruise climb*.

Example 1-6. Calculate the endurance factor and range factor at Mach 0.8 and 40-kft altitude of hypothetical fighter aircraft HF-1 at 90 percent of maximum gross takeoff weight and a load factor of 1.

Solution.

$$q = \frac{\gamma}{2}\delta P_{\text{ref}} M_0^2 = 0.7 \times 0.1858 \times 2116 \times 0.8^2 = 176.1 \text{ lbf/ft}^2$$

From Fig. 1-24 at $M = 0.8$, $C_{D0} = 0.012$, $K_1 = 0.20$, and $K_2 = 0$.

$$C_L = \frac{nW}{qS_w} = \frac{1 \times 0.9 \times 40{,}000}{176.1 \times 720} = 0.2839$$

$$C_D = K_1 C_L^2 + K_2 C_L + C_{D0} = 0.2(0.2839^2) + 0.012 = 0.0281$$

Using Eq. (1-36*b*), we have

$$\text{TSFC} = (1.0 + 0.35 M_0)\sqrt{\theta} = (1.0 + 0.35 \times 0.8)\sqrt{0.7519} = 1.110 \text{(lbm/hr)/lbf}$$

Thus
$$\text{EF} = \frac{C_L}{C_D(\text{TSFC})}\frac{g_c}{g_0} = \frac{0.2839}{0.0281 \times 1.110}\frac{32.174}{32.174} = 9.102 \text{ hr}$$

$$\text{RF} = \frac{C_L}{D_C}\frac{V}{\text{TSFC}}\frac{g_c}{g_0}$$

$$= \frac{0.2839}{0.0281}\frac{0.8 \times 0.8671 \times 1116 \text{ ft/sec}}{1.110 \text{(lbm/hr)/lbf}}\frac{3600 \text{ sec/hr}}{6080 \text{ ft/nm}}\frac{32.174}{32.174}$$

$$= 4170 \text{ nm}$$

Example 1-7. Determine the variation in endurance factor and range factor for the two hypothetical aircraft HF-1 and HP-1.

a. The endurance factor EF is plotted versus Mach number and altitude in Fig. 1-30 for our hypothetical fighter aircraft HF-1 at 90 percent of maximum gross takeoff weight. Note that the best endurance Mach number (minimum fuel consumption) increases with altitude and the best fuel consumption occurs at altitudes of 30 and 36 kft. The range factor is plotted versus Mach number and altitude in Fig. 1-31 for the HF-1 at 90 percent of maximum gross takeoff weight. Note that the best cruise Mach number (minimum fuel consumption) increases with altitude and the best fuel consumption occurs at an altitude of 36 kft and Mach number of 0.8.

b. The endurance factor is plotted versus Mach number and altitude in Fig. 1-32 for our hypothetical passenger aircraft HP-1 at 95 percent of maximum gross takeoff weight. Note that the best endurance Mach number (minimum fuel

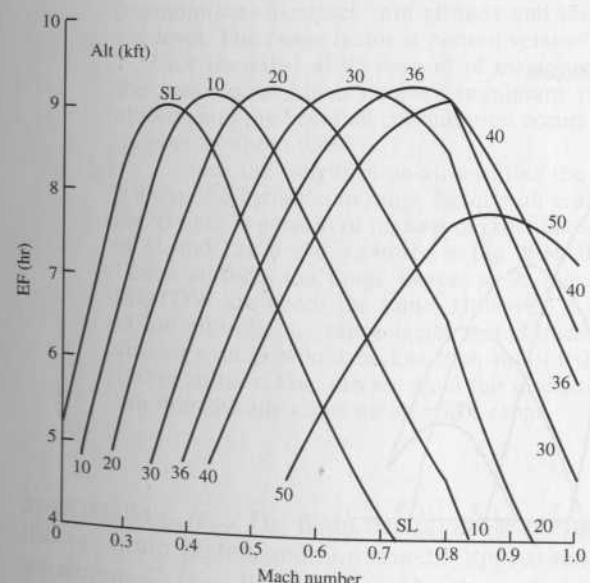


FIGURE 1-30
Endurance factor for HF-1 aircraft.

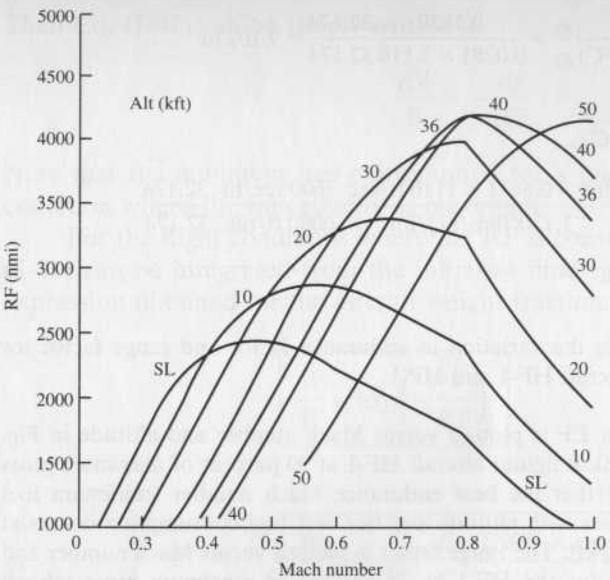


FIGURE 1-31
Range factor for HF-1 aircraft.

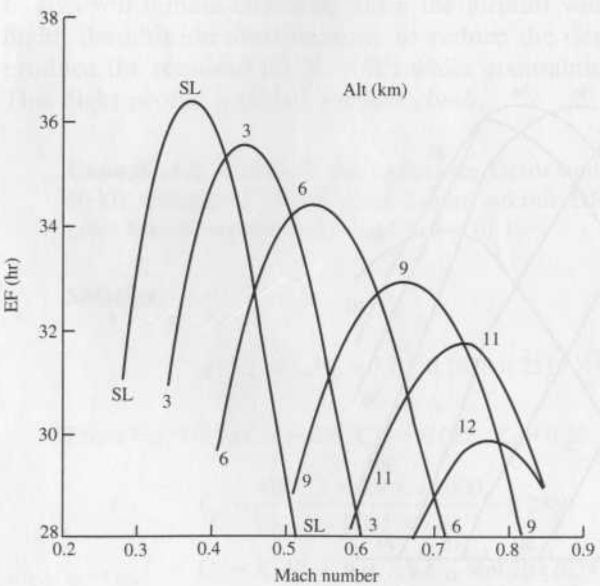


FIGURE 1-32
Endurance factor for HP-1 aircraft.

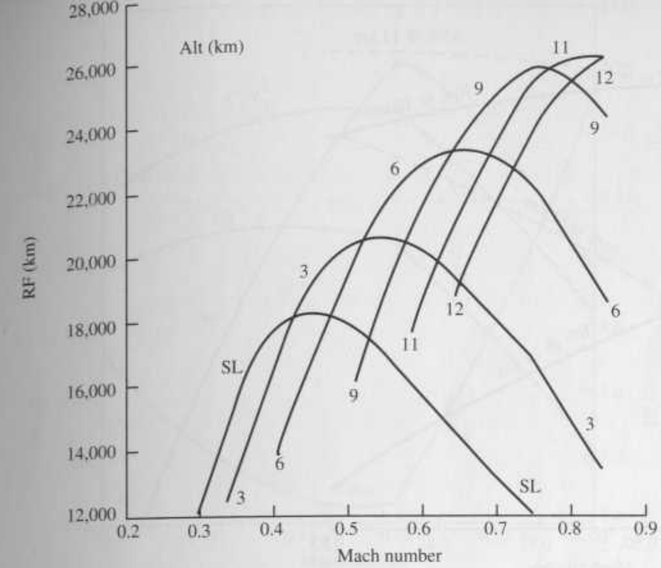


FIGURE 1-33
Range factor for HP-1 aircraft for various altitudes.

consumption) increases with altitude and the best fuel consumption occurs at sea level. The range factor is plotted versus Mach number and altitude in Fig. 1-33 for the HP-1 at 95 percent of maximum gross takeoff weight. Note that the best cruise Mach number (minimum fuel consumption) increases with altitude and the best fuel consumption occurs at an altitude of 11 km and Mach number of about 0.83.

Since the weight of an aircraft like the HP-1 can vary considerably over a flight, the variation in range factor with cruise Mach number was determined for 95 and 70 percent of maximum gross takeoff weight (MGTOW) at altitudes of 11 and 12 km and is plotted in Fig. 1-34. If the HP-1 flew at 0.83 Mach and 12-km altitude, the range factors at 95 percent MGTOW and at 70 percent MGTOW are about the same. However, if the HP-1 flew at 0.83 Mach and 11-km altitude, the range factor would decrease with aircraft weight and the aircraft's range would be less than that of the HP-1 flown at 0.83 Mach and 12-km altitude. One can see from this discussion that the proper cruise altitude can dramatically affect an aircraft's range.

MAXIMUM C_L/C_D. For flight conditions requiring minimum fuel consumption, the optimum flight condition can be approximated by that corresponding to maximum C_L/C_D. From Eq. (1-32), the maximum C_L/C_D (minimum C_D/C_L) can be found by taking the derivative of the following expression, setting it

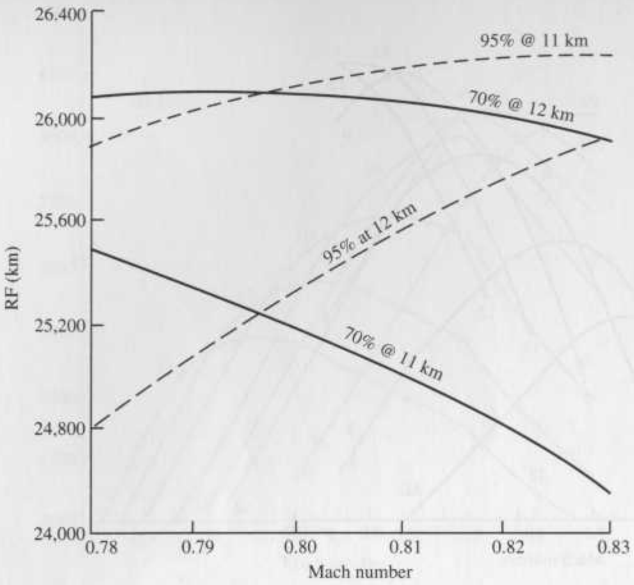


FIGURE 1-34
Range factor for HP-1 aircraft at 70 and 95% MGTOW.

equal to zero, and solving for the C_L that gives minimum C_D/C_L:

$$\frac{C_D}{C_L} = K_1 C_L + K_2 + \frac{C_{D0}}{C_L} \tag{1-46}$$

The lift coefficient that gives maximum C_L/C_D (minimum C_D/C_L) is

$$\boxed{C_L^* = \sqrt{\frac{C_{D0}}{K_1}}} \tag{1-47}$$

and maximum C_L/C_D is given by

$$\boxed{\left(\frac{C_L}{C_D}\right)^* = \frac{1}{2\sqrt{C_{D0}K_1} + K_2}} \tag{1-48}$$

The drag D, range factor, endurance factor, and C_L/C_D versus Mach number at an altitude are plotted in Fig. 1-35 for the HF-1 aircraft and in Fig. 1-36 for the HP-1. Note that the maximum C_L/C_D occurs at Mach 0.8 for the HF-1 and at Mach 0.75 for the HP-1—the same Mach numbers where drags are minimum. The endurance factor is a maximum at a substantially lower Mach number than that corresponding to $(C_L/C_D)^*$ for the HF-1 due to the high TSFC and its increase with Mach number [see Eq. (1-36*b*)]. The endurance factor for the HP-1 is a maximum at the same Mach number that

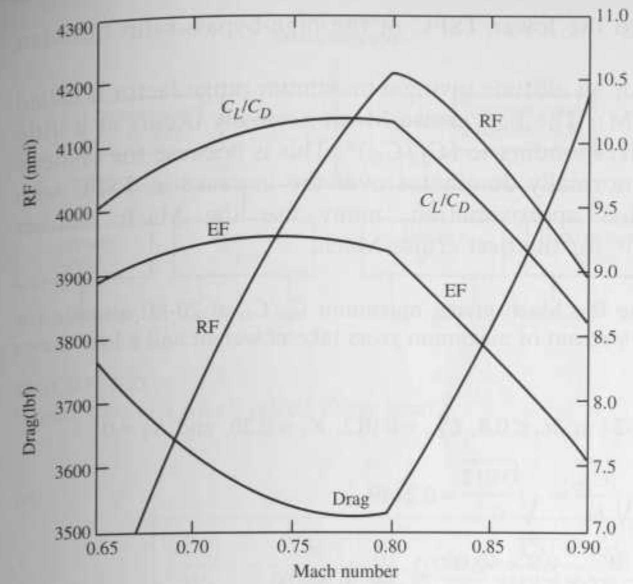


FIGURE 1-35
Comparison of drag C_L/C_D, endurance factor, and range factor for the HF-1 at 36-kft altitude.

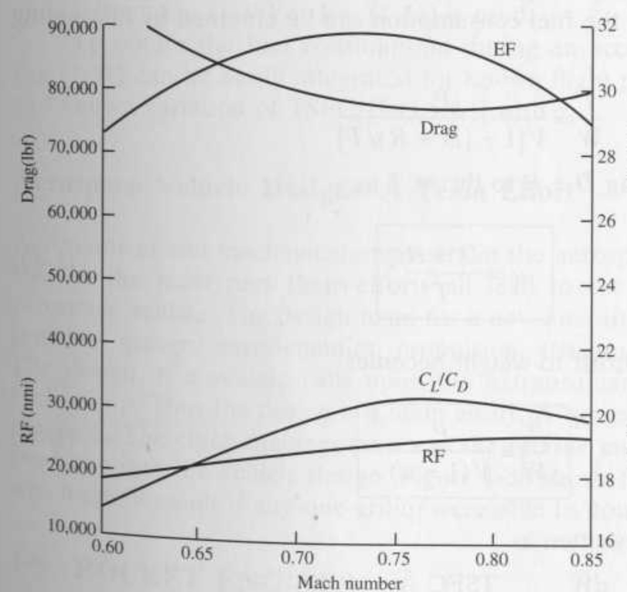


FIGURE 1-36
Comparison of drag C_L/C_D, endurance factor, and range factor for the HP-1 at 11-km altitude.

C_L/C_D is maximum due to the lower TSFC of the high-bypass-ratio turbofan engine [see Eq. (1-36*a*)].

The Mach number for an altitude giving a maximum range factor is called the *best cruise Mach* (BCM). The best cruise Mach normally occurs at a little higher Mach than that corresponding to $(C_L/C_D)^*$. This is because the velocity term in the range factor normally dominates over the increase in TSFC with Mach number. As a first approximation, many use the Mach number corresponding to $(C_L/C_D)^*$ for the best cruise Mach.

Example 1-8. Calculate the Mach giving maximum C_L/C_D at 20-kft altitude for the HF-1 aircraft at 90 percent of maximum gross takeoff weight and a load factor of 1.

***Solution*.** From Fig. 1-24 at $M_0 < 0.8$, $C_{D0} = 0.012$, $K_1 = 0.20$, and $K_2 = 0$.

$$C_L^* = \sqrt{\frac{C_{D0}}{K_1}} = \sqrt{\frac{0.012}{0.2}} = 0.2449$$

$$q = \frac{W}{C_L S_w} = \frac{0.9 \times 40{,}000}{0.2449 \times 720} = 204.16 \text{ lbf/ft}^2$$

$$M_0 = \sqrt{\frac{q}{(\gamma/2)\delta P_{\text{ref}}}} = \sqrt{\frac{204.16}{0.7 \times 0.4599 \times 2116}} = 0.547$$

ACCELERATED FLIGHT. For flight conditions when thrust T is greater than drag D, an expression for the fuel consumption can be obtained by first noting from Eq. (1–28) that

$$\frac{T}{W} = \frac{P_s}{V[1 - (D + R)/T]}$$

We define the ratio of drag $D + R$ to thrust T as

$$u \equiv \frac{D + R}{T} \tag{1-49}$$

The above equation for thrust to weight becomes

$$\frac{T}{W} = \frac{P_s}{V(1 - u)} \tag{1-50}$$

Now Eq. (1-35) can be rewritten as

$$\frac{dW}{W} = -\frac{\text{TSFC}}{V(1 - u)}\frac{g_0}{g_c} P_s\, dt$$

Since $P_s\, dt = dz_e$, the above equation can be expressed in its most useful forms

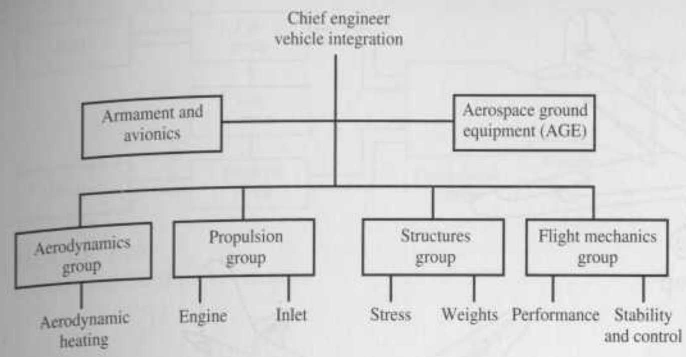


FIGURE 1-37
Organization of a typical vehicle design team.

as

$$\frac{dW}{W} = -\frac{\text{TSFC}}{V(1 - u)}\frac{g_0}{g_c}\, dz_e = -\frac{\text{TSFC}}{V(1 - u)}\frac{g_0}{g_c}\, d\left(h + \frac{V^2}{2g}\right) \tag{1-51}$$

The term $1 - u$ represents the fraction of engine power that goes to increasing the aircraft energy z_e, and u represents that fraction that is lost to aircraft drag $D + R$. Note that this equation applies for cases when u is not unity. When u is unity, either Eq. (1-39) or Eq. (1-44) is used.

To obtain the fuel consumption during an acceleration flight condition, Eq. (1-51) can be easily integrated for known flight paths (values of V and z_e) and known variation of $\text{TSFC}/[V(1 - u)]$ with z_e.

Aerospace Vehicle Design—A Team Effort

Aeronautical and mechanical engineers in the aerospace field do many things, but for the most part their efforts all lead to the design of some type of aerospace vehicle. The design team for a new aircraft may be divided into four principal groups: aerodynamics, propulsion, structures, and flight mechanics. The design of a vehicle calls upon the extraordinary talents of engineers in each group. Thus the design is a team effort. A typical design team is shown in Fig. 1-37. The chief engineer serves as the referee and integrates the efforts of everyone into the vehicle design. Figure 1-38 shows the kind of aircraft design which might result if any one group were able to dominate the others.

1-6 ROCKET ENGINES

Non-air-breathing propulsion systems are characterized by the fact that they carry both fuel and the oxidizer within the aerospace vehicle. Such systems

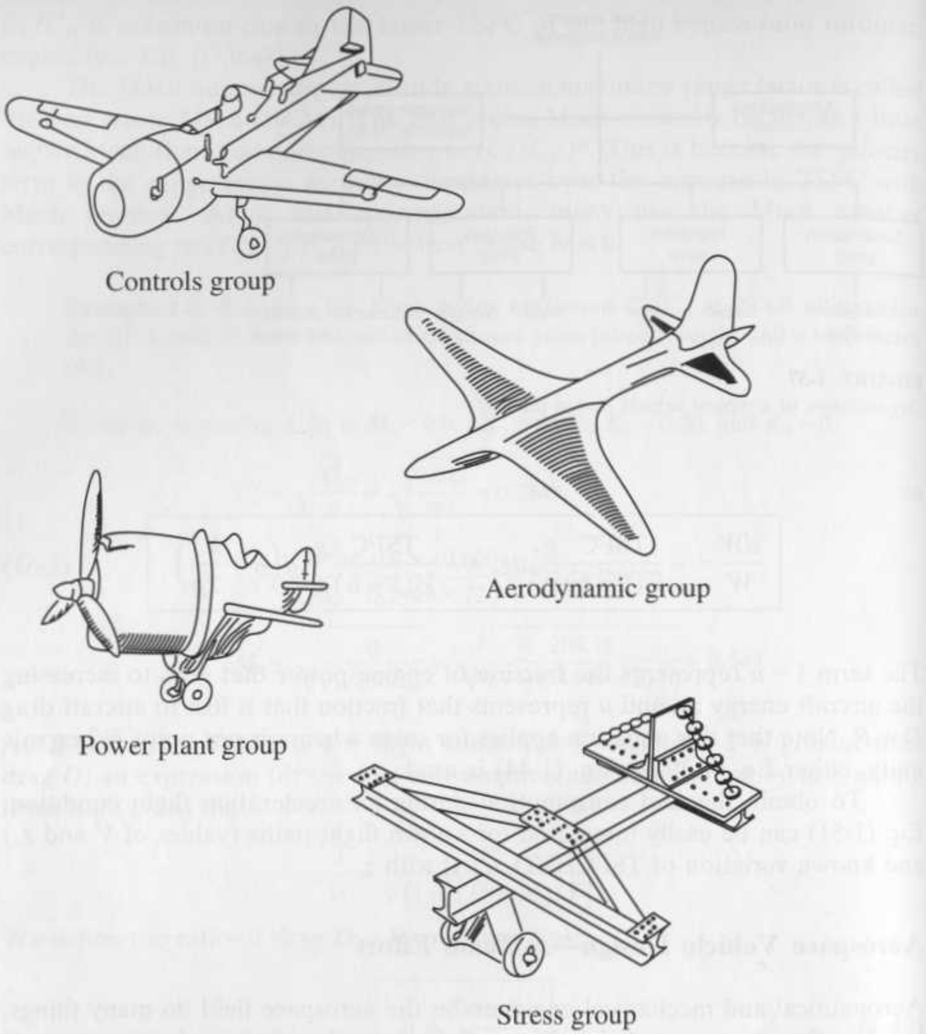


FIGURE 1-38
Aircraft designs.

thus may be used anywhere in space as well as in the atmosphere. Figure 1-39 shows the essential features of a liquid-propellant rocket system. Two propellants (an oxidizer and a fuel) are pumped into the combustion chamber where they ignite. The nozzle accelerates the products of combustion to high velocities and exhausts them to the atmosphere or space.

A solid-propellant rocket motor is the simplest of all propulsion systems. Figure 1-40 shows the essential features of this type of system. In this system, the fuel and oxidizer are mixed together and cast into a solid mass called the *grain*. The grain, usually formed with a hole down the middle called the

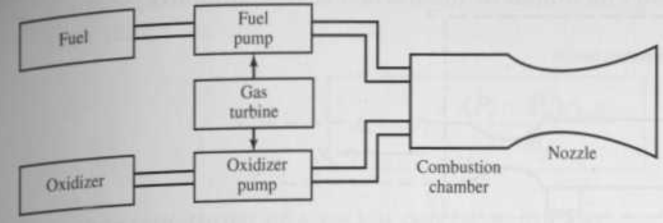


FIGURE 1-39
Liquid-propellant rocket motor.

perforation, is firmly cemented to the inside of the combustion chamber. After ignition, the grain burns radially outward, and the hot combustion gases pass down the perforation and are exhausted through the nozzle.

The absence of a propellant feed system in the solid-propellant rocket is one of its major advantages. Liquid rockets, on the other hand, may be stopped and later restarted, and their thrust may be varied somewhat by changing the speed of the fuel and oxidizer pumps.

Rocket Engine Thrust

A natural starting point in understanding the performance of a rocket is the examination of the static thrust. Application of the momentum equation developed in Chap. 2 will show that the static thrust is a function of the propellant flow rate $\dot{m}_p$, the exhaust velocity V_e and pressure P_e, the exhaust area A_e, and the ambient pressure P_a. Figure 1-41 shows a schematic of a stationary rocket to be considered for analysis. We assume the flow to be one-dimensional, with a steady exit velocity V_e and propellant flow rate $\dot{m}_p$. About this rocket we place a control volume σ whose control surface intersects the exhaust jet perpendicularly through the exit plane of the nozzle. Thrust acts in the direction opposite to the direction of V_e. The reaction to the thrust F necessary to hold the rocket and control volume stationary is shown in Fig. 1-41.

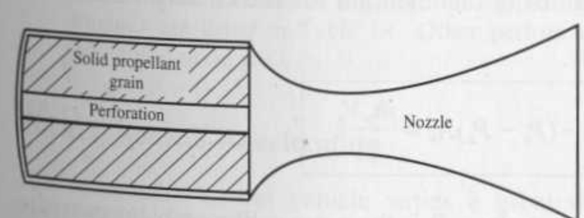


FIGURE 1-40
Solid-propellant rocket motor.

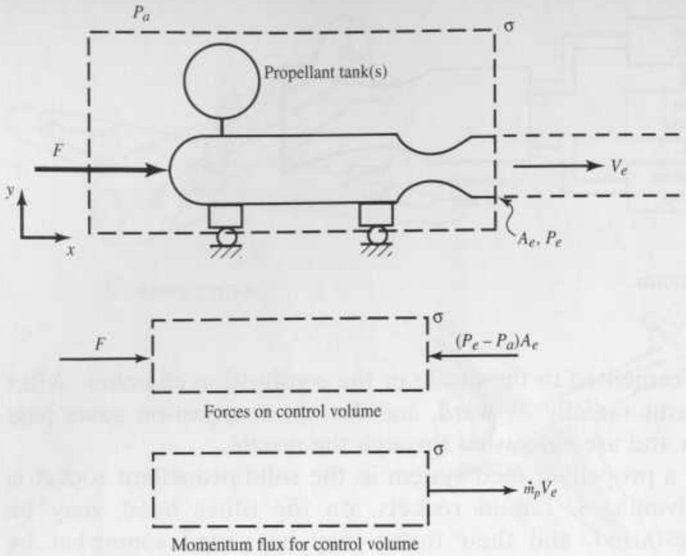


FIGURE 1-41
Schematic diagram of static rocket engine.

The momentum equation applied to this system gives the following:

1. Sum of forces acting on the outside surface of the control volume:

$$\sum F_x = F - (P_e - P_a)A_e$$

2. The net rate of change of momentum for the control volume:

$$\Delta(\text{momentum}) = \dot{M}_{\text{out}} = \frac{\dot{m}_p V_e}{g_c}$$

Since the sum of the forces acting on the outside of the control volume is equal to the net rate of change of the momentum for the control volume, we have

$$F - (P_e - P_a)A_e = \frac{\dot{m}_p V_e}{g_c} \tag{1-52}$$

If the pressure in the exhaust plane P_e is the same as the ambient pressure P_a, the thrust is given by $F = \dot{m}_p V_e / g_c$. The condition $P_e = P_a$ is called *on-design* or *optimum expansion* because it corresponds to maximum thrust for the given

chamber conditions. It is convenient to define an *effective exhaust velocity C* such that

$$C \equiv V_e + \frac{(P_e - P_a)A_e g_c}{\dot{m}_p} \tag{1-53}$$

Thus the static thrust of a rocket can be written as

$$F = \frac{\dot{m}_p C}{g_c} \tag{1-54}$$

Specific Impulse

The *specific impulse* I_{sp} for a rocket is defined as the thrust per unit of propellant weight flow

$$I_{sp} \equiv \frac{F}{\dot{w}_p} = \frac{F}{\dot{m}_p}\frac{g_c}{g_0} \tag{1-55}$$

where g_0 is the acceleration due to gravity at sea level. The unit of I_{sp} is the second. From Eqs. (1-54) and (1-55), the specific impulse can also be written as

$$I_{sp} = \frac{C}{g_0} \tag{1-56}$$

Example 1-9. Find the specific impulse of the space shuttle main engine (SSME) which produces 470,000 lbf in a vacuum with a propellant weight flow of 1030 lbf/sec. By using Eq. (1-55), we find that the SSME has a specific impulse I_{sp} of 456 sec (= 470,000/1030) in vacuum.

An estimate of the variation in thrust with altitude for the space shuttle main engine is shown in Fig. 1-42. The typical specific impulses for some rocket engines are listed in Table 1-6. Other performance data for rocket engines are contained in App. C.

Rocket Vehicle Acceleration

The mass of a rocket vehicle varies a great deal during flight due to the consumption of the propellant. The velocity that a rocket vehicle attains during powered flight can be determined by considering the vehicle in Fig. 1-43.

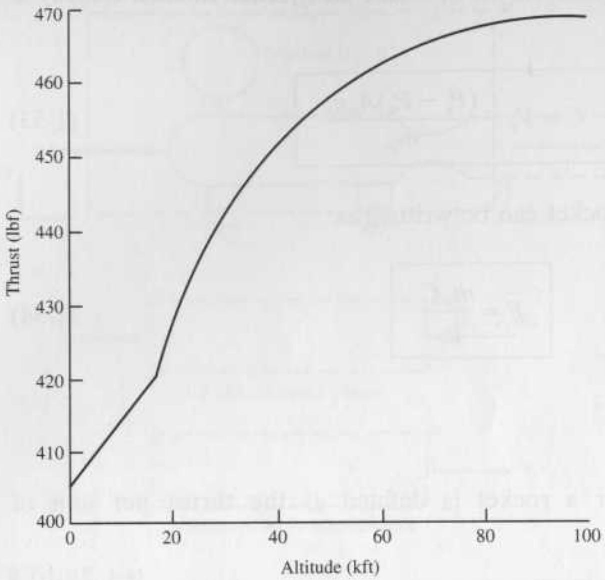


FIGURE 1-42
Rocket thrust variation with altitude.

The figure shows an accelerating rocket vehicle in a gravity field. At some time, the mass of the rocket is m and its velocity is V. In an infinitesimal time dt, the rocket exhausts an incremental mass dm_p with an exhaust velocity V_e relative to the rocket as the rocket velocity changes to $V + dV$. The net change in momentum of the control volume σ is composed of the momentum out of the rocket at the exhaust plus the change of the momentum of the rocket. The momentum out of the rocket in the V direction is $-V_e\,dm_p$, and the change in the momentum of the rocket in the V direction is $m\,dV$. The forces acting on

TABLE 1-6
Ranges of specific impulse I_{sp} for typical rocket engines

Fuel/oxidizer	I_{sp} (sec)
Solid propellant	250
Liquid O_2: kerosene (RP)	310
Liquid O_2: H_2	410
Nuclear fuel: H_2 propellant	840

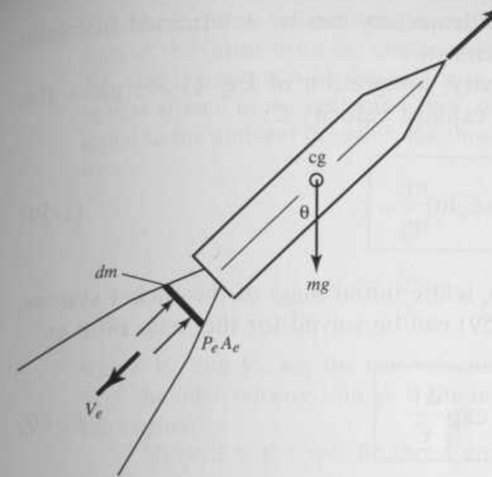


FIGURE 1-43
Rocket vehicle in flight.

the control volume σ are composed of the net pressure force, the drag D, and the gravitational force. The sum of these forces in the V direction is

$$\sum F_V = (P_e - P_a)A_e - D - \frac{mg}{g_c}\cos\theta$$

The resultant impulse on the rocket $(\sum F_V)\,dt$ must equal the momentum change of the system $\Delta(\text{momentum}) = (-V_e\,dm_p + m\,dV)/g_c$. Thus

$$\left[(P_e - P_a)A_e - D - \frac{mg}{g_c}\cos\theta\right]dt = \frac{-V_e\,dm_p + m\,dV}{g_c}$$

From the above relationship, the momentum change of the rocket $(m\,dV)$ is

$$\frac{m\,dV}{g_c} = \left[(P_e - P_a)A_e - D - \frac{mg}{g_c}\cos\theta\right]dt + \frac{V_e\,dm_p}{g_c} \tag{1-57}$$

Since $dm_p = \dot{m}_p\,dt = -(dm/dt)\,dt$, then Eq. (1-57) can be written as

$$\frac{m\,dV}{g_c} = \left[(P_e - P_a)A_e + \frac{\dot{m}_p V_e}{g_c} - D - \frac{mg}{g_c}\cos\theta\right]dt$$

By using Eq. (1-53), the above relationship becomes

$$\frac{m\,dV}{g_c} = \left(\frac{\dot{m}_p}{g_c}C - D - \frac{mg}{g_c}\cos\theta\right)dt$$

or

$$\boxed{dV = -C\frac{dm}{m} - \frac{Dg_c}{m}dt - g\cos\theta\,dt} \tag{1-58}$$

The velocity of a rocket along its trajectory can be determined from the above equation if C, D, g, and θ are known.

In the absence of drag and gravity, integration of Eq. (1-58) gives the following, assuming constant effective exhaust velocity C:

$$\Delta V = C \ln \frac{m_i}{m_f} \tag{1-59}$$

where ΔV is the change in velocity, m_i is the initial mass of the rocket system, and m_f is the final mass. Equation (1-59) can be solved for the mass ratio as

$$\frac{m_i}{m_f} = \exp \frac{\Delta V}{C} \tag{1-60}$$

Example 1-10. We want to estimate the mass ratio (final to initial) of an H_2-O_2 ($C = 4000$ m/sec) rocket for an earth orbit ($\Delta V = 8000$ m/sec), neglecting drag and gravity. Using Eq. (1-59), we obtain $m_f/m_i = e^{-2} = 0.132$, or a single-stage rocket would be about 13 percent payload and structure and 87 percent propellant.

PROBLEMS

1-1. Calculate the uninstalled thrust for Example 1-1, using Eq. (1-6).

1-2. Develop the following analytical expressions for a turbojet engine:

a. When the fuel flow rate is very small in comparison with the air mass flow rate, the exit pressure is equal to ambient pressure, and the installation loss coefficients are zero, then the installed thrust T is given by

$$T = \frac{\dot{m}_0}{g_c}(V_e - V_0)$$

b. For the above conditions, the thrust specific fuel consumption is given by

$$\text{TSFC} = \frac{Tg_c/\dot{m}_0 + 2V_0}{2\eta_T h_{PR}}$$

c. For $V_0 = 0$ and 500 ft/sec, plot the above equation for TSFC [in (lbm/hr)/lbf] versus specific thrust $T/\dot{m}_0$ [in lbf/(lbm/sec)] for values of specific thrust from 0 to 120. Use $\eta_T = 0.4$ and $h_{PR} = 18{,}400$ Btu/lbm.

d. Explain the trends.

1-3. Repeat 1-2*c*, using SI units. For $V_0 = 0$ to 150 m/sec, plot TSFC [in (mg/sec)/N] versus specific thrust $T/\dot{m}_0$ [in N/(kg/sec)] for values of specific thrust from 0 to 1200. Use $\eta_T = 0.4$ and $h_{PR} = 42{,}800$ kJ/kg.

1-4. A J57 turbojet engine is tested at sea-level, static, standard-day conditions ($P_0 = 14.696$ psia, $T_0 = 518.7$°R, and $V_0 = 0$). At one test point, the thrust is 10,200 lbf while the airflow is 164 lbm/sec and the fuel flow is 8520 lbm/hr. Using these data, estimate the exit velocity V_e for the case of exit pressure equal to ambient pressure ($P_0 = P_e$).

1-5. The thrust for a turbofan engine with separate exhaust streams is equal to the sum of the thrust from the engine core F_C and the thrust from the bypass stream F_B. The bypass ratio of the engine α is the ratio of the mass flow through the bypass stream to the core mass flow, or $\alpha \equiv \dot{m}_B/\dot{m}_C$. When the exit pressures are equal to the ambient pressure, the thrusts of the core and bypass stream are given by

$$F_C = \frac{1}{g_c}[(\dot{m}_C + \dot{m}_f)V_{Ce} - \dot{m}_C V_0]$$

$$F_B = \frac{\dot{m}_B}{g_c}(V_{Be} - V_0)$$

where V_{Ce} and V_{Be} are the exit velocities from the core and bypass, respectively, V_0 is the inlet velocity, and $\dot{m}_f$ is the mass flow rate of fuel burned in the core of the engine.

Show that the specific thrust and thrust specific fuel consumption can be expressed as

$$\frac{F}{\dot{m}_0} = \frac{1}{g_c}\left(\frac{1 + \dot{m}_f/\dot{m}_C}{1+\alpha} V_{Ce} + \frac{\alpha}{1+\alpha} V_{Be} - V_0\right)$$

$$S = \frac{\dot{m}_f}{F} = \frac{\dot{m}_f/\dot{m}_C}{(F/\dot{m}_0)(1+\alpha)}$$

where $\dot{m}_0 = \dot{m}_C + \dot{m}_B$.

1-6. The CF6 turbofan engine has a rated thrust of 40,000 lbf at a fuel flow rate of 13,920 lbm/hr at sea-level static conditions. If the core airflow rate is 225 lbm/sec and the bypass ratio is 6.0, what are the specific thrust [lbf/(lbm/sec)] and thrust specific fuel consumption [(lbm/hr)/lbf]?

1-7. The JT9D high-bypass-ratio turbofan engine at maximum static power ($V_0 = 0$) on a sea-level, standard day ($P_0 = 14.696$ psia, $T_0 = 518.7$°R) has the following data: the air mass flow rate through the core is 247 lbm/sec, the air mass flow rate through the fan bypass duct is 1248 lbm/sec, the exit velocity from the core is 1190 ft/sec, the exit velocity from the bypass duct is 885 ft/sec and the fuel flow rate into the combustor is 15,750 lbm/hr. Estimate the following for the case of exit pressures equal to ambient pressure ($P_0 = P_e$):

a. The thrust of the engine

b. The thermal efficiency of the engine (heating value of jet fuel is about 18,400 Btu/lbm)

c. The propulsive efficiency and thrust specific fuel consumption of the engine

1-8. Repeat Prob. 1-7, using SI units.

1-9. One advanced afterburning fighter engine, whose performance is depicted in Figs. 1-14*a* through 1-14*e*, is installed in the HF-1 fighter aircraft. Using the aircraft drag data of Fig. 1-26*b*, determine and plot the variation of weight specific excess power (P_s in feet per second) versus flight Mach number for level flight ($n = 1$) at 36-kft altitude. Assume the installation losses are constant with values of $\phi_{\text{inlet}} = 0.05$ and $\phi_{\text{noz}} = 0.02$.

1-10. Determine the takeoff speed of the HF-1 aircraft.

1-11. Determine the takeoff speed of the HP-1 aircraft at 90 percent of maximum gross takeoff weight.

1-12. Derive Eqs. (1-47) and (1-48) for maximum C_L/C_D. Start by taking the derivative of Eq. (1-46) with respect to C_L and finding the expression for the lift coefficient that gives maximum C_L/C_D.

1-13. Show that for maximum C_L/C_D, the corresponding drag coefficient C_D is given by

$$C_D = 2C_{D0} + K_2\sqrt{\frac{C_{D0}}{K_1}}$$

1-14. An aircraft with a wing area of 800 ft² is in level flight ($n = 1$) at maximum C_L/C_D. Given that the drag coefficients for the aircraft are $C_{D0} = 0.02$, $K_2 = 0$, and $K_1 = 0.2$, find

a. The maximum C_L/C_D and the corresponding values of C_L and C_D

b. The flight altitude [use Eqs. (1-29) and (1-30*b*)] and aircraft drag for an aircraft weight of 45,000 lbf at Mach 0.8

c. The flight altitude and aircraft drag for an aircraft weight of 35,000 lbf at Mach 0.8

d. The range for an installed engine thrust specific fuel consumption rate of 0.8 (lbm/hr)/lbf, if the 10,000-lbf difference in aircraft weight between parts *b* and *c* above is due only to fuel consumption

1-15. An aircraft weighing 110,000 N with a wing area of 42 m² is in level flight ($n = 1$) at the maximum value of C_L/C_D. Given that the drag coefficients for the aircraft are $C_{D0} = 0.03$, $K_2 = 0$, and $K_1 = 0.25$, find the following:

a. The maximum C_L/C_D and the corresponding values of C_L and C_D

b. The flight altitude [use Eqs. (1-29) and (1-30*b*)] and aircraft drag at Mach 0.5

c. The flight altitude and aircraft drag at Mach 0.75

1-16. The Breguet range equation [Eq. (1-45*b*)] applies for a cruise climb flight profile with constant range factor RF. Another range equation can be developed for a level cruise flight profile with varying RF. Consider the case where we keep C_L, C_D, and TSFC constant and vary the flight velocity with aircraft weight by the expression

$$V = \sqrt{\frac{2g_c W}{\rho C_L S_w}}$$

Using the subscripts *i* and *f* for the initial and final flight conditions, respectively, show the following:

a. Substitution of this expression for flight velocity into Eq. (1-42) gives

$$\frac{dW}{\sqrt{W}} = -\frac{\sqrt{W_i}}{\text{RF}_i}\,ds$$

b. Integration of the above between the initial *i* and final *f* conditions gives

$$\frac{W_f}{W_i} = \left[1 - \frac{s}{2(\text{RF}_i)}\right]^2$$

c. For a given weight fraction W_f/W_i, the maximum range *s* for this level cruise flight corresponds to starting the flight at the maximum altitude (minimum density) and maximum value of $\sqrt{C_L}/C_D$.

d. For the drag coefficient equation of Eq. (1-32), maximum $\sqrt{C_L}/C_D$ corresponds to $C_L = (1/6K_1)(\sqrt{12K_1C_{D0} + K_2^2} - K_2)$.

1-17. An aircraft begins a cruise at a wing loading W/S_w of 100 lbf/ft² and Mach 0.8. The drag coefficients are $K_1 = 0.056$, $K_2 = -0.008$, and $C_{D0} = 0.014$, and the fuel consumption TSFC is constant at 0.8 (lbm/hr)/lbf. For a weight fraction W_f/W_i of 0.9, determine the range and other parameters for two different types of cruise.

a. For a cruise climb (max. C_L/C_D) flight path, determine C_L, C_D, initial and final altitudes, and range.

b. For a level cruise (max. $\sqrt{C_L}/C_D$) flight path, determine C_L, C_D, altitude, initial and final velocities, and range.

1-18. An aircraft weighing 70,000 lbf with a wing area of 1000 ft² is in level flight ($n = 1$) at 30-kft altitude. Using the drag coefficients of Fig. 1-24 and the TSFC model of Eq. (1-36*b*), find the following:

a. The maximum C_L/C_D and the corresponding values of C_L, C_D, and Mach number (*Note:* Since the drag coefficients are a function of Mach number and it is an unknown, you must first guess a value for the Mach number to obtain the drag coefficients. Try a Mach number of 0.8 for your first guess.)

b. The C_L, C_D, C_L/C_D, range factor, endurance factor, and drag for flight Mach numbers of 0.74, 0.76, 0.78, 0.80, 0.81, and 0.82

c. The best cruise Mach (maximum RF)

d. The best loiter Mach (maximum EF)

1-19. An aircraft weighing 200,000 N with a wing area of 60 m² is in level flight ($n = 1$) at 9-km altitude. Using the drag coefficients of Fig. 1-24 and TSFC model of Eq. (1-36*b*), find the following:

a. The maximum C_L/C_D and the corresponding values of C_L, C_D, and Mach number (*Note:* Since the drag coefficients are a function of the Mach number and it is an unknown, you must first guess a value for the Mach number to obtain the drag coefficients. Try a Mach number of 0.8 for your first guess.)

b. The C_L, C_D, C_L/C_D, range factor, endurance factor, and drag for flight Mach numbers of 0.74, 0.76, 0.78, 0.80, 0.81, and 0.82

c. The best cruise Mach (maximum RF)

d. The best loiter Mach (maximum EF)

1-20. What is the specific impulse in seconds of the JT9D turbofan engine in Prob. 1-7?

1-21. A rocket motor is fired in place on a static test stand. The rocket exhausts 100 lbm/sec at an exit velocity of 2000 ft/sec and pressure of 50 psia. The exit area of the rocket is 0.2 ft². For an ambient pressure of 14.7 psia, determine the effective exhaust velocity, the thrust transmitted to the test stand, and the specific impulse.

1-22. A rocket motor under static testing exhausts 50 kg/sec at an exit velocity of 800 m/sec and pressure of 350 kPa. The exit area of the rocket is 0.02 m². For an ambient pressure of 100 kPa, determine the effective exhaust velocity, the thrust transmitted to the test stand, and the specific impulse.

1-23. The propellant weight of an orbiting space system amounts to 90 percent of the system gross weight. Given that the system rocket engine has a specific impulse of 300 sec, determine:

a. The maximum attainable velocity if all the propellant is burned and the system's initial velocity is 7930 m/sec *b*. The propellant mass flow rate, given that the rocket engine thrust is 1,670,000 N

1-24. A chemical rocket motor wtih a specific impulse of 400 sec is used in the final stage of a multistage launch vehicle for deep-space exploration. This final stage has a mass ratio (initial to final) of 6, and its single rocket motor is first fired while it orbits the earth at a velocity of 26,000 ft/sec. The final stage must reach a velocity of 36,700 ft/sec to escape the earth's gravitational field. Determine the percentage of fuel that must be used to perform this maneuver (neglect gravity and drag).

1-D1 GAS TURBINE DESIGN PROBLEM 1 (HP-1 AIRCRAFT)

Background

You are to determine the thrust and fuel consumption requirements of the two engines for the hypothetical passenger aircraft, the HP-1. The twin-engine aircraft will cruise at 0.83 Mach and be capable of the following requirements:

1. Takeoff at maximum gross takeoff weight W_{TO} from an airport at 1.6-km pressure altitude on a hot day (38°C) uses a 3650-m (12-kft) runway. The craft is able to maintain a 2.4 percent single-engine climb gradient in the event of engine failure at liftoff.
2. It transports 253 passengers and luggage (90 kg each) over a still-air distance of 11,120 km (6000 nmi). It has 30 min of fuel in reserve at end (loiter).
3. It attains an initial altitude of 11-km at beginning of cruise ($P_s = 1.5$ m/sec).
4. The single-engine craft cruises at 5-km altitude at 0.45 Mach ($P_s = 1.5$ m/sec).

All the data for the HP-1 contained in Example 1-2 apply. Preliminary mission analysis of the HP-1 using the methods of Ref. 12 for the 11,120-km flight with 253 passengers and luggage (22,770-kg payload) gives the following preliminary fuel use:

Description	Distance (km)	Fuel used (kg)
Taxi		200*
Takeoff		840*
Climb and acceleration	330	5,880*
Cruise	10,650	50,240
Descent	140	1,090*
Loiter (30 min at 9-km altitude)		2,350
Land and taxi		600*
	11,120	61,200

* These fuel consumptions can be considered to be constant.

Analysis of takeoff indicates that each engine must produce an installed thrust of 214 kN on a hot day (38°C) at 0.1 Mach and 1.6-km pressure altitude. To provide for reasonable-length landing gear, the maximum diameter of the engine inlet is limited to 2.2 m. Based on standard design practice (see Chap. 10), the maximum mass flow rate per unit area is given by

$$\frac{\dot{m}}{A} = 231.8\frac{\delta_0}{\sqrt{\theta_0}} \qquad \text{(kg/sec)/m}^2$$

Thus on a hot day (38°C) at 0.1 Mach and 1.6-km pressure altitude, $\theta = (38 + 273.1)/288.2 = 1.079$, $\theta_0 = 1.079 \times 1.002 = 1.081$, $\delta = 0.8256$, $\delta_0 = 0.8256 \times 1.007 = 0.8314$, and the maximum mass flow through the 2.2-m-diameter inlet is 704.6 kg/sec.

Calculations

1. If the HP-1 starts out the cruise at 11-km with a weight of 1,577,940 N, find the allowable TSFC for the distance of 10,650 km for the following cases.
   a. Assume the aircraft performs a cruise climb (flies at a constant C_D/C_L). What is its altitude at the end of the cruise climb?
   b. Assume the aircraft cruises at a constant altitude of 11 km. Determine C_D/C_L at the start and end of cruise. Using the average of these two values, calculate the allowable TSFC.
2. Determine the loiter (endurance) Mach numbers for altitudes of 10, 9, 8, 7, and 6 km when the HP-1 aircraft is at 64 percent of W_{TO}.
3. Determine the aircraft drag at the following points in the HP-1 aircraft's 11,120-km flight based on the fuel consumptions listed above:
   a. Takeoff, $M = 0.23$, sea level
   b. Start of cruise, $M = 0.83$, 11 km
   c. End of cruise climb, $M = 0.83$, altitude = ? ft
   d. End of 11-km cruise, $M = 0.83$, 11 km
   e. Engine out (88 percent of W_{TO}), $M = 0.45$, 5 km

1-D2 GAS TURBINE DESIGN PROBLEM 2 (HF-1 AIRCRAFT)

Background

You are to determine the thrust and fuel consumption requirements of the two engines for the hypothetical fighter aircraft HF-1. This twin-engine fighter will supercruise at 1.6 Mach and will be capable of the following requirements:

1. Takeoff at maximum gross takeoff weight W_{TO} from a 1200-ft (366-m) runway at sea level on a standard day.
2. Supercruise at 1.6 Mach and 40-kft altitude for 250 nmi (463 km) at 92 percent of W_{TO}.
3. Perform 5*g* turns at 1.6 Mach and 30-kft altitude at 88 percent of W_{TO}.
4. Perform 5*g* turns at 0.9 Mach and 30-kft altitude at 88 percent of W_{TO}.
5. Perform the maximum mission listed below.

All the data for the HF-1 contained in Example 1-2 apply. Preliminary mission analysis of the HF-1 using the methods of Ref. 12 for the maximum mission gives the following preliminary fuel use:

Description	Distance (nm)	Fuel used (lbm)
Warmup, taxi, takeoff		700*
Climb and acceleration to 0.9 Mach and 40 kft	35	1,800*
Accelerate from 0.9 to 1.6 Mach	12	700*
Supercruise at 1.6 Mach and 40 kft	203	4,400
Deliver payload of 2000 lbf	0	0*
Perform one 5g turn at 1.6 Mach and 30 kft	0	1,000*
Perform two 5g turns at 0.9 Mach and 30 kft	0	700*
Climb to best cruise altitude and 0.9 Mach	23	400*
Cruise climb at 0.9 Mach	227	1,600
Loiter (20 min at 30-kft altitude)		1,100
Land		0*
	500	12,400

* These fuel consumptions can be considered to be constant.

Analysis of takeoff indicates that each engine must produce an installed thrust of 23,500 lbf on a standard day at 0.1 Mach and sea-level altitude. To provide for optimum integration into the airframe, the maximum area of the engine inlet is limited to 5 ft². Based on standard design practice (see Chap. 10), the maximum mass flow rate per unit area for subsonic flight conditions is given by

$$\frac{\dot{m}}{A} = 47.5\frac{\delta_0}{\sqrt{\theta_0}} \quad (\text{lbm/sec})/\text{ft}^2$$

Thus at 0.1 Mach and sea-level standard day, $\theta = 1.0$, $\theta_0 = 1.002$, $\delta = 1.0$, $\delta_0 = 1.007$, and the maximum mass flow through the 5-ft² inlet is 238.9 lbm/sec. For supersonic flight conditions, the maximum mass flow rate per unit area is simply the density of the air ρ times its velocity V.

Calculations

1. If the HF-1 starts the supercruise at 40 kft with a weight of 36,800 lbf, find the allowable TSFC for the distance of 203 nmi for the following cases:
   a. Assume the aircraft performs a cruise climb (flies at a constant C_D/C_L). What is its altitude at the end of the cruise climb?
   b. Assume the aircraft cruises at a constant altitude of 40 kft. Determine C_D/C_L at the start and end of cruise. Using the average of these two values, calculate the allowable TSFC.
2. Find the best cruise altitude for the subsonic return cruise at 0.9 Mach and 70.75 percent of W_{TO}.
3. Determine the loiter (endurance) Mach numbers for altitudes of 32, 30, 28, 26, and 24 kft when the HF-1 aircraft is at 67 percent of W_{TO}.
4. Determine the aircraft drag at the following points in the HF-1 aircraft's maximum mission based on the fuel consumptions listed above:
   a. Takeoff, $M = 0.172$, sea level
   b. Start of supercruise, $M = 1.6$, 40 kft
   c. End of supercruise climb, $M = 1.6$, altitude = ? ft
   d. End of 40-kft supercruise, $M = 1.6$, 40 kft
   e. Start of subsonic cruise, $M = 0.9$, altitude = best cruise altitude
   f. Start of loiter, altitude = 30 kft

CHAPTER 2

THERMODYNAMICS REVIEW

2-1 INTRODUCTION

The operation of gas turbine engines and of rocket motors is governed by the laws of mechanics and thermodynamics. The field of mechanics includes the mechanics of both fluids and solids. However, since the process occurring in most propulsion devices involves a flowing fluid, our emphasis will be fluid mechanics or, more specifically, gas dynamics.

With the aid of definitions and experimentally observed phenomena, logical deductions have been made over the years leading to the fundamental laws of mechanics and thermodynamics. Initially the development of these sciences was based on intuition and the accumulation of many different, but not always unrelated, theorems and rules. Frequently the understanding of certain concepts and phenomena was hindered by ambiguous and conflicting definitions. Today, as a result of years of work in mechanics and thermodynamics, we can present an efficient and logical introduction to these sciences based on the schematic outline of Fig. 2-1. In this figure, the terms *fundamental laws, theorems,* and *corollaries* have the following meanings. A *fundamental law* is a statement that can be neither deduced logically from definitions nor established by a finite number of experimental observations. A fundamental law is usually a generalization of experimental results beyond the region covered by the experiments themselves. A *theorem* is a statement whose validity depends upon the validity of a given set of laws. A *corollary* is a more or less self-evident statement following a definition, law, or theorem.

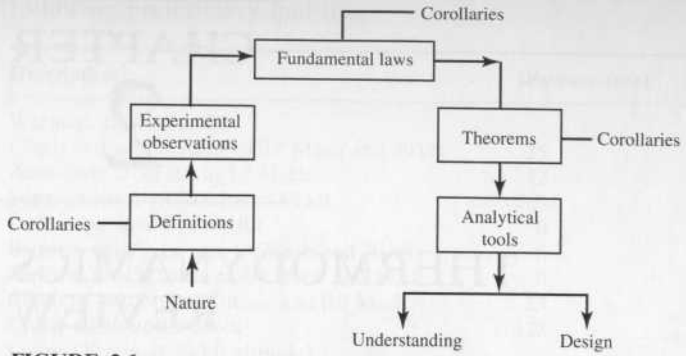


FIGURE 2-1
Interrelationship of definitions, laws, and theorems.

Our approach, then, will be as follows:

1. Definitions will be given which enable us to describe the phenomena of interest.
2. With a necessary and sufficient set of terms so defined, we will indicate how experimental observations of these defined quantities—alone or as they interact with each other—lead to the statement of certain laws of nature.
3. From definitions and laws, corollaries and theorems will be stated and analytical tools shaped. These tools will be used in the study of propulsion systems and the gas flow through components of propulsion devices.

This chapter begins by setting forth the definitions upon which many experimental observations in mechanics and aeronautics are based. The concepts of mass, energy, entropy, and momentum are then introduced, and the basic laws are developed for a system (control mass or closed-system) and control volume (open system). The following chapters use these basic laws in developing analytical tools for the study of one-dimensional gas dynamics, rocket propulsion, and aircraft propulsion.

2-2 DEFINITIONS

Before introducing the concepts of mass, energy, entropy, and momentum, we consider some basic definitions.

System and Control Volume

The system and the control volume play the part in the mechanics of fluids that a free body serves in the mechanics of rigid bodies. In fact, the free body of mechanics is simply a special case of a system.

A *system* is any collection of matter of fixed identity within a prescribed boundary. The boundary of a system is not necessarily rigid; hence, the volume of a system may change. Everything external to the system is called the *surroundings*, and the surface which separates the system from its surroundings is called the *system boundary*. A system, then, is the body or substance which one focuses attention upon in order to observe its behavior alone or as it interacts with the surroundings. Consider Newton's second law of motion in the form $F = ma$. The mass m in this equation is the mass of a system, F is the resultant force (interaction) exerted by the surroundings on the system, and a is the acceleration of the center of mass of the system.

Sometimes it is more convenient to analyze a fluid flow problem by fixing one's attention on a region through which fluid is flowing rather than by studying a fluid system. For this reason we introduce the concept of a control volume.

A *control volume* is any prescribed volume in space bounded by a *control surface* through which matter may flow and across which interactions with the surroundings may occur. Often in our study of fluid flows, we will use the control volume approach rather than the system approach. Each approach is equally valid (Fig. 2-2), and the method selected for a particular problem is simply a matter of convenience.

Classes of Forces

We can identify two classes of forces: boundary or contact forces and distant-acting or body forces. *Boundary forces* act on the boundaries of systems. A boundary force is the force of one system upon another at the point of contact of the two system boundaries. In order for system A to exert a boundary force on system B of magnitude F_{AB}, the boundary of A must be in contact with the boundary of B.

A *body force* is due to distant-acting influences such as gravity, magnetic

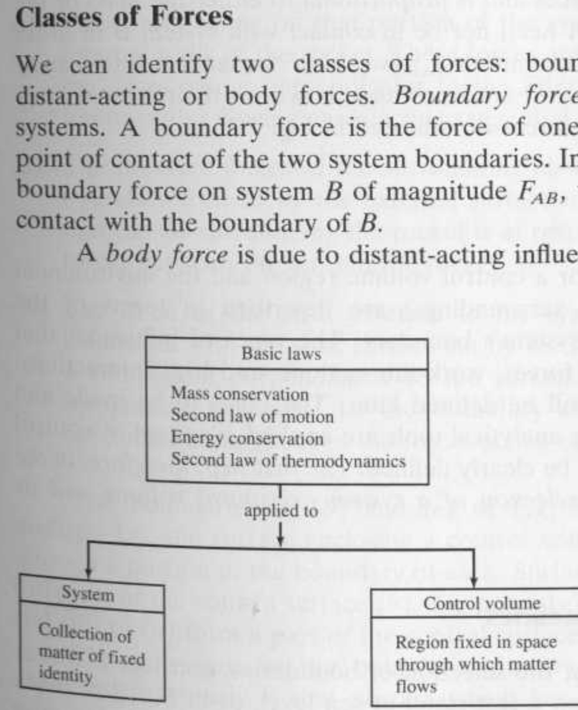
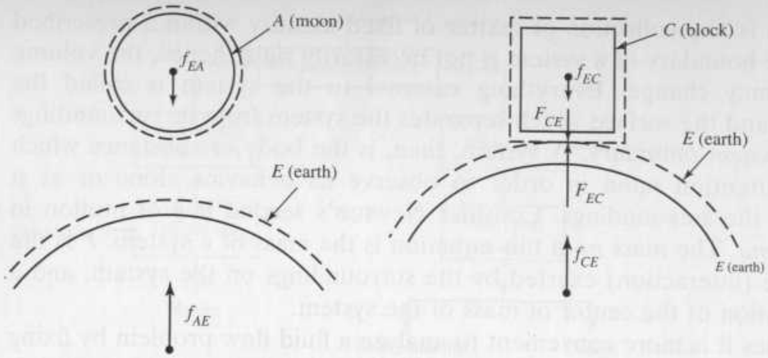


FIGURE 2-2
First step in application of basic laws is the selection of a system and its boundary or a control volume and its boundary.



(*a*) Earth and moon exert equal and opposite body forces upon eah other. The body forces act at the center of gravity of each.

(*b*) A block resting on the earth has a body force f_{CE} (its weight) and exerts a body force f_{CE} on the earth. These body forces give rise to the contact forces F_{CE} and F_{EC} at the point of contact of *C* and *E*.

FIGURE 2-3
Body and contact forces.

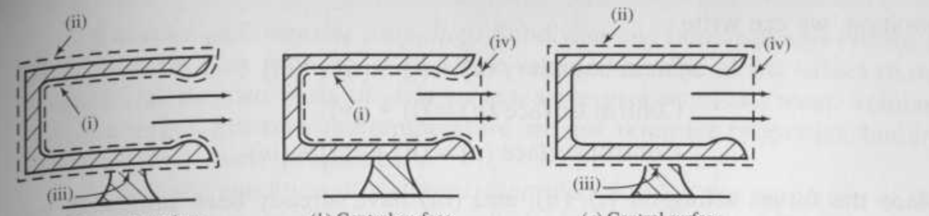
effects, and electrodynamic forces and is proportional to either the mass or the volume of the body. System *A* need not be in contact with system *B* in order for *A* to exert a body force of magnitude f_{AB} (we use *F* for contact forces and *f* for body forces) on *B*. If *A* and *B* are in contact, however, their mutual body forces can give rise to contact forces as indicated in Fig. 2-3.

Influences and Boundaries

Influences between a system or a control volume region and the environment external to the system (the surroundings) are described in terms of the phenomena occurring at the system's boundary. The types of influences that will concern us are boundary forces, work interactions, and heat interactions. Work and heat interactions will be defined later. The point to be made and emphasized here is that before analytical tools are applied, a system or control volume and its boundary must be clearly defined. *The first step, therefore, in the solution of a problem is the selection of a system or control volume and its boundary.*

Example: Selection of Boundaries

To illustrate the importance of the selection of boundaries, consider a rocket engine mounted horizontally on a test stand at sea level, as in Fig. 2-4. If we wish to examine the various forces acting on the rocket, we must select a



(*a*) System boundary (*b*) Control surface (*c*) Control surface

FIGURE 2-4
Three different boundaries for examining forces on a rocket engine.

system or control volume and identify its boundary. Each of three boundaries shown in Fig. 2-4 identifies either a system or a control volume having forces acting thereon.

Let the boundary in (*a*) of the figure define a *system*. The forces of the surroundings on the system are conveniently examined in terms of the portions of the system boundary coincident with (i) the rocket's internal surface, (ii) the rocket's external surface, and (iii) the surface of the strut cut by the boundary. The forces are as follows:

(i) The forces acting on that portion of the system boundary formed by the internal walls of the rocket. These forces are due to the pressure forces of the gases acting perpendicular to the interal surfaces and the frictional forces of the flowing gases acting tangential to the internal surfaces.

(ii) The pressure forces of the surrounding air on that portion of the system boundary formed by the external surface of the rocket. We will assume that the air surrounding the rocket is at rest so that there are no frictional forces between the air and the rocket.

(iii) The force of the strut external to the system acting on the boundary coincident with the strut surface cut by the boundary. This force will have a component perpendicular to the surface counteracting gravity and a component tangential to the boundary due to the imbalance in the horizontal component of the forces acting on the surfaces of (i) and (ii).

The boundaries in (*b*) and (*c*) of Fig. 2-4 each represent a *control surface*—i.e., the surface enclosing a control volume—because mass may flow through a portion of the boundary of each. Surface (i) of system (*a*) is identical with part of the control surface (*b*). We note also that surfaces identified as (ii) and (iii) of (*a*) form a part of the control surface (*c*).

The only part of the boundaries in (*b*) and (*c*) that has not been examined previously is that portion which lies across the exit of the rocket nozzle. Let us designate this portion of the control surface as (iv). With this notation, we can write

$$\text{System boundary } (a) = \text{(i)} + \text{(ii)} + \text{(iii)}$$
$$\text{Control surface } (b) = \text{(i)} + \text{(iv)}$$
$$\text{Control surface } (c) = \text{(ii)} + \text{(iii)} + \text{(iv)}$$

Since the forces acting on (i), (ii), and (iii) have already been discussed, we need only to examine the forces acting on (iv) to complete the discussion.

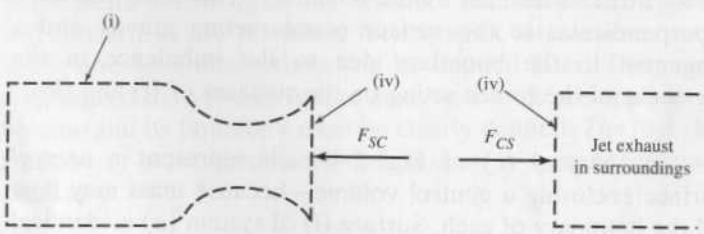
When a sketch is made of a control surface as in (*b*) and (*c*) with a gas flowing across the surface, it represents an instantaneous picture of the flow situation. In this instantaneous picture, we imagine momentarily that the boundary (iv) is occupied by an infinitely thin and massless sheet of material having the instantaneous speed of the fluid at (iv). Now we ask, what are the forces on this sheet and, hence, on the control surface boundary (iv)? The forces on the sheet are the pressure forces of the gas adjacent to it. These forces will be as shown in Fig. 2-5, where *external* and *internal* refer to the outside and the inside of the control volume, respectively. If we are interested in the force of the surroundings on (iv), then we observe that its value is the product of the surface area of (iv) and the pressure at (iv), and that it has a direction, as indicated in the figure. The force of the control volume on the surroundings at (iv) is opposite and equal to this external force.

Other Definitions

Work interaction is an interaction between two systems as a result of a boundary force between the two systems displacing the common boundary through a distance.

Heat interaction is an interaction between two systems as the result of a temperature difference between the two systems.

A *property* is any observable characteristic of a system. Some examples of properties are temperature, volume, pressure, and velocity.



F_{SC} = internal force of control volume *C* on surroundings *S*.

F_{CS} = external force of *S* on *C*.

FIGURE 2-5
Forces acting on that portion of a control surface through which gas is flowing.

Extensive and intensive properties: Subdivide any homogeneous system *A* into two parts. Any property of *A* whose value is the sum of the values of the property for the two parts of *A* is an extensive property (i.e., mass, volume, kinetic energy). Pressure and temperature are not extensive properties, but are examples of intensive properties.

State is the condition of a system, identified through the properties of the system. Two states of a system are called *identical* if every property of the system is the same in both instances.

Process describes how a system changes from one state to another. A process is fully described only when end states, path, and interactions are specified.

An *adiabatic process* is any process in which there are no heat interactions.

An *isolated system* is a system which can have no interactions with any other system.

Entropy S is a property of matter that measures the degree of randomization or disorder at the microscopic level. The natural state of affairs is for entropy to be produced by all processes. The notion that entropy can be produced, but never destroyed, is the second law of thermodynamics. Entropy changes can be quantified by use of the Gibbs equation

$$dS = \frac{dU + P\,dV}{T} \quad \text{or} \quad dS = \frac{dH - V\,dP}{T}$$

2-3 SIMPLE COMPRESSIBLE SYSTEM

The state of a system is described by specifying the value of the properties of the system. Some properties which may be used in describing the state of a system are given in Table 2-1. If the specification of one of the properties in the table fixes the value of a second property, the two properties are called *dependent.* Density and specific volume ($\rho = 1/v$) form an example of two dependent properties. Two properties are *independent* if the specification of

TABLE 2-1
Some thermodynamic properties

Primitive		Derived	
Extensive	**Intensive**	**Extensive**	**Intensive**
Mass m	Density ρ	Energy E	Specific energy e
—	Pressure P	Kinetic energy E_K	Specific kinetic energy $\frac{V^2}{2g_c}$
—	Temperature T	Potential energy E_P	Specific potential energy $\frac{zg}{g_c}$
Volume V	Specific volume v	Internal energy U	Specific internal energy u

one does not fix the value of the other. Temperature and potential energy are two independent properties.

The number of properties required to fix the state of a system is a measure of the system's complexity. We can reduce the complexity of a system by:

a. Restricting the type of material making up the system.
b. Limiting the modes of behavior of the system.

We use both (*a*) and (*b*) in defining a simple system which is easily subject to analysis.

A simple compressible system is:

a. A substance which is homogeneous and invariant in chemical composition, and

b. A system in the absence of motion, force fields (gravity, electric, etc.), capillarity effects, and distortion of solid phases. Let us refer to those modes of behavior listed in (*b*) immediately above as *b*-effects. Gaseous air in the absence of *b*-effects is an example of a simple compressible system. Notice that properties which are related to velocity or position in a gravitational field (momentum, kinetic energy, potential energy) need not be specified in fixing the state of a simple compressible system.

The number of properties required to fix the state of a simple compressible system is very limited. The state of a pure substance in the absence of *b*-effects is *fixed by specifying any two independent intensive properties of the system.*

Once the state of the system is fixed, the values of all other properties of the system are fixed. If, for example, the two independent properties P and ρ of a simple gas system of mass m are specified, then all remaining properties (T, S, U, etc.) are fixed. When we say a derived property such as U is fixed, we mean, of course, that its value relative to some arbitrary datum-state is fixed.

As the restriction on each mode of behavior in the *b*-effects is removed, one more property must be specified to fix the state of the system. Thus the state of a gas of mass m in the presence of motion, but in the absence of other *b*-effects, is fixed by specifying its pressure, temperature, and a third property that fixes its speed or velocity. This third property may be velocity directly or other properties dependent upon velocity such as Mach number, or the properties called *total temperature* and *total pressure* which we will define and use later.

2-4 EQUATIONS OF STATE

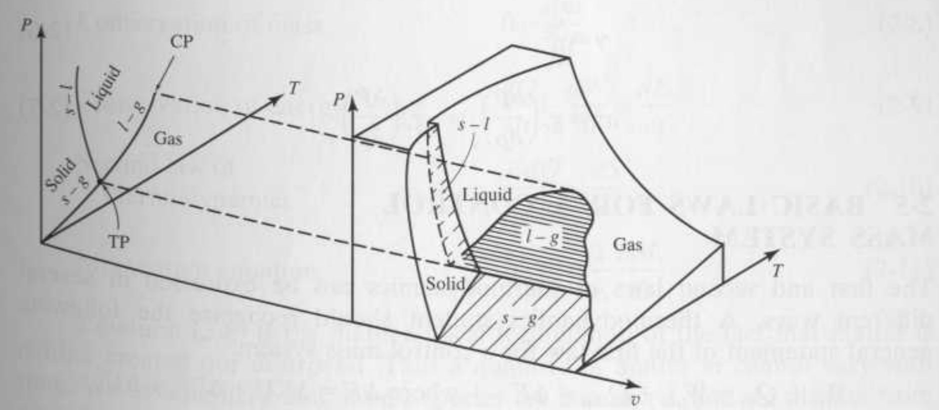
The specification of any two independent properties will fix the state of a simple system and, therefore, the values of all other properties of the system. The values of the other properties may be found through equations of state.

By making two coordinates of a three-dimensional axis system correspond to two independent properties of a simple system and letting the third coordinate represent any dependent thermodynamic property, a three-dimensional thermodynamic surface representing the relation between the three properties can be constructed from measured values of the three properties in equilibrium states. If the three properties related in this manner are P, v, and T, the resulting thermodynamic surface is called the *P-v-T surface.* The P-v-T surface for a unit mass of water is shown in Fig. 2-6. Any point on the P-v-T surface of Fig. 2-6 represents an element of the solution of the function

$$f(P, v, T) = 0 \tag{2-1}$$

A function relating one dependent and two independent thermodynamic properties of a simple system of unit mass is called an *equation of state.* When the three properties are P, v, and T as in Eq. (2-1), the equation is called the *thermal equation of state.* In general, we cannot write the functional relationship Eq. (2-1) in the form of an equation in which specified values of the two properties will allow us to determine the value of the third. Although humans may not know what the functional relation Eq. (2-1) is for a given system, one does exist and nature always knows what it is. When the solution set of Eq. (2-1) cannot be determined from relatively simple equations, tables which list the values of P, v, and T (elements of the solution set) satisfying the function may be prepared. This has been done for water (in all its phases), air, and most common gases.

The functional relation between the energy u of a simple system of unit



On the ruled surfaces, P and T are dependent properties.
CP = critical point: state beyond which vapor and liquid phases are indistinguishable.
TP = triple point: junction of the solid, liquid, and vapor phase boundaries.

FIGURE 2-6
The P-v-T surface for water.

mass and any two independent properties for the set P, $v(=1/\rho)$, T is called the *energy equation of state.* This equation can be written functionally as

$$u = u(T, v)$$

$$\text{or} \quad u = u(P, v) \tag{2-2}$$

$$\text{or} \quad u = u(P, T)$$

As with the thermal equation of state, we may not be able to write an analytical expression for any of the functional relations of (2-2). The important thing is that energy is a property; hence, the functional relations exist.

If the solution sets of the thermal and energy equations of state of a simple system of unit mass are known, all thermodynamic properties of the system can be found when any two of the three properties P, v, T are specified. From the solution set, we can form a tabulation of v and u against specified values of P and T for all states of the system. From these known values of P, T, v, and u, we can determine any other property of the simple system. For example, the value of the property *enthalpy* h is found for any state of the system by combining the tabulated values of P, v, and u for that state by

$$h \equiv u + Pv \tag{2-3}$$

Four other definitions are listed here for use in the later sections of this chapter: specific heat at constant volume c_v, specific heat at constant pressure c_p, ratio of specific heats γ, and the speed of sound a:

$$c_v \equiv \left(\frac{\partial u}{\partial T}\right)_v \tag{2-4}$$

$$c_p \equiv \left(\frac{\partial h}{\partial T}\right)_P \tag{2-5}$$

$$\gamma \equiv \frac{c_p}{c_v} \tag{2-6}$$

$$a^2 \equiv g_c\left(\frac{\partial P}{\partial \rho}\right)_s = \gamma g_c\left(\frac{\partial P}{\partial \rho}\right)_T \tag{2-7}$$

2-5 BASIC LAWS FOR A CONTROL MASS SYSTEM

The first and second laws of thermodynamics can be expressed in several different ways. A thermodynamics student should recognize the following general statement of the first law for a control mass system:

$$W_{\text{in}} + Q_{\text{in}} = W_{\text{out}} + Q_{\text{out}} + \Delta E \qquad \text{where } \Delta E = \Delta \text{KE} + \Delta \text{PE} + \Delta U$$

The general form of the second law of thermodynamics for a control mass system can be expressed as follows:

$$\Delta S + \left(\frac{Q}{T}\right)_{\text{out}} - \left(\frac{Q}{T}\right)_{\text{in}} \geq 0$$

Both of these expressions allow for work and heat interactions both in and out of a control mass. However, in the study of propulsion, an interesting convention is observed. All heat interactions are assumed to be Q_{in}, while all work interactions are assumed to be W_{out}. Therefore, if you were to analyze a system with a Q_{out}, you would have to consider a Q_{out} as a *negative* Q_{in}. In a similar manner, a W_{in} would be a negative W_{out}. With this convention in mind, the first and second laws for a control mass system can be rewritten as follows:

$$\text{First law} \qquad Q = W + \Delta E \quad \text{or} \quad Q - W = \Delta E$$

$$\text{Second law} \qquad \Delta S - \frac{Q}{T} \geq 0$$

The laws of mechanics and thermodynamics, as written in the first instance for a control mass system, are cumbersome to apply to fluid flow problems where one wishes to study a region through which fluid is flowing rather than fix one's attention upon a fixed amount of mass (control mass system). It is extremely useful at the outset, therefore, to convert these laws as written for a control mass system to a form directly applicable to the study of flow through a region fixed in space. This region we may call an *open system* (as is often done in thermodynamics and chemical engineering) or a *control volume,* which is the name customarily used in fluid mechanics and aerodynamics. We follow this latter use. A *control volume* is any prescribed volume in space bounded by a control surface across which matter may flow and heat interactions and work interactions may occur.

We desire, then, to develop control volume relations from the basic laws as written for a system of fixed mass. First, however, we consider the basic laws of interest.

$$\text{Conservation of mass} \qquad 0 = \frac{dm}{dt} \tag{2-8}$$

$$\text{Conservation of energy} \qquad \frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt} \tag{2-9}$$

$$\text{Second law of thermodynamics} \qquad \frac{1}{T}\frac{dQ}{dt} \leq \frac{dS}{dt} \tag{2-10}$$

$$\text{Momentum equation} \qquad \sum F_x = \frac{1}{g_c}\frac{dM_x}{dt} \tag{2-11}$$

Equation (2-8) is the mathematical formulation of the fact that matter is neither created nor destroyed. Thus a quantity of matter m cannot vary with time. We assume here that atomic species are conserved, and we dismiss from consideration nuclear reactions. Equation (2-9) relates the change of energy E of a system to the heat interaction Q into a system and the work interaction W out of a system as it proceeds in time dt between two states infinitesimally different from each other. From the second law of thermodynamics and the definition of entropy, we obtain Eq. (2-10), which states that the rate of entropy production dS/dt must be greater than or equal to zero (i.e., the rate of entropy creation in a mass system must be greater than or equal to the rate of inflow associated with the heat interaction). Finally, Newton's second law of motion [Eq. (2-11)] states that the instantaneous rate of change of the momentum M_x in the x direction of a system of fixed mass is equal to the sum of the forces in the x direction acting on the mass at that instant. The subscript x is used to emphasize that momentum and force are vector quantities.

By writing Eqs. (2-8), (2-9), (2-10), and (2-11) in the manner chosen, a similarity or unification of the right-hand side of each equation is evident. Each equation has on the right-hand side the time derivative of a property—mass, energy, entropy, and momentum, respectively. Each of these property time derivatives applies to a mass of fixed identity—a control mass system. In the next section, expressions will be developed that relate each derivative to quantities associated with a control volume. Thus the right-hand sides of Eqs. (2-8) through (2-11) will be put into a form directly applicable to flow through a control volume.

2-6 RELATIONS BETWEEN THE SYSTEM AND CONTROL VOLUME

Suppose that fluid is flowing through the control volume σ in Fig. 2-7 along the streamlines shown. Let the mass contrained within σ at any time be designated as m_σ. At some initial time t_1, suppose that a system is defined to be the mass m of fluid contained in σ (Fig. 2-7). At some later time t_2, this mass system will have moved to the position shown by the boundary S in Fig. 2-8. To relate the system m to the control volume σ, we must evaluate the time derivative of m in terms of control volume quantities. By definition,

$$\frac{dm}{dt} = \lim_{\delta t \to 0} \frac{m_{t_2} - m_{t_1}}{\delta t} \tag{2-12}$$

where $\delta t = t_2 - t_1$

m_{t_1} = mass system at time t_1

m_{t_2} = mass system at time t_2

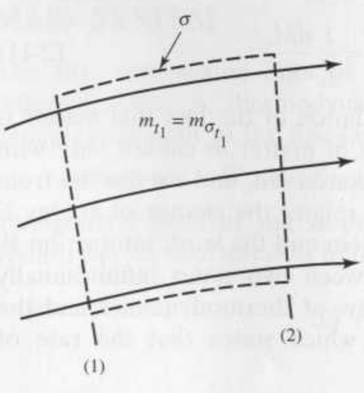


FIGURE 2-7
Mass system within σ at time t_1.

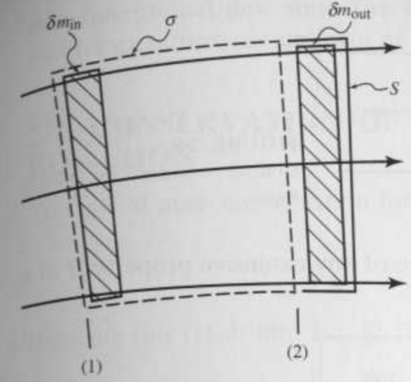


FIGURE 2-8
Mass system not completely in σ at time t_2.

At time t_1, the mass system m completely fills the control volume so that

$$m_{t_1} = m_{\sigma t_1} \tag{2-13}$$

At the later time t_2, the mass system has moved so that a small portion of the mass system, denoted by δm_{out}, has moved out of the control volume at section 2 while the remaining portion of the mass still occupies most of the control volume. During the time interval δt, note that a small element of mass δm_{in} has entered σ through section 1, but this mass is not part of our system of interest. Thus we have

$$m_{t_2} = m_{\sigma t_2} + \delta m_{out} - \delta m_{in} \tag{2-14}$$

Substitution of Eqs. (2-13) and (2-14) into Eq. (2-12) and rearranging terms yield

$$\frac{dm}{dt} = \lim_{\delta t \to 0} \left(\frac{m_{\sigma t_2} - m_{\sigma t_1}}{\delta t} + \frac{\delta m_{out} - \delta m_{in}}{\delta t} \right) \tag{2-15}$$

The derivative depends, therefore, on two items. The first represents the rate of accumulation of mass within σ which we denote by dm_σ/dt. The second represents the net outflow of mass from σ which we denote by $\dot{m}_{out} - \dot{m}_{in}$, where $\dot{m}$ represents the mass flux or flow rate through a control surface. Thus Eq. (2-15) becomes

$$\boxed{\frac{dm}{dt} = \frac{dm_\sigma}{dt} + \dot{m}_{out} - \dot{m}_{in}} \tag{2-16}$$

Equation (2-16) is the desired result relating to the system and control volume approaches. In words, the equation reads: The time rate of change of the mass of a system *at the instant it is within a control volume* σ is equal to the accumulation rate of mass within the control volume plus the net mass flux out of the control volume.

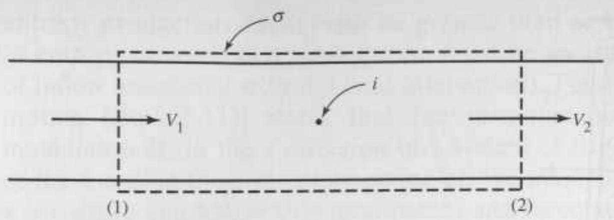


FIGURE 2-9
Control volume for steady flow.

The result expressed in Eq. (2-16) is true of any extensive property R of a system. We may write, therefore,

$$\boxed{\frac{dR}{dt} = \frac{dR_\sigma}{dt} + \dot{R}_{\text{out}} - \dot{R}_{\text{in}}} \tag{2-17}$$

This is the general result we may use to obtain a control volume equation from any system equation involving the rate of change of an extensive property.

DEFINITION OF STEADY FLOW. Consider the flow of fluid through the control volume σ shown in Fig. 2-9. If the properties of the fluid at any point i in the control volume do not vary with time, the flow is called *steady flow*. For such flows we may conclude

$$\frac{dR_\sigma}{dt} = 0 \quad \text{in steady flow} \tag{2-18}$$

DEFINITION OF ONE-DIMENSIONAL FLOW. If the intensive stream properties at a permeable control surface section normal to the flow directions are uniform, the flow is called *one-dimensional*. Many flows in engineering may be treated as steady one-dimensional flows. The term *one-dimensional* is synonymous in this use with *uniform* and applies only at a control surface section. Thus the overall flow through a control volume may be in more than one dimension and still be uniform (one-dimensional flow) at permeable sections of the control surface normal to the flow direction. The flow in Fig. 2-10 is called one-dimensional flow since the intensive properties, such as velocity, density, and temperature, are uniform at sections 1 and 2.

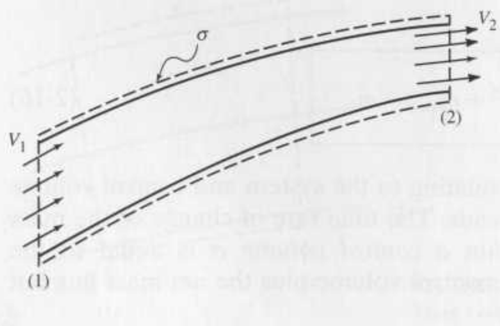


FIGURE 2-10
One-dimensional flow through a convergent duct. The flow is uniform at sections 1 and 2, hence one-dimensional, even though the flow direction may vary elsewhere in the flow.

2-7 CONSERVATION OF MASS EQUATION

The law of mass conservation for any system A is simply

$$\frac{dm_A}{dt} = 0 \tag{2-19}$$

Entering this result into Eq. (2-17) yields

$$\frac{dm_\sigma}{dt} + \dot{m}_{\text{out}} - \dot{m}_{\text{in}} = 0 \tag{2-20}$$

which is known as the *conservation of mass equation*. For steady flows through any control volume, Eq. (2-20) simplifies to

$$\boxed{\dot{m}_{\text{out}} = \dot{m}_{\text{in}}} \tag{2-21}$$

If the flow is steady and one-dimensional through a control volume with a single inlet and exit such as shown in Fig. 2-10, $\dot{m} = \rho A V_n$ (where V_n is the velocity component normal to A), and

$$\boxed{\rho_1 A_1 V_{1n} = \rho_2 A_2 V_{2n}} \tag{2-22}$$

2-8 STEADY FLOW ENERGY EQUATION

The energy equation written as a rate question is

$$\dot{Q}_A - \dot{W}_A = \frac{dE_A}{dt} \tag{2-9}$$

where $\dot{Q}_A$ and $\dot{W}_A$ are the rates at which heat and work interactions are occurring at the system boundary. If all the work is due to shaft work, $\dot{W}_A$ represents the shaft power delivered by system A as it receives energy due to a heat interaction at rate $\dot{Q}_A$ and as the energy E of A changes at rate dE_A/dt. It is convenient to have Eq. (2-9) in a form involving terms associated with the boundary of a control volume. With this in mind, we consider steady one-dimensional flow of a fluid (pure substance in the presence of motion and gravity) through a control volume and surface σ (Fig. 2-11). Fluid crosses σ at the in and out stations only. A shaft work interaction $\dot{W}_x$ and heat interaction $\dot{Q}$ occur at the boundary of σ. In addition, flow work interaction occurs at the in and out stations of σ due to the pressure forces at these stations moving

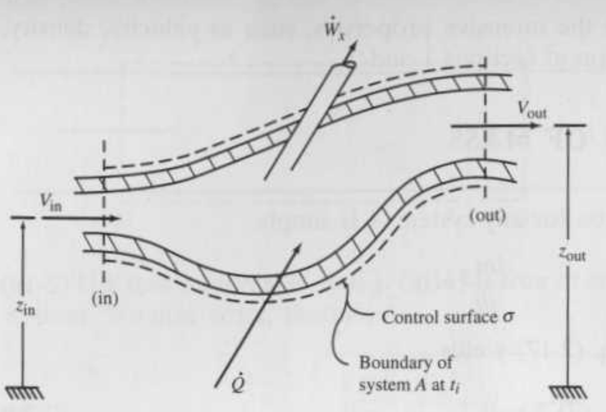


FIGURE 2-11
Steady flow through control volume σ—flow picture at any time t_1.

with the fluid. Since energy is an extensive property, we may write for any instant of time t_i that system A is in σ

$$\frac{dE_A}{dt} = \frac{dE_\sigma}{dt} + \dot{E}_{out} - \dot{E}_{in}$$

Therefore, for steady one-dimensional flow, the right-hand term of Eq. (2-9) becomes

$$\left(\frac{dE_A}{dt}\right)_{t_i} = \left(\frac{dE_\sigma}{dt}\right)_{t_i}^{\nearrow 0} + \dot{E}_{out} - \dot{E}_{in} = (e\dot{m})_{out} - (e\dot{m})_{in}$$

or, with $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$,

$$\left(\frac{dE_A}{dt}\right)_{t_i} = \dot{m}(e_{out} - e_{in}) \tag{i}$$

where $e = u + V^2/(2g_c) + gz/g_c$. For the heat interaction term of Eq. (2-9) we have, for system A at time t_i,

$$(\dot{Q}_A)_{t_i} = \dot{Q} \tag{ii}$$

where $\dot{Q}$ represents the heat interaction rate at the boundary of σ.

For a rigid boundary of the control volume σ, the work interaction term of Eq. (2-9) is due to two effects—the shaft work and flow work rates of A at time t_i. Thus,

$$(\dot{W}_A)_{t_i} = \dot{W}_x + \dot{W}_{flow}$$

where

$$\dot{W}_{flow} = (Pv\dot{m})_{out} - (Pv\dot{m})_{in}$$

from $\dot{W}_{flow}$ = pressure force × velocity = $PA(V) = P(A\,dx/dm)(dm/dt) = Pv\dot{m}$. Now $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$, so

$$(\dot{W}_A)_{t_i} = \dot{W}_x + \dot{m}[(Pv)_{out} - (Pv)_{in}] \tag{iii}$$

When Eqs. (i), (ii), and (iii) are placed in the energy equation for time t_i, we get

$$\dot{Q} - \{\dot{W}_x + \dot{m}[(Pv)_{out} - (Pv)_{in}]\} = \dot{m}(e_{out} - e_{in}) \tag{iv}$$

This is the result sought—an equation in terms of quantities evaluated at the control surface. Some rearrangement of Eq. (iv) will put the equation in a more conventional form. First we assemble all terms involving $\dot{m}$ on the right side of the equation:

$$\boxed{\dot{Q} - \dot{W}_x = \dot{m}(Pv + e)_{out} - \dot{m}(Pv + e)_{in}} \tag{2-23}$$

Then we divide by $\dot{m}$ and use the expression $e = u + V^2/(2g_c) + gz/g_c$ to get

$$q - w_x = \left(Pv + u + \frac{V^2}{2g_c} + \frac{gz}{g_c}\right)_{out} - \left(Pv + u + \frac{V^2}{2g_c} + \frac{gz}{g_c}\right)_{in}$$

where q and w_x are heat and shaft work interactions per unit mass flow through σ. In the last equation, the properties u, P, and v appear again in the arrangement $u + Pv$ which we have called *enthalpy* h. Using h, we obtain the usual form of the *steady flow energy equation*

$$\boxed{q - w_x = \left(h + \frac{v^2}{2g_c} + \frac{gz}{g_c}\right)_{out} - \left(h + \frac{V^2}{2g_c} + \frac{gz}{g_c}\right)_{in}} \tag{2-24}$$

where all terms have the dimensions of energy per unit mass. If Eq. (2-24) is multiplied by $\dot{m}$, then we get

$$\dot{Q} - \dot{W}_x = \dot{m}\left(h + \frac{V^2}{2g_c} + \frac{gz}{g_c}\right)_{out} - \dot{m}\left(h + \frac{V^2}{2g_c} + \frac{gz}{g_c}\right)_{in} \tag{2-25}$$

where the dimensions of $\dot{Q}$, $\dot{W}_x$, etc., are those of power or energy per unit time.

Example 2-1. The first step in the application of the steady flow energy equation is a clear definition of a control surface σ. This is so because each term in the equation refers to a quantity at the boundary of a control volume. Thus, to use the equation, one needs only to examine the control surface and identify the applicable terms of the equation.

In the application of Eq. (2-24) to specific flow situations, many of the terms are zero or may be neglected. The following example will illustrate this point.

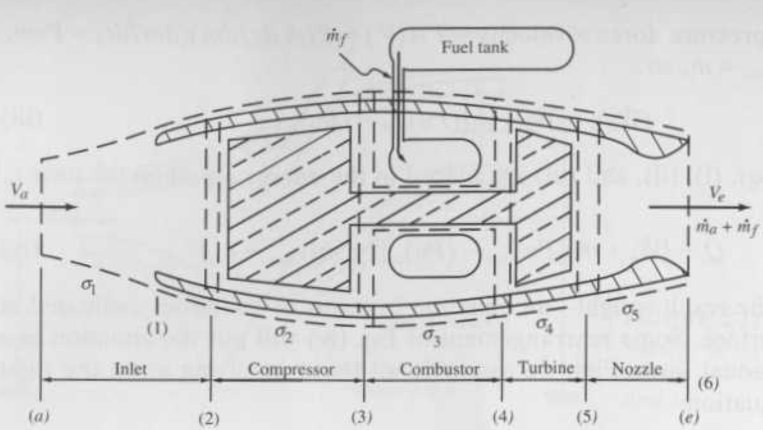


FIGURE 2-12*a*
Control volume for analyzing each component of a turbojet engine.

Consider a turbojet aircraft engine as shown in Fig. 2-12*a*. We divide the engine into the control volume regions:

σ_1: inlet
σ_2: compressor
σ_3: combustion chamber
σ_4: turbine
σ_5: nozzle

Let us apply the steady flow energy equation to each of these control volumes. In all cases the potential energy change $(gz/g_c)_{out} - (gz/g_c)_{in}$ is zero and will be ignored.

It is advisable in using the steady flow energy equation to make two sketches of the applicable control surface σ, showing the heat and shaft work interactions (q, w_x) in one sketch and the fluxes of energy $[h, V^2/(2g_c)]$ in the second sketch. {The term $[h + V^2/(2g_c)]_{out}$ is a flux per unit mass flow of internal energy u, kinetic energy $V^2/(2g_c)$, and flow work Pv. We will use the expression *flux of energy* to include the flow work flux also.}

a. Inlet and nozzle: σ_1 *and* σ_5. There are no shaft work interactions at control surfaces σ_1 and σ_5. Heat interactions are negligible and may be taken as zero. Therefore, the steady flow energy equation, as depicted in Fig. 2-12*b*, for the inlet or nozzle control surfaces gives the result

$$0 = \left(h + \frac{V^2}{2g_c}\right)_{out} - \left(h + \frac{V^2}{2g_c}\right)_{in}$$

Numerical example: Nozzle. Let the gases flowing through the nozzle control volume σ_5 be perfect with $c_p g_c = 6000\ \text{ft}^2/(\text{sec}^2 \cdot {}^\circ\text{R})$. Determine V_6 for $T_5 = 1800°R$, $V_5 = 400$ ft/sec, and $T_6 = 1200$°R.

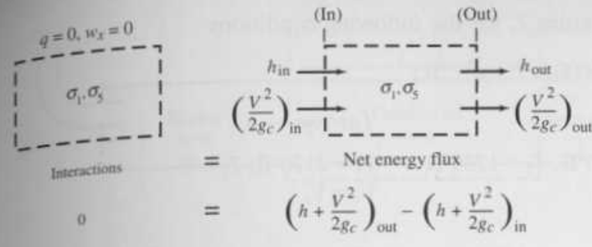


FIGURE 2-12*b*
Energy equation applied to control volumes σ_1 and σ_5.

Solution. From the steady flow energy equation with 5 and 6 the in and out stations respectively, we have

$$h_6 + \frac{V_6^2}{2g_c} = h_5 + \frac{V_5^2}{2g_c}$$

and
$$V_6 = \sqrt{2g_c(h_5 - h_6) + V_5^2} = \sqrt{2c_p g_c(T_5 - T_6) + V_5^2}$$

or
$$= \sqrt{2(6000)(1800 - 1200) + 400^2}$$

so
$$= 2700\ \text{ft/sec}$$

b. Compressor and turbine: σ_2 *and* σ_4. The heat interactions at control surfaces σ_2 and σ_4 are negligibly small. Shaft work interactions are present because each control surface cuts a rotating shaft. The steady flow energy equation for the compressor or for the turbine is depicted in Fig. 2-12*c* and gives

$$-w_x = \left(h + \frac{V^2}{2g_c}\right)_{out} - \left(h + \frac{V^2}{2g_c}\right)_{in}$$

Numerical example: Compressor and turbine. For an equal mass flow† through the compressor and turbine of 185 lb/sec, determine the compressor power and

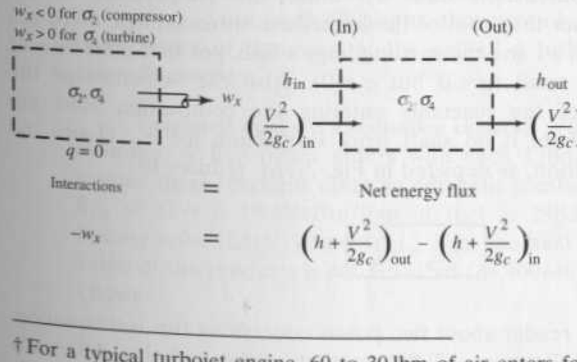


FIGURE 2-12*c*
Energy equation applied to control volumes σ_2 and σ_4.

† For a typical turbojet engine, 60 to 30 lbm of air enters for each 1 lbm of fuel consumed. It is, therefore, reasonable to assume approximately equal mass flow rates through the compressor and turbine.

the turbine exit temperature T_5 for the following conditions:

$$c_p g_c = 6000 \text{ ft}^2/(\text{sec}^2 \cdot {}^\circ\text{R})$$

Compressor	*Turbine*
$T_2 = 740^\circ\text{R},\ T_3 = 1230^\circ\text{R}$	$T_4 = 2170^\circ\text{R},\ T_5 = ?$
$V_2 = V_3$	$V_4 = V_5$

Solution. The compressor power $\dot{W}_c = (\dot{m}w_x)_{\sigma_2}$ is, with $V_2 = V_3$,

$$\begin{aligned}\dot{W}_c &= -\dot{m}(h_3 - h_2) = -\dot{m}c_p(T_3 - T_2)\\ &= -(185 \text{ lbm/sec})\frac{6000(\text{ft/sec})^2}{32.174 \text{ ft} \cdot \text{lbf}/(\text{lbm} \cdot \text{sec}^2)}(1230 - 740)\\ &= -16.9 \times 10^6 \text{ ft} \cdot \text{lbf/sec} \times \frac{1 \text{ hp}}{550 \text{ ft} \cdot \text{lbf/sec}}\\ &= -30{,}700 \text{ hp}\end{aligned}$$

The minus sign means the compressor shaft is delivering energy to the air in σ_2.

The turbine drives the compressor so that the turbine power $\dot{W}_t = (\dot{m}w_x)_{\sigma_4}$ is equal in magnitude to the compressor power. Thus $\dot{W}_t = -\dot{W}_c$, where, from the energy equation,

$$\dot{W}_t = \dot{m}(h_4 - h_5) \qquad \text{and} \qquad \dot{W}_c = -\dot{m}(h_3 - h_2)$$

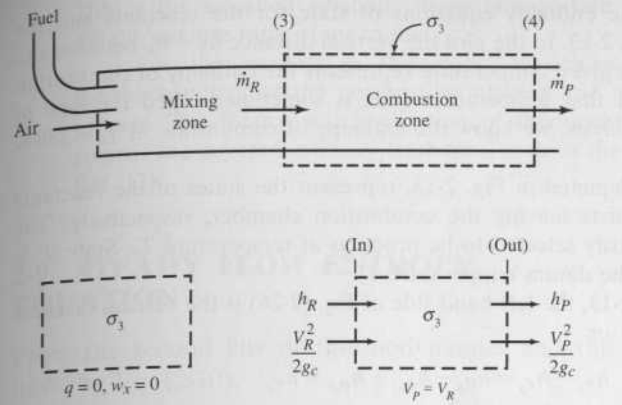
Thus $\qquad \dot{m}c_p(T_5 - T_4) = -\dot{m}c_p(T_3 - T_2)$

and $\qquad \begin{aligned}T_5 &= T_4 - (T_3 - T_2)\\ &= 2170^\circ\text{R} - (1230^\circ\text{R} - 740^\circ\text{R}) = 1680^\circ\text{R} = 1220^\circ\text{F}\end{aligned}$

c. Combustion chamber: σ_3. Let us assume that the fuel and air entering the combustion chamber mix physically in a mixing zone (Fig. 2-12*d*) to form what we will call *reactants* (denoted by subscript *R*). The reactants then enter a combustion zone where combustion occurs, forming *products* of combustion (subscript *P*) which leave the combustion chamber. We apply the steady flow energy equation to combustion zone σ_3. Since the temperature in the combustion zone is higher than that of the immediate surroundings, there is a heat interaction between σ_3 and the surroundings which, per unit mass flow of reactants, is negligibly small ($q < 0$ but $q \approx 0$). Also the velocities of the products leaving and of the reactants entering the combustion zone are approximately equal. There is no shaft work interaction for σ_3. Hence the steady flow energy equation, as depicted in Fig. 2-12*d*, reduces to

$$\boxed{h_{R_3} = h_{P_4}}$$

We must caution the reader about two points concerning this last equation. First, we cannot use the relation $c_p\,\Delta T$ for computing the enthalpy difference between two states of a system when the chemical aggregation of the two states differs. Second, we must measure the enthalpy of each term in the equation



Interactions = Net energy flux

$$0 = \left(h_P + \frac{V_P^2}{2g_c}\right)_{\text{out}} - \left(h_R + \frac{V_R^2}{2g_c}\right)_{\text{in}}$$

FIGURE 2-12*d*
Energy equation applied to control volume σ_3.

relative to the same datum state. To place emphasis on the first point, we have introduced the additional subscripts *R* and *P* to indicate that the chemical aggregations of states 3 and 4 are different.

To emphasize the second point, we select as our common enthalpy datum a state *d* having the chemical aggregation of the products at a datum temperature T_d. Then, introducing the datum state enthalpy $(h_P)_d$ into the last equation above, we have

$$\boxed{h_{R_3} - h_{P_d} = h_{P_4} - h_{P_d}} \tag{2-26}$$

Equation (2-26) can be used to determine the temperature of the products of combustion leaving an adiabatic combustor for given inlet conditions. If the combustor is not adiabatic, Eq. (2-26), adjusted to include the heat interaction term *q* on the left-hand side, is applicable. Let us treat the reactants and products as perfect gases and illustrate the use of Eq. (2-26) in determining the temperature of the gases at the exit of a turbojet combustion chamber via an example problem.

Numerical example: Combustion chamber. For the turbojet engine combustion chamber, 45 lbm of air enters with each 1 lbm of JP-4 (kerosene) fuel. Let us assume these reactants enter an adiabatic combustor at 1200°R. The heating value h_{PR} of JP-4 is 18,400 Btu/lbm of fuel at 298 K. [This is also called the *lower heating value* (LHV) of the fuel.] Thus the heat released $(\Delta H)_{298\text{ K}}$ by the fuel per 1 lbm of the products is 400 Btu/lbm (18,400/46) at 298 K. The following data are known:

$$c_{pP} = 0.267 \text{ Btu}/(\text{lbm} \cdot {}^\circ\text{R}) \qquad \text{and} \qquad c_{pR} = 0.240 \text{ Btu}/(\text{lbm} \cdot {}^\circ\text{R})$$

Determine the temperature of the products leaving the combustor.

Solution. A plot of the enthalpy equations of state for the reactants and the products is given in Fig. 2-13. In the plot the vertical distance $h_R - h_P$ between the curves of h_R and h_P at a given temperature represents the enthalpy of combustion ΔH of the reactants at that temperature (this is sometimes called the *heat of combustion*). In our analysis, we know the enthalpy of combustion at $T_d = 298$ K (536.4°R).

States 3 and 4, depicted in Fig. 2-13, represent the states of the reactants entering and the products leaving the combustion chamber, respectively. The datum state d is arbitrarily selected to be products at temperature T_d. State d' is the reactants' state at the datum temperature T_d.

In terms of Fig. 2-13, the left-hand side of Eq. (2-26) is the vertical distance between states 3 and d, or

$$h_{R_3} - h_{P_d} = h_{R_3} - h_{R_{d'}} + h_{R_{d'}} - h_{P_d}$$

and since $\Delta h_R = c_{pR}\,\Delta T$ and $h_{R_{d'}} - h_{P_d} = (\Delta H)_{T_d}$

then
$$h_{R_3} - h_{P_d} = c_{pR}(T_3 - T_d) + (\Delta H)_{T_d} \quad \text{(i)}$$

Similarly, the right-hand side of Eq. (2-26) is

$$h_{P_4} - h_{P_d} = c_{pP}(T_4 - T_d) \quad \text{(ii)}$$

Substituting Eqs. (i) and (ii) in Eq. (2-26), we get

$$c_{pR}(T_3 - T_d) + (\Delta H)_{T_d} = c_{pP}(T_4 - T_d) \quad (2\text{-}27)$$

We can solve this equation for T_4, which is the temperature of the product gases leaving the combustion chamber. Solving Eq. (2-27) for T_4, we get

$$T_4 = \frac{c_{pR}(T_3 - T_d) + (\Delta H)_{Td}}{c_{pP}} + T_d$$

$$= \frac{0.240(1200 - 536.4) + 400}{0.267} + 536.4$$

$$= 2631°\text{R} \ (2171°\text{F})$$

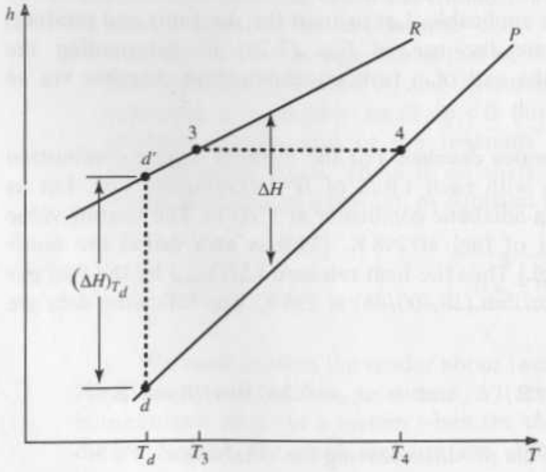


FIGURE 2-13
Enthalpy versus temperature for reactants and products treated as perfect gases.

This is the so-called adiabatic flame temperature of the reactants for a 45:1 weight/mixture ratio of air to fuel.

For the analysis in this book, we choose to sidestep the complex thermochemistry of the combustion process and model it as a simple heating process. The theory and application of thermochemistry to combustion in jet engines are covered in many textbooks, such as the classical text by Penner (see Ref. 13).

2-9 STEADY FLOW ENTROPY EQUATION

From the second law of thermodynamics and the definition of entropy, we have, by Eq. (2-10),

$$\frac{1}{T}\frac{dQ}{dt} \le \frac{dS}{dt} \quad (2\text{-}10)$$

The quantity dQ/dt is a boundary phenomenon for a control volume flow as well as for a system (control mass). Interpreting, therefore, dQ/dt as the heat flux through those parts of the control surface impervious to the flow of matter and using Eq. (2-17) with $R = S$, we obtain

$$\frac{dS_\sigma}{dt} + \dot{S}_{\text{out}} - \dot{S}_{\text{in}} \ge \frac{1}{T}\frac{dQ}{dt} \quad (2\text{-}28)$$

where
$$\dot{S} = \dot{m}s$$

This is called the *entropy equation for control volume flow*; dS_σ/dt represents the entropy production rate within the control volume while $\dot{S}_{\text{out}}$ and $\dot{S}_{\text{in}}$ represent the entropy flux into and out of the control volume through the control surface, respectively; dQ/dt is the heat flux through the control surface; and T is the temperature of the fluid adjacent to the control surface.

Steady Flow

For steady flow, Eq. (2-28) becomes

$$\dot{S}_{\text{out}} - \dot{S}_{\text{in}} \ge \frac{1}{T}\frac{dQ}{dt}$$

And, for steady and adiabatic flow through a control volume, this reduces to the statement that the entropy flux out is greater than or equal to the entropy flux in:

$$\dot{S}_{\text{out}} \ge \dot{S}_{\text{in}}$$

For one outlet section (2) and one inlet section (1), this and the continuity condition yield

$$s_2 \ge s_1$$

2-10 MOMENTUM EQUATION

Newton's second law of motion in the form

$$\sum F_A = \frac{1}{g_c}\frac{dM_A}{dt} \tag{2-11}$$

is a system equation relating the net force on system A to the time rate of change of the extensive property momentum M.† We can write an equivalent control volume equation for the second law of motion, using $R = M$ in the general relation

$$\frac{dR_A}{dt} = \frac{dR_\sigma}{dt} + \dot{R}_{\text{out}} - \dot{R}_{\text{in}}$$

to get
$$\sum F_A = \sum F_\sigma = \frac{1}{g_c}\frac{dM_A}{dt} = \frac{1}{g_c}\left(\frac{dM_\sigma}{dt} + \dot{M}_{\text{out}} - \dot{M}_{\text{in}}\right)$$

or
$$\boxed{\sum F_\sigma = \frac{1}{g_c}\left(\frac{dM_\sigma}{dt} + \dot{M}_{\text{out}} - \dot{M}_{\text{in}}\right)} \tag{2-29}$$

In words, Eq. (2-29) says that the net force acting on a fixed control volume σ is equal to the time rate of increase of momentum within σ plus the net flux of momentum from σ. This is the very important momentum equation. It means that the sum of the forces acting on a system A at the instant A occupies control volume σ equals the rate of change of momentum in σ plus the net flux of momentum out of σ. This equation is in fact a *vector* equation, which implies that it must be applied in a specified direction in order to solve for an unknown quantity.

Applying control volume equations to a steady flow problem gives useful results with only a knowledge of conditions at the control surface. Nothing needs to be known about the state of the fluid interior to the control volume. The following examples illustrate the use of the steady one-dimensional flow condition and the momentum equation. We suggest that the procedure of sketching a control surface (and showing the applicable fluxes through the surface and the applicable forces acting on the surface) be followed whenever a control volume equation is used. We illustrate this procedure in the solutions that follow.

Example 2-2. Water ($\rho = 1000\,\text{kg/m}^3$) is flowing at a steady rate through a convergent duct as illustrated in Fig. 2-14(a). For the data given in the figure, find the force of the fluid $F_{\sigma D}$ acting on the convergent duct D between stations 1 and 2.

† $M = \int_A V\,dm$ for a fluid system A. This reduces to $M = mV$ for a rigid system of mass m.

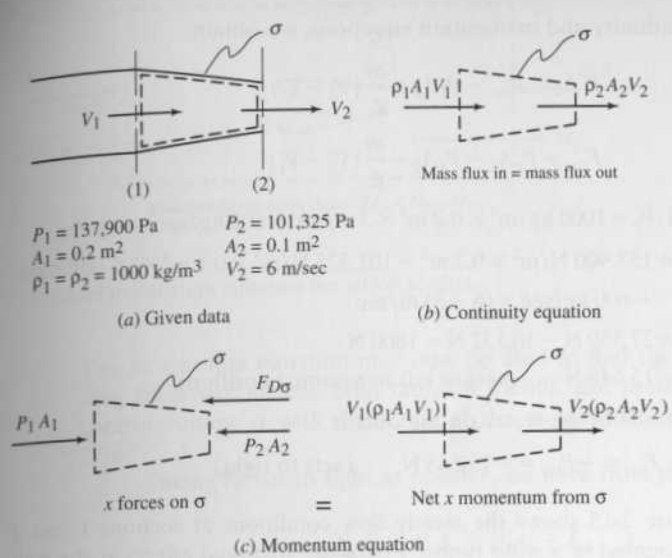


FIGURE 2-14
Flow through a convergent duct.

Solution. We first select the control volume σ such that the force of interest is acting at the control surface. Since we want the force interaction between D and the flowing water, we choose a control surface coincident with the inner wall surface of D bounded by the permeable surfaces 1 and 2, as illustrated in Fig. 2-14(a). By applying the steady one-dimensional continuity equation, Eq. (2-21), as depicted in Fig. 2-14(b), we find V_1 as follows:

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$
$$V_1 = \frac{A_2}{A_1} V_2 \qquad (\rho_1 = \rho_2)$$
$$= 3\,\text{m/sec}$$

With V_1 determined, we can apply the momentum Eq. (2-29) to σ and find the force of the duct walls on σ, denoted by $F_{D\sigma}$ ($F_{\sigma D} = F_{D\sigma}$). By symmetry, $F_{D\sigma}$ is a horizontal force so the horizontal (x) components of forces and momentum fluxes will be considered. The x forces acting on σ are depicted in Fig. 2-14(c) along with the x momentum fluxes through σ.

From Fig. 2-14(c), we have

Momentum equation $\quad P_1A_1 - F_{D\sigma} - P_2A_2 = \dfrac{1}{g_c}[(\rho_2 A_2 V_2)V_2 - (\rho_1 A_1 V_1)V_1]$

And by Fig. 2-14(b),

Continuity equation $\quad \rho_1 A_1 V_1 = \rho_2 A_2 V_2 = \dot{m}$

Combining the continuity and momentum equations, we obtain

$$F_1A_1 - F_{D\sigma} - P_2A_2 = \frac{\dot m}{g_c}(V_2 - V_1)$$

or

$$F_{D\sigma} = P_1A_1 - P_2A_2 - \frac{\dot m}{g_c}(V_2 - V_1)$$

And with $\dot m = \rho_1 A_1 V_1 = 1000\ \text{kg/m}^3 \times 0.2\ \text{m}^2 \times 3\ \text{m/sec} = 600\ \text{kg/sec}$, we have

$$F_{D\sigma} = 137{,}900\ \text{N/m}^2 \times 0.2\ \text{m}^2 - 101{,}325\ \text{N/m}^2 \times 0.1\ \text{m}^2$$
$$- 600\ \text{kg/sec} \times (6-3)\ \text{m/sec}$$

or $$= 27{,}580\ \text{N} - 10{,}132\ \text{N} - 1800\ \text{N}$$

and $$= 15{,}648\ \text{N} \quad \text{(acts to left in assumed position)}$$

Finally, the force of the water on the duct is

$$F_{\sigma D} = -F_{D\sigma} = -15{,}648\ \text{N} \quad \text{(acts to right)}$$

Example 2-3. Figure 2-15 shows the steady flow conditions at sections 1 and 2 about an airfoil mounted in a wind tunnel where the frictional effects at the wall are negligible. Determine the section drag coefficient C_d of this airfoil.

Solution. Since the flow is steady, the continuity equation may be used to find the unknown velocity V_B as follows:

$$(\rho AV)_1 = (\rho AV)_2 \quad (\text{that is, } \dot m_1 = \dot m_2)$$

or $$\rho_1 A_1 V_A = \rho_2(\tfrac{2}{3}V_B + \tfrac{1}{3}V_C)A_2$$

but $$\rho_1 = \rho_2 \quad \text{and} \quad A_1 = A_2$$

$$V_A = \tfrac{2}{3}V_B + \tfrac{1}{3}V_C$$

thus $$V_B = \tfrac{3}{2}V_A - \tfrac{1}{2}V_C = 31.5\ \text{m/sec}$$

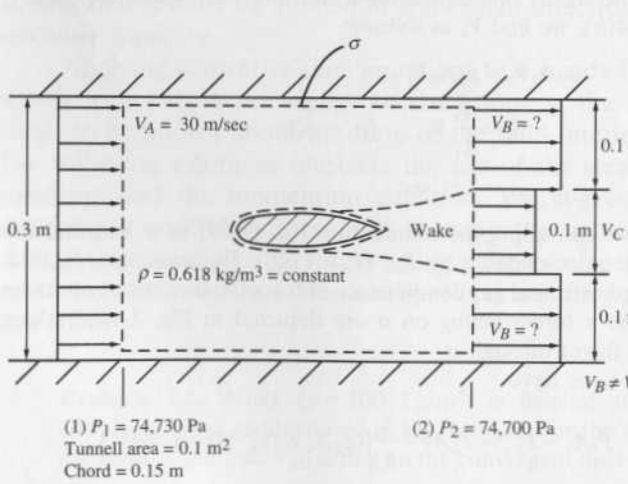


FIGURE 2-15
Wind tunnel drag determination for an Airfoil section.

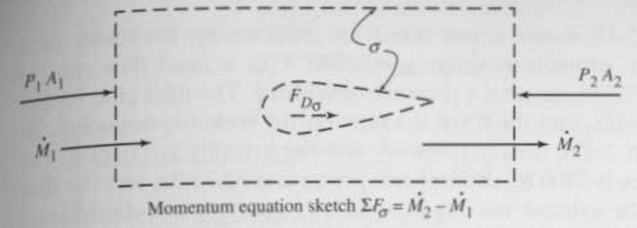


FIGURE 2-16
Sketch of momentum equation for airfoil section.

The momentum equation may now be used to find the drag on the airfoil. This drag force will include both the skin friction and pressure drag. We sketch the control volume σ with the terms of the momentum equation as shown in Fig. 2-16.

Taking forces to right as positive, we have from the sketch above

$$\sum F_\sigma = P_1A_1 - P_2A_2 + F_{D\sigma}$$

$$\dot M_1 = (\rho_1 A_1 V_A)V_A = \rho_1 A_1 V_A^2$$

$$\dot M_2 = \rho_2\left(\frac{2}{3}A_2\right)V_B^2 + \rho_2\left(\frac{1}{3}A_2\right)V_C^2$$

$$= \rho_2 A_2\left(\frac{2}{3}V_B^2 + \frac{1}{3}V_C^2\right)$$

For $\rho = \rho_1 = \rho_2$ and $A = A_1 = A_2$,

$$-F_{D\sigma} = (P_1 - P_2)A + \frac{\rho A}{g_c}\left(V_A^2 - \frac{2}{3}V_B^2 - \frac{1}{3}V_C^2\right)$$

or $$= (74{,}730 - 74{,}700)0.1 + 0.618$$

$$\times 0.1\left[30^2 - \frac{2}{3}(31.5^2) - \frac{1}{3}(27^2)\right]$$

$$= 3.0\ \text{N} - 0.278\ \text{N}$$

$$= 2.722\ \text{N} \qquad \therefore F_{D\sigma} \text{ acts to left}$$

$$F_{\sigma D} = \text{drag force for section}$$

$$= -F_{D\sigma} \quad \text{and} \quad F_{\sigma D} \text{ acts to left}$$

$$F'_D = \frac{F_{\sigma D}}{b} = \frac{2.722\ \text{N}}{0.333\ \text{m}} = 8.174\ \text{N/m}$$

$$C_d = \frac{F'_D}{qc} \quad \text{and} \quad q = \frac{\rho V_\infty^2}{2g_c} \quad \text{where } V_\infty = V_A$$

$$C_d = \frac{F'_D}{[(\rho V_\infty^2)/(2g_c)]c} = \frac{8.174}{(0.618 \times 30^2/2)0.15} = 0.196$$

Example 2-4. Figure 2-17 shows a test stand for determining the thrust of a liquid-fuel rocket. The propellants enter at section 1 at a mass flow rate of 15 kg/sec, a velocity of 30 m/sec, and a pressure of 0.7 MPa. The inlet pipe for the propellants is very flexible, and the force it exerts on the rocket is negligible. At the nozzle exit, section 2, the area is 0.064 m^2, and the pressure is 110 kPa. The force read by the scales is 2700 N, atmospheric pressure is 82.7 kPa, and the flow is steady. Determine the exhaust velocity at section 2, assuming one-dimensional flow exists. Mechanical frictional effects may be neglected.

Solution. First, determine the force on the lever by the rocket to develop a 2700 N scale reading. This may be done by summing moments about the fulcrum point 0 [see Fig. 2-18(*a*)]. Next, draw an external volume around the rocket as shown below, and indicate the horizontal forces and momentum flux [see Fig. 2-18(*b*)].

Gage pressure is used at the exit plane because of the cancellation of atmospheric pressure forces everywhere except at the exit plane where the pressure exceeds atmospheric by an amount equal to $P_2 - P_\infty$, or P_{2g}. Applying the momentum equation to the control volume σ shown in Fig. 2-18(*b*), we obtain

$$\sum F_\sigma = \frac{\dot{M}_{\text{out}_x}}{g_c}$$

Thus

$$F_R - P_{2g}A_2 = \frac{\dot{m}_2 V_2}{g_c}$$

Since the flow is steady, the continuity equation yields $\dot{m}_1 = \dot{m}_2 = 15$ kg/sec. Therefore,

$$V_2 = \frac{F_R - P_{2g}A_2}{\dot{m}_2/g_c}$$

$$= \frac{10{,}800\ \text{N} - [(110 - 82.7)\times 10^3\ \text{N/m}^2](0.064\ \text{m}^2)}{15\ \text{kg/sec}}$$

$$= 603.5\ \text{m/sec}$$

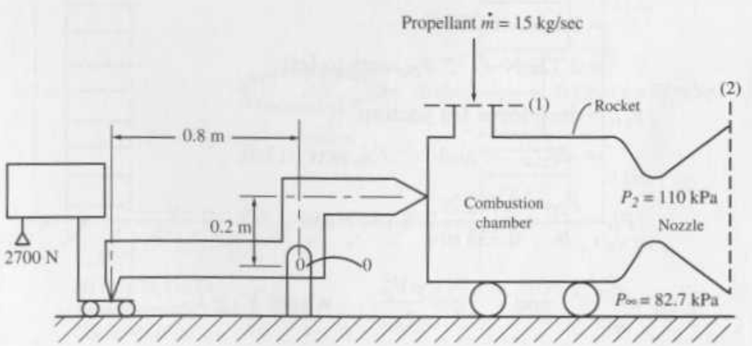


FIGURE 2-17
Liquid-fuel rocket test setup.

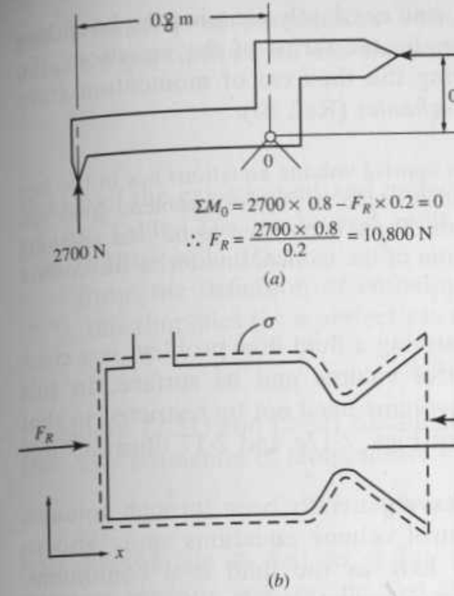


FIGURE 2-18
Moment balance and control volume σ for rocket.

2-11 SUMMARY OF LAWS FOR FLUID FLOW

Table 2-2 gives a convenient summary of the material covered so far in this chapter in the form of a tabulation of the mass, energy, entropy, and momentum equations for a system (control mass) and for control volume flow. For steady flow, all terms of the control volume equations refer to quantities evaluated at the control surface (neglecting body forces). Thus, to use the

TABLE 2-2
Summary of laws

	Closed system of mass	Control volume flow
Mass conservation	$\frac{dm}{dt} = 0$	$\frac{dm}{dt} = \frac{dm_\sigma}{dt} + \dot{m}_{\text{out}} - \dot{m}_{\text{in}}$
First law of thermodynamics	$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt}$	$\dot{Q} - \dot{W}_x = \dot{m}(Pv + e)_{\text{out}} - \dot{m}(Pv + e)_{\text{in}} + \frac{dE_\sigma}{dt}$
Second law of thermodynamics	$\frac{dS}{dt} \geq \frac{1}{T}\frac{dQ}{dt}$	$\frac{dS_\sigma}{dt} + \dot{S}_{\text{out}} - \dot{S}_{\text{in}} \geq \frac{1}{T}\frac{dQ}{dt}$
Second law of motion	$\sum F_x = \frac{dM_x}{dt}$	$\sum F_\sigma = \frac{dM_\sigma}{dt} + \dot{M}_{\text{out}} - \dot{M}_{\text{in}}$

control volume equations for steady flow, one need only examine the boundary of the control region and identify the applicable terms of the equations. To paraphrase Prandtl and Tiejens concerning the theorem of momentum from their *Fundamentals of Hydro and Aeromechanics* (Ref. 14):

> The undoubted value of the steady flow control volume equations lies in the fact that their application enables one to obtain results in physical problems from just a knowledge of the boundary conditions. There is no need to be told anything about the state of fluid, or the mechanism of the motion, interior to the control volume.

Needless to say, the first step in analyzing a fluid flow problem is a clear statement or understanding of the control volume and its surface. In this respect, note that the mass in the control volume need not be restricted to that of a flowing fluid. The control volumes of Figs. 2-12*a* and 2-17 illustrate this point.

The flows analyzed in this chapter have generally been through volumes of other than infinitesimal size. The control volume equations apply also to infinitesimally sized control volumes as long as the fluid is a continuum. Examples of the use of an infinitesimal control volume will be given in Chap. 3.

The basic laws discussed in this chapter represent a powerful set of analytical tools which form the starting point in the analysis of any continuum fluid flow problem. Equations (2-8) through (2-11), or Eqs. (2-20), (2-24), (2-28), and (2-29) plus an equation of state relating the thermodynamic properties of the substance under consideration will form the basis of all the analytical work to follow.

Definitions of new quantities may be introduced, but no further fundamental laws will be required. Since the relations presented in this chapter form the starting point of all analytical studies to follow, time spent on the homework problems of this chapter, which are designed to bring out a basic understanding of the fundamental equations, will be well invested.

2-12 PERFECT GAS

General Characteristics

The thermodynamic equations of state for a perfect gas are

$$\boxed{P = \rho R T} \tag{2-30}$$

$$\boxed{u = u(T)} \tag{2-31}$$

where P is the thermodynamic pressure, ρ is the density, R is the gas constant, T is the thermodynamic temperature, and u is the internal energy per unit mass

and a function of temperature only. The gas constant R is related to the universal gas constant R_u and the molecular weight of the gas M by

$$R = \frac{R_u}{M}$$

Values of the gas constant and molecular weight for typical gases are presented in Table 2-3 in several unit systems; $R_u = 8.31434\ \text{kJ/(kmol}\cdot\text{K)} = 1.98718\ \text{Btu/(mol}\cdot{}^\circ\text{R)}$.

From the definition of enthalpy per unit mass h of a substance in Eq. (2-3), this simplifies for a perfect gas to

$$h = u + RT \tag{2-32}$$

Equations (2-31) and (2-32) combined show that the enthalpy per unit mass is also only a function of temperature $h = h(T)$. Differentiating Eq. (2-32) gives

$$dh = du + R\,dT \tag{2-33}$$

The differentials dh and du in Eq. (2-33) are related to the specific heat at constant pressure and specific heat at constant volume [see definitions in Eqs. (2-4) and (2-5)], respectively, as follows:

$$dh = c_p\,dT$$

$$du = c_v\,dT$$

Note that both specific heats can be functions of temperature. These equations can be integrated from state 1 to state 2 to give

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v\,dT \tag{2-34}$$

$$h_2 - h_1 = \int_{T_1}^{T_2} c_p\,dT \tag{2-35}$$

Substitution of the equations for dh and du into Eq. (2-33) gives the relationship between specific heats for a perfect gas

$$\boxed{c_p = c_v + R} \tag{2-36}$$

And γ is the ratio of the specific heat at constant pressure to the specific heat at constant volume, or

$$\boxed{\gamma \equiv \frac{c_p}{c_v}} \tag{2-6}$$

TABLE 2-3
Properties of ideal gases at 298.15 K (536.67°R)

Gas	Molecular weight	c_p [kJ/(kg · K)]	c_p [Btu/(lbm °R)]	R [kJ/(kg · K)]	R [(ft · lbf)/(lbm °R)]	γ
Air	28.07	1.004	0.240	0.286	53.34	1.40
Argon	39.94	0.523	0.125	0.208	38.69	1.67
Carbon dioxide	44.01	0.845	0.202	0.189	35.1	1.29
Carbon monoxide	28.01	1.042	0.249	0.297	55.17	1.40
Hydrogen	2.016	14.32	3.42	4.124	766.5	1.40
Nitrogen	28.02	1.038	0.248	0.296	55.15	1.40
Oxygen	32.00	0.917	0.217	0.260	48.29	1.39
Sulfur dioxide	64.07	0.644	0.154	0.130	24.1	1.25
Water vapor	18.016	1.867	0.446	0.461	85.78	1.33

The following relationships result from using Eqs. (2-36) and (2-6):

$$\frac{R}{c_v} = \gamma - 1 \tag{2-37}$$

$$\frac{R}{c_p} = \frac{\gamma - 1}{\gamma} \tag{2-38}$$

The Gibbs equation relates the entropy per unit mass s to the other thermodynamic properties of a substance. It can be written as

$$ds = \frac{du + P\,d(1/\rho)}{T} = \frac{dh - (1/\rho)\,dP}{T} \tag{2-39}$$

For a perfect gas, the Gibbs equation can be written simply as

$$ds = c_v \frac{dT}{T} + R \frac{d(1/\rho)}{1/\rho} \tag{2-40}$$

or

$$ds = c_p \frac{dT}{T} - R \frac{dP}{P} \tag{2-41}$$

These equations can be integrated between states 1 and 2 to yield the following expressions for the change in entropy $s_2 - s_1$:

$$s_2 - s_1 = \int_{T_1}^{T_2} c_v \frac{dT}{T} + R \ln \frac{\rho_1}{\rho_2} \tag{2-42}$$

$$s_2 - s_1 = \int_{T_1}^{T_2} c_p \frac{dT}{T} - R \ln \frac{P_2}{P_3} \tag{2-43}$$

If the specific heats are known functions of temperature for a perfect gas, then Eqs. (2-34), (2-35), (2-42), and (2-43) can be integrated from a reference state and tabulated for further use in what are called *gas tables.*

The equation for the speed of sound in a perfect gas is easily obtained by use of Eqs. (2-7) and (2-30) to give

$$a = \sqrt{\gamma R g_c T} \tag{2-44}$$

Calorically Perfect Gas

A *calorically perfect gas* is a perfect gas with constant specific heats (c_p and c_v). In this case, the expressions for changes in internal energy u, enthalpy h, and entropy s simplify to the following:

$$u_2 - u_1 = c_v(T_2 - T_1) \tag{2-45}$$

$$h_2 - h_1 = c_p(T_2 - T_1) \tag{2-46}$$

$$s_2 - s_1 = c_v \ln\frac{T_2}{T_1} - R\ln\frac{\rho_2}{\rho_1} \tag{2-47}$$

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \tag{2-48}$$

Equations (2-47) and (2-48) can be rearranged to give the following equations for the temperature ratio T_2/T_1:

$$\frac{T_2}{T_1} = \left(\frac{\rho_2}{\rho_1}\right)^{R/c_v} \exp\frac{s_2 - s_1}{c_v}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{R/c_p} \exp\frac{s_2 - s_1}{c_p}$$

From Eqs. (2-37) and (2-38), these expressions become

$$\frac{T_2}{T_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma-1} \exp\frac{s_2 - s_1}{c_v} \tag{2-49}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} \exp\frac{s_2 - s_1}{c_p} \tag{2-50}$$

Isentropic Process

For an isentropic process ($s_2 = s_1$), Eqs. (2-49), (2-50), and (2-30) yield the following equations:

$$\boxed{\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma}} \tag{2-51}$$

$$\boxed{\frac{T_2}{T_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma-1}} \tag{2-52}$$

$$\boxed{\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}} \tag{2-53}$$

Note that Eqs. (2-51), (2-52), and (2-53) apply only to a calorically perfect gas undergoing an isentropic process.

Example 2-5. Air initially at 20°C and 1 atm is compressed reversibly and adiabatically to a final pressure of 15 atm. Find the final temperature.

Solution. Since the process is isentropic from initial to final state, Eq. (2-51) can be used to solve for the final temperature. The ratio of specific heats for air is 1.4.

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} = (20 + 273.15)\left(\frac{15}{1}\right)^{0.4/1.4}$$

$$= 293.15 \times 2.1678 = 635.49 \text{ K } (362.34°\text{C})$$

Example 2-6. Air is expanded isentropically through a nozzle from $T_1 = 3000°\text{R}$, $V_1 = 0$, and $P_1 = 10$ atm to $V_2 = 3000$ ft/sec. Find the exit temperature and pressure.

Solution. Application of the first law of thermodynamics to the nozzle gives the following for a calorically perfect gas:

$$c_pT_1 + \frac{V_1^2}{2g_c} = c_pT_2 + \frac{V_2^2}{2g_c}$$

This equation can be rearranged to give T_2:

$$T_2 = T_1 - \frac{V_2^2 - V_1^2}{2g_cc_p} = 3000 - \frac{3000^2}{2 \times 32.174 \times 186.76}$$

$$= 3000 - 748.9 = 2251.1°\text{R}$$

Solving Eq. (2-51) for P_2 gives

$$P_2 = P_1\left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)} = 10\left(\frac{2251.1}{3000}\right)^{3.5} = 3.66 \text{ atm}$$

Mollier Diagram for a Perfect Gas

The Mollier diagram is a thermodynamic state diagram with the coordinates of enthalpy and entropy s. Since the enthalpy of a perfect gas depends upon temperature alone,

$$dh = c_p\,dT$$

temperature can replace enthalpy as the coordinate of a Mollier diagram for a perfect gas. When temperature T and entropy s are the coordinates of a

Mollier diagram, we call it a *T-s diagram*. We can construct lines of constant pressure and density in the *T-s* diagram by using Eqs. (2-42) and (2-43). For a calorically perfect gas, Eqs. (2-47) and (2-48) can be written between any state and the entropy reference state ($s = 0$) as

$$s = c_v \ln \frac{T}{T_{\text{ref}}} - R \ln \frac{\rho}{\rho_{\text{ref}}}$$

$$s = c_p \ln \frac{T}{T_{\text{ref}}} - R \ln \frac{P}{P_{\text{ref}}}$$

where T_{ref}, P_{ref}, and ρ_{ref} are the values of temperature, pressure, and density, respectively, when $s = 0$. Since the most common working fluid in gas turbine engines is air, Fig. 2-19 was constructed for air by using the above equations with these data:

$$c_p = 1.004\ \text{kJ/(kg} \cdot \text{K)} \qquad T_{\text{ref}} = 288.2\ \text{K} \qquad \rho_{\text{ref}} = 1.225\ \text{kg/m}^3$$

$$R = 0.286\ \text{kJ/(kg} \cdot \text{K)} \qquad P_{\text{ref}} = 1\ \text{atm} = 101{,}325\ \text{Pa}$$

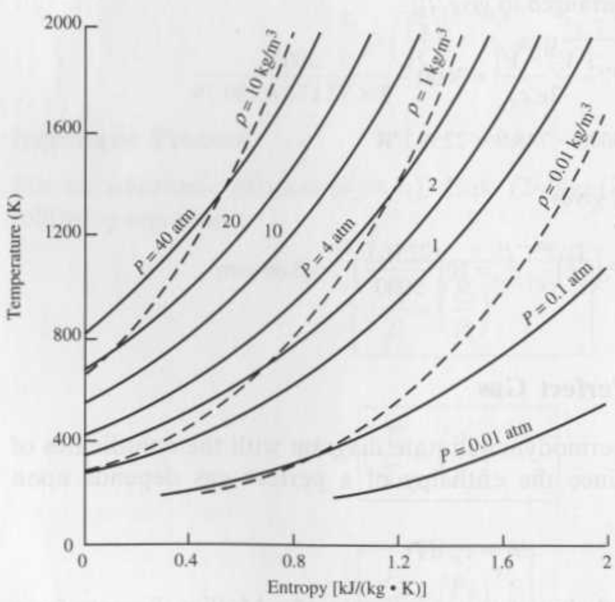


FIGURE 2-19
A *T-s* diagram for air as a calorically perfect gas.

Mixtures of Perfect Gases

We consider a mixture of perfect gases, each obeying the perfect gas equation

$$PV = NR_u T$$

where N is the number of moles and R_u is the universal gas constant. The mixture is idealized as independent perfect gases, each having the temperature T and occupying the volume V. The partial pressure of gas i is

$$P_i = N_i R_u \frac{T}{V}$$

According to the Gibbs-Dalton law, the pressure of the gas mixture of n constituents is the sum of the partial pressures of each constituent:

$$P = \sum_{i=1}^{n} P_i \tag{2-54}$$

The total number of moles N of the gas is

$$N = \sum_{i=1}^{n} N_i \tag{2-55}$$

The ratio of the number of moles of constituent i to the total number of moles in the mixture is called the *mole fraction* χ_i. By using the above equations, the mole fraction of constituent i can be shown to equal the ratio of the partial pressure of constituent i to the pressure of the mixture:

$$\chi_i = \frac{N_i}{N} = \frac{P_i}{P} \tag{2-56}$$

The Gibbs-Dalton law also states that the internal energy, enthalpy, and entropy of a mixture are equal, respectively, to the sum of the internal energies, the enthalpies, and the entropies of the constituents when each alone occupies the volume of the mixture at the mixture temperature. Thus we can write for a mixture of n constituents:

Energy $$U = \sum_{i=1}^{n} U_i = \sum_{i=1}^{n} m_i u_i \tag{2-57}$$

Enthalpy $$H = \sum_{i=1}^{n} H_i = \sum_{i=1}^{n} m_i h_i \tag{2-58}$$

Entropy $$S = \sum_{i=1}^{n} S_i = \sum_{i=1}^{n} m_i s_i \tag{2-59}$$

where m_i is the mass of constituent i.

The specific heats of the mixture follow directly the definitions of c_p and c_v and the above equations. For a mixture of n constituents, the specific heats are

$$c_p = \frac{\sum_{i=1}^{n} m_i c_{pi}}{m} \quad \text{and} \quad c_v = \frac{\sum_{i=1}^{n} m_i c_{vi}}{m} \tag{2-60}$$

where m is the total mass of the mixture.

Gas Tables

In the case of a perfect gas with nonconstant specific heats, the variation of the specific heat at constant pressure c_p is normally modeled by several terms of a power series in temperature T. This expression is used in conjunction with the general equations presented above and the new equations that are developed below to generate a gas table for a particular gas (see Ref. 15).

For convenience, we define

$$h \equiv \int_{T_{\text{ref}}}^{T} c_p \, dT \tag{2-61}$$

$$\phi \equiv \int_{T_{\text{ref}}}^{T} c_p \frac{dT}{T} \tag{2-62}$$

$$P_r \equiv \exp \frac{\phi - \phi_0}{R} \tag{2-63}$$

$$v_r \equiv \exp\left(-\frac{1}{R} \int_{T_0}^{T} c_v \frac{dT}{T} \right) \tag{2-64}$$

where P_r and v_r are called the *reduced pressure* and *reduced volume*, respectively. Using the definition of ϕ from Eq. (2-62) in Eq. (2-43) gives

$$s_2 - s_1 = \phi_2 - \phi_1 - R \ln \frac{P_2}{P_1} \tag{2-65}$$

For an isentropic process between states 1 and 2, Eq. (2-65) reduces to

$$\phi_2 - \phi_1 = R \ln \frac{P_2}{P_1}$$

which can be rewritten as

$$\left(\frac{P_2}{P_1}\right)_{s=\text{const}} = \exp \frac{\phi_2 - \phi_1}{R} = \frac{\exp(\phi_2/R)}{\exp(\phi_1/R)}$$

Using Eq. (2-63), we can express this pressure ratio in terms of the reduced pressure P_r as

$$\left(\frac{P_2}{P_1}\right)_{s=\text{const}} = \frac{P_{r2}}{P_{r1}} \tag{2-66}$$

Likewise, it can be shown that

$$\left(\frac{v_2}{v_1}\right)_{s=\text{const}} = \frac{v_{r2}}{v_{r1}} \tag{2-67}$$

For a perfect gas, the properties h, P_r, u, v_r, and ϕ are functions of T, and these can be calculated by starting with a polynomial for c_p. Say we have the seventh-order polynomial

$$c_p = A_0 + A_1 T + A_2 T^2 + A_3 T^3 + A_4 T^4 + A_5 T^5 + A_6 T^6 + A_7 T^7 \tag{2-68}$$

The equations for h and ϕ as functions of temperature follow directly from using Eqs. (2-61) and (2-62):

$$h = h_{\text{ref}} + A_0 T + \frac{A_1}{2} T^2 + \frac{A_2}{3} T^3 + \frac{A_3}{4} T^4 + \frac{A_4}{5} T^5 + \frac{A_5}{6} T^6 + \frac{A_6}{7} T^7 + \frac{A_7}{8} T^8 \tag{2-69}$$

$$\phi = \phi_{\text{ref}} + A_0 \ln T + A_1 T + \frac{A_2}{2} T^2 + \frac{A_3}{3} T^3 + \frac{A_4}{4} T^4 + \frac{A_5}{5} T^5 + \frac{A_6}{6} T^6 + \frac{A_7}{7} T^7 \tag{2-70}$$

After we define reference values, the variations of P_r and v_r follow from Eqs. (2-63), (2-64), and the above.

Appendix D

Typically, air flows through the inlet and compressor of the gas turbine engine whereas products of combustion flow through the engine components downstream of a combustion process. Most gas turbine engines use hydrocarbon fuels of composition $(CH_2)_n$. We can use the above equations to estimate the properties of these gases, given the ratio of the mass of fuel burned to the mass

mass of air. For convenience, we use the fuel/air ratio f, defined as

$$f = \frac{\text{mass of fuel}}{\text{mass of air}} \tag{2-71}$$

The maximum value of f is 0.0676 for the hydrocarbon fuels of composition $(CH_2)_n$.

Given the values of c_p, h, and ϕ for air and the values of combustion products, the values of c_p, h, and ϕ for the mixture follow directly from the mixture equations [Eqs. (2-57) through (2-60)] and are given by

$$R = \frac{1.9857117 \text{ But/(lbm}\cdot{}^\circ\text{R)}}{28.97 - f \times 0.946186} \tag{2-72a}$$

$$c_p = \frac{c_{p\,\text{air}} + f c_{p\,\text{prod}}}{1+f} \tag{2-72b}$$

$$h = \frac{h_{\text{air}} + f h_{\text{prod}}}{1+f} \tag{2-72c}$$

$$\phi = \frac{\phi_{\text{air}} + f \phi_{\text{prod}}}{1+f} \tag{2-72d}$$

Appendix D is a table of the properties h and P_r as functions of the temperature and fuel/air ratio f for air and combustion products [air with hydrocarbon fuels of composition $(CH_2)_n$] at low pressure (perfect gas). These data are based on the above equations and the constants given in Table 2-4, which are valid over the temperature range of 300 to 4000°R. These constants come from the gas turbine engine modeling work of Capt. John S. McKinney

TABLE 2-4
Constants for air and combustion products used in App. D and program AFPROP (Ref. 16)

Air alone		Combustion products of air and $(CH_2)_n$ fuels	
A_0	2.5020051×10^{-1}	A_0	7.3816638×10^{-2}
A_1	$-5.1536879 \times 10^{-5}$	A_1	1.2258630×10^{-3}
A_2	6.5519486×10^{-8}	A_2	$-1.3771901 \times 10^{-6}$
A_3	$-6.7178376 \times 10^{-12}$	A_3	$9.9686793 \times 10^{-10}$
A_4	$-1.5128259 \times 10^{-14}$	A_4	$-4.2051104 \times 10^{-13}$
A_5	$7.6215767 \times 10^{-18}$	A_5	$1.0212913 \times 10^{-16}$
A_6	$-1.4526770 \times 10^{-21}$	A_6	$-1.3335668 \times 10^{-20}$
A_7	$1.0115540 \times 10^{-25}$	A_7	$7.2678710 \times 10^{-25}$
h_{ref}	−1.7558886 But/lbm	h_{ref}	30.58153 But/lbm
ϕ_{ref}	0.0454323 But/(lbm · °R)	ϕ_{ref}	0.6483398 But/(lbm · °R)

(U.S. Air Force) while assigned to the Air Force's Aero Propulsion Laboratory (Ref. 16), and they continue to be widely used in the industry. Appendix D uses a reference value of 2 for P_r at 600°R and $f = 0$.

Computer Program AFPROP

The computer program AFPROP was written by using the above constants for air and products of combustion from air with $(CH_2)_n$. The program can calculate the four primary thermodynamic properties at a state (P, T, h, and s) given the fuel/air ratio f and two independent thermodynamic properties (say, P and h).

To show the use of the gas tables, we will resolve Examples 2-5 and 2-6, using the gas tables of App. D. These problems could also be solved by using the computer program AFPROP.

Example 2-7. Air initially at 100°F and 1 atm is compressed reversibly and adiabatically to a final pressure of 15 atm. Find the final temperature.

Solution. Since the process is isentropic from initial to final state, Eq. (2-66) can be used to solve for the final reduced pressure. From App. D at 20°C (293.15 K) and $f = 0$, $P_r = 1.2768$ and

$$\frac{P_{r2}}{P_{r1}} = \frac{P_2}{P_1} = 15$$

$$P_{r2} = 15 \times 1.2768 = 19.152$$

From App. D for $P_{r2} = 19.152$, the final temperature is 354.42°C (627.57 K). This is 7.9 K lower than the result obtained in Example 2-5 for air as a calorically perfect gas.

Example 2-8. Air is expanded isentropically through a nozzle from $T_1 = 3000$°R, $V_1 = 0$, and $P_1 = 10$ atm to $V_2 = 3000$ ft/sec. Find the exit temperature and pressure.

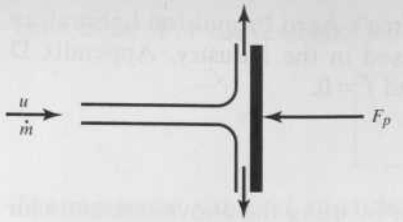
Solution. Application of the first law of thermodyanics to the nozzle gives the following for a calorically perfect gas:

$$h_1 + \frac{V_1^2}{2g_c} = h_2 + \frac{V_2^2}{2g_c}$$

From App. D at $f = 0$ and $T_1 = 3000$°R, $h_1 = 790.46$ Btu/lbm and $P_{r1} = 938.6$. Solving the above equation for h_2 gives

$$h_2 = h_1 - \frac{V_2^2 - V_1^2}{2g_c} = 790.46 - \frac{3000^2}{2 \times 32.174 \times 778.16}$$

$$= 790.46 - 179.74 = 610.72 \text{ But/lbm}$$



FIGURES P2-1 and P2-2

For $h = 610.72$ Btu/lbm and $f = 0$, App. D gives $T_2 = 2377.7$°R and $P_{r2} = 352.6$. Using Eq. (2-66), we solve for the exit pressure

$$P_2 = P_1 \frac{P_{r2}}{P_{r1}} = 10\left(\frac{352.6}{938.6}\right) = 3.757 \text{ atm}$$

These results for temperature and pressure at station 2 are higher by 126.6°R and 0.097 atm, respectively, than those obtained in Example 2-6 for air as a calorically perfect gas.

PROBLEMS

2-1. A stream of air with velocity of 500 ft/sec and density of 0.07 lbm/ft^3 strikes a stationary plate and is deflected 90°. Select an appropriate control volume and determine the force F_p necessary to hold the plate stationary. Assume that atmospheric pressure surrounds the jet and that the initial jet diameter is 1.0 in.

2-2. An airstream with density of 1.25 kg/m^3 and velocity of 200 m/sec strikes a stationary plate and is deflected 90°. Select an appropriate control volume and determine the force F_p necessary to hold the plate stationary. Assume that atmospheric pressure surrounds the jet and that the initial jet diameter is 1.0 cm.

2-3. Consider the flow shown in Fig. P2-3 of an incompressible fluid. The fluid enters (station 1) a constant-area circular pipe of radius r_0 with uniform velocity V_1 and pressure P_1. The fluid leaves (station 2) with the parabolic velocity profile V_2 given by

$$V_2 = V_{max}\left[1 - \left(\frac{r}{r_0}\right)^2\right]$$

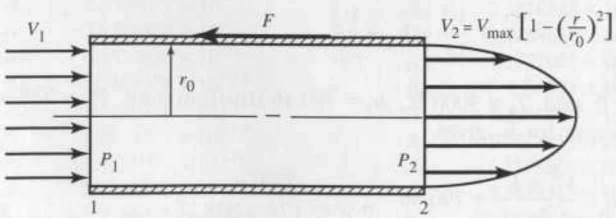


FIGURE P2-3

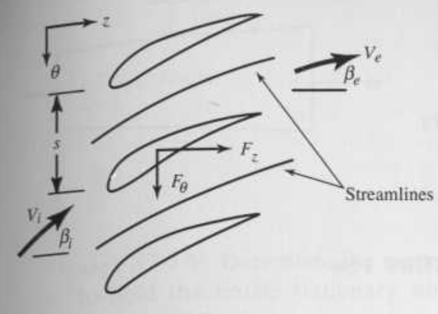


FIGURE P2-4

and uniform pressure P_2. Using the conservation of mass and momentum equations, show that the force F necessary to hold the pipe in place can be expressed as

$$F = \pi r_0^2\left(P_1 - P_2 + \frac{\rho V_1^2}{3g_c}\right)$$

2-4. Consider the flow of an incompressible fluid through a two-dimensional cascade as shown in Fig. P2-4. The airfoils are spaced at a distance s and have unit depth into the page. Application of the conservation of mass requires $V_i \cos \beta_i = V_e \cos \beta_e$.

a. From the tangential momentum equation, show that

$$F_\theta = \frac{\dot{m}}{g_c}(V_i \sin \beta_i - V_e \sin \beta_e)$$

b. From the axial momentum equation, show that

$$F_z = s(P_e - P_i)$$

c. Show that the axial force can be written as

$$F_z = s\left[\frac{\rho}{2g_c}(V_i^2 \sin^2 \beta_i - V_e^2 \sin^2 \beta_e) - (P_{ti} - P_{te})\right]$$

2-5. When a free jet is deflected by a blade surface, a change of momentum occurs and a force is exerted on the blade. If the blade is allowed to move at a velocity, power may be derived from the moving blade. This is the basic principle of the impulse turbine. The jet of Fig. P2-5, which is initially horizontal, is deflected by a fixed blade. Assuming the same pressure surrounds the jet, show that the

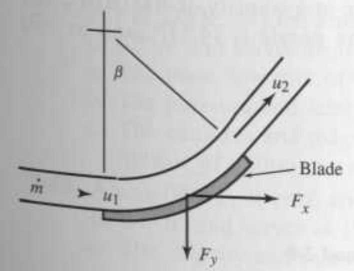


FIGURE P2-5

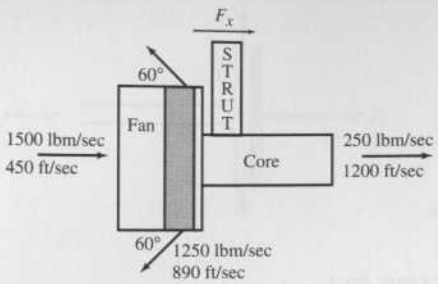


FIGURE P2-6

horizontal (F_x) and vertical forces (F_y) by the fluid on the blade are given by

$$F_x = \frac{\dot{m}(u_1 - u_2 \cos\beta)}{g_c} \quad \text{and} \quad F_y = \frac{\dot{m}u_2 \sin\beta}{g_c}$$

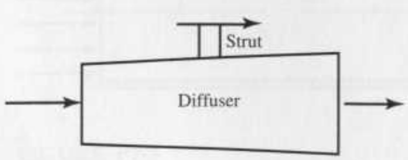
Calculate the force F_y for a mass flow rate of 100 lbm/sec, $u_1 = u_2 = 2000$ ft/sec, and $\beta = 60°$.

2-6. One method of reducing an aircarft's landing distance is through the use of thrust reversers. Consider the turbofan engine in Fig. P2-6 with thrust reverser of the bypass airstream. It is given that 1500 lbm/sec of air at 60°F and 14.7 psia enters the engine at a velocity of 450 ft/sec and that 1250 lbm/sec of bypass air leaves the engine at 60° to the horizontal, velocity of 890 ft/sec, and pressure of 14.7 psia. The remaining 250 lbm/sec leaves the engine core at a velocity of 1200 ft/sec and pressure of 14.7 psia. Determine the force on the strut F_x. Assume the outside of the engine sees a pressure of 14.7 psia.

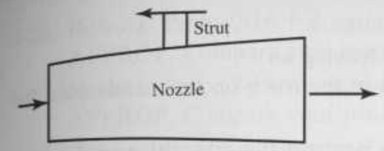
2-7. Air with a density of 0.027 lbm/ft³ enters a diffuser at a velocity of 2470 ft/sec and a static pressure of 4 psia. The air leaves the diffuser at a velocity of 300 ft/sec and a static pressure of 66 psia. The entrance area of the diffuser is 1.5 ft², and its exit area is 1.7 ft². Determine the magnitude and direction of the strut force necessary to hold the diffuser stationary when this diffuser is operated in an atmospheric pressure of 4 psia.

2-8. It is given that 50 kg/sec of air enters a diffuser at a velocity of 750 m/sec and a static pressure of 20 kPa. The air leaves the diffuser at a velocity of 90 m/sec and a static pressure of 330 kPa. The entrance area of the diffuser is 0.25 m², and its exit area is 0.28 m². Determine the magnitude and direction of the strut force necessary to hold the diffuser stationary when this diffuser is operated in an atmospheric pressure of 20 kPa.

2-9. It is given that 100 lbm/sec of air enters a nozzle at a velocity of 600 ft/sec and a static pressure of 70 psia. The air leaves the nozzle at a velocity of 4000 ft/sec and static pressure of 2 psia. The entrance area of the nozzle is 14.5 ft², and its exit area is 30 ft². Determine the magnitude and direction of the strut force necessary to hold the nozzle stationary when this nozzle is operated in an atmospheric pressure of 4 psia.



FIGURES P2-7 and 2-8



FIGURES P2-9 and 2-10

2-10. Air with a density of 0.98 kg/m³ enters a nozzle at a velocity of 180 m/sec and a static pressure of 350 kPa. The air leaves the nozzle at a velocity of 1200 m/sec and a static pressure of 10 kPa. The entrance area of the nozzle is 1.0 m², and its exit area is 2.07 m². Determine the magnitude and direction of the strut force necessary to hold the nozzle stationary when this nozzle is operated in an atmospheric pressure of 10 kPa.

2-11. For a calorically perfect gas, show that $P + \rho V^2/g_c$ can be written as $P(1 + \gamma M^2)$. Note that the Mach number M is defined as the velocity V divided by the speed of sound a.

2-12. Air at 1400 K, 8 atm, and 0.3 Mach expands isentropically through a nozzle to 1 atm. Assuming a calorically perfect gas, find the exit temperature and the inlet and exit areas for a mass flow rate of 100 kg/sec.

2-13. It is given that 250 lbm/sec of air at 2000°F, 10 atm, and 0.2 Mach expands isentropically through a nozzle to 1 atm. Assuming a calorically perfect gas, find the exit temperature and the inlet and exit areas.

2-14. Air at 518.7°R is isentropically compressed from 1 to 10 atm. Assuming a calorically perfect gas, determine the exit temperature and the compressor's input power for a mass flow rate of 150 lbm/sec.

2-15. It is given that 50 kg/sec of air at 288.2 K is isentropically compressed from 1 to 12 atm. Assuming a calorically perfect gas, determine the exit temperature and the compressor's input power.

2-16. Air at −55°F, 4 psia, and $M = 2.5$ enters an isentropic diffuser with an inlet area of 1.5 ft² and leaves at $M = 0.2$. Assuming a calorically perfect gas, determine:
a. The mass flow rate of the entering air
b. The pressure and temperature of the leaving air
c. The exit area and magnitude and direction of the force on the diffuser (assume outside of diffuser sees 4 psia)

2-17. Air at 225 K, 28 kPa, and $M = 2.0$ enters an isentropic diffuser with an inlet area of 0.2 m² and leaves at $M = 0.2$. Assuming a calorically perfect gas, determine:
a. The mass flow rate of the entering air
b. The pressure and temperature of the leaving air
c. The exit area and magnitude and direction of the force on the diffuser (assume outside of diffuser sees 28 kPa)

2-18. Air at 1800°F, 40 psia, and $M = 0.4$ enters an isentropic nozzle with an inlet area of 1.45 ft² and leaves at 10 psia. Assuming a calorically perfect gas, determine:
a. The velocity and mass flow rate of the entering air

b. The temperature and Mach number of the leaving air
c. The exit area and magnitude and direction of the force on the nozzle (assume outside of nozzle sees 10 psia)

2-19. Air at 1500 K, 300 kPa, and $M = 0.3$ enters an isentropic nozzle with an inlet area of 0.5 m^2 and leaves at 75 kPa. Assuming a calorically perfect gas, determine:
a. The velocity and mass flow rate of the entering air
b. The temperature and Mach number of the leaving air
c. The exit area and magnitude and direction of the force on the nozzle (assume outside of nozzle sees 75 kPa)

2-20. It is given that 100 lb/sec of air enters a steady flow compressor at 1 atm and 68°F. It leaves at 20 atm and 800°F. If the process is adiabatic, find the input power, specific volume at exit, and change in entropy. Is the process reversible? (Assume a calorically perfect gas.)

2-21. It is given that 50 kg/sec of air enters a steady flow compressor at 1 atm and 20°C. It leaves at 20 atm and 427°C. If the process is adiabatic, find the input power, specific volume at exit, and change in entropy. Is the process reversible? (Assume a calorically perfect gas.)

2-22. It is given that 200 lb/sec of air enters a steady flow turbine at 20 atm and 3400°R. It leaves at 10 atm. For a turbine efficiency of 85 percent, determine the exit temperature, output power, and change in entropy. (Assume a calorically perfect gas.)

2-23. It is given that 80 kg/sec of air enters a steady flow turbine at 30 atm and 2000 K. It leaves at 15 atm. For a turbine efficiency of 85 percent, determine the exit temperature, output power, and change in entropy. (Assume a calorically perfect gas.)

2-24. Air at 140°F, 300 psia, and 300 ft/sec enters a long insulated pipe of uniform diameter. At the exit, the pressure has dropped to 102.9 psia. Assuming a calorically perfect gas, determine:
a. The temperature and velocity at the exit [*Hint:* Since both the mass flow rate and the area are constant for this problem, ρV is constant. Also, from the first law of thermodynamics, $c_p T + V^2/(2g_c)$ is constant. Using the first law of thermodynamics, substitute for the exit velocity and solve for the exit temperature.]
b. The change in entropy

2-25. Air at 300 K, 20 atm, and 70 m/sec enters a long insulated pipe of uniform diameter. At the exit, the pressure has dropped to 6.46 atm. Assuming a calorically perfect gas, determine:
a. The temperature and velocity at the exit [*Hint:* Since both the mass flow rate and the area are constant for this problem, ρV is constant. Also, from the first law of thermodynamics, $c_p T + V^2/(2g_c)$ is constant. Using the first law of thermodynamics, substititue for the exit velocity and solve for the exit temperature.]
b. The change in entropy

2-26. Rework Prob. 2-13 for variable specific heats, using App. D or the program AFPROP. Compare your results to Prob. 2-13.

2-27. Rework Prob. 2-15 for variable specific heats, using App. D or the program AFPROP. Compare your results to Prob. 2-15.

2-28. Rework Prob. 2-16 for variable specific heats, using App. D or the program AFPROP. Compare your results to Prob. 2-16.

2-29. Rework Prob. 2-19 for variable specific heats, using App. D or the program AFPROP. Compare your results to Prob. 2-19.

2-30. Rework Prob. 2-20 for variable specific heats, using App. D or the program AFPROP. Compare your results to Prob. 2-20.

2-31. Rework Prob. 2-23 for variable specific heats, using App. D or the program AFPROP. Compare your results to Prob. 2-23.